First Year Higher Secondary Model Examination, June 2022

Answer Key

Answer any 6 questions. Each carries 3 scores.

1. a) No. Of subsets of a set with n elements = 2^n

$2^2=4$

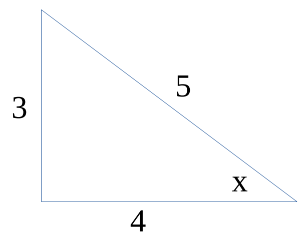
option ii) 4 **(1)**

b) {1,2}, {1}, {2}, ϕ **(1)**

c) $\{x : 6 < x \le 12, x \in R\}$ **(1)**

2. a) 45° **(1)**

b) **(2)**



$\cos x = -\frac{4}{5}$ and $\tan x = -\frac{3}{4}$

3. a) $a_1 = 5x1+1=6$ **(1)**

$a_2 = 5x2+1=11$

$a_3 = 5x3+1=16$

$a_4 = 5x4+1=21$

b) Here a = 6 and d = 11-6 = 5 **(2)**

$S_n = \frac{n}{2}(2a+(n-1)d)$

ie, $S_n = \frac{n}{2}(12+(n-1)5) = \frac{n}{2}(5n+7) = \frac{5n^2+7n}{2}$

4. a) $m = \frac{y_2-y_1}{x_2-x_1} = \frac{5-1}{4-2} = 2$ **(1)**

b) Three points A,B and C are collinear , then Slope of AB = slope of BC **(2)**

$\frac{1--1}{2-x} = \frac{5-1}{4-2}$

$\frac{2}{2-x} = 2$

$1 = 2-x$

x=1

5. Equation of a circle with centre (h,k) and radius r is $(x-h)^2+(y-k)^2=r^2$ **(3)**

Let (h,0) be the centre, since Centre lies on x-axis. And its radius 5.

Equation become, $(x-h)^2+y^2=5^2$

Also the circle passing through the point (2,3)

Then , $(2-h)^2+3^2=5^2$

$(2-h)^2+9=25$

$(2-h)^2=16$

$2-h=\pm 4$

If 2-h=4, then h= -2.

If 2-h= -4, then h = 6.

Therefore the equations of circles are,

$(x+2)^2+y^2=5^2$ and $(x-6)^2+y^2=5^2$

6. a) 8 octants **(1)**

b) Distance formula,

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ **(2)**

$= \sqrt{(1--1)^2+(-3-3)^2+(4--4)^2}$

$= \sqrt{2^2+6^2+8^2}$

$= \sqrt{104}$

7. a) $\lim_{x\to 4} \frac{4x+3}{x-2} = \frac{4x4+3}{4-2} = \frac{19}{2}$ **(1)**

b) $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{\left(\frac{x^3-1}{x-1}\right)}{\left(\frac{x^2-1}{x-1}\right)}$ **(2)**

$= \frac{\lim_{x\to 1}\left(\frac{x^3-1}{x-1}\right)}{\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)} = \frac{3}{2}$

8. a) $\sqrt{7}$ is not irrational **(1)**

b) Converse **(2)**

If a number n^2 is even , then n is even.

Contrapositive

If a number n^2 is not even , then n is not even.

Answer any 6 questions. Each carries 4 scores.

9. a) B **(1)**

b) i) $A' = \{1,3,5,7,9\}$ **(1)**

$B' = \{1,4,6,8,9\}$

ii) $A \cup B = \{2,3,4,5,6,7,8\}$ **(1)**

iii) $(A \cup B)' = \{1,9\}$ **(1)**

$A' \cap B' = \{1,9\}$

Therefore $(A \cup B)' = A' \cap B'$ **(3)**

10. a) $R = \{(1,3),(2,6),(3,9),(4,12)\}$

$Domain = \{1,2,3,4\}$

$Range = \{3,6,9,12\}$

b) $f(0) = 2x0-5 = -5$ **(1)**

11. a) For n=1 **(1)**

LHS = 1

RHS = $\frac{3^1-1}{2} = 1$

LHS=RHS, P(1) is true.

b) Assume that P(k) is true. **(3)**

$P(k) = 1+3+3^2+...+3^{k-1} = \frac{3^k-1}{2}$

Now $P(k+1) = 1+3+3^2+...+3^{k-1}+3^k$

$= \frac{3^k-1}{2}+3^k$

$= \frac{3^k-1+2.3^k}{2}$

$= \frac{3^{k+1}-1}{2}$

P(k+1) is true.

Hence P(n) is true by induction.

12. a) $\frac{7!}{5!}=7x6=42$ **(1)**

b) No.of arrangements = ${}^{n}p_{r}$ **(2)**

= ${}^{9}p_{4}$

= 9 x 8 x 7 x 6 = 3024

c) 1 **(1)**

13. a) 5 terms **(1)**

b) **(3)**

$$(a+b)^n={}^nC_0a^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+...+{}^nC_nb^n$$

$$\left(x^2+\frac{3}{x}\right)^4={}^4C_0(x^2)^4+{}^4C_1(x^2)^3\left(\frac{3}{x}\right)+{}^4C_2(x^2)^2\left(\frac{3}{x}\right)^2$$

$$+{}^4C_3(x^2)\left(\frac{3}{x}\right)^3+{}^4C_4\left(\frac{3}{x}\right)^4$$

$$=1.x^8+4.x^6.\frac{3}{x}+6.x^4.\frac{9}{x^2}+4.x^2.\frac{27}{x^3}+1.\frac{81}{x^4}$$

$$=x^8+12x^5+54x^2+\frac{108}{x}+\frac{81}{x^4}$$

14. Let $\frac{a}{r}$, a , ar be three terms of a GP. **(4)**

Then their product is 1.

So, $\frac{a}{r}.a.ar=1$

ie, $a^3=1$

a =1

Also their sum is $\frac{39}{10}$

i e, $\frac{a}{r}+a+ar=\frac{39}{10}$

$\frac{1}{r}+1+r=\frac{39}{10}$

$10(1+r+r^2)=39r$

$10r^2-29r+10=0$

$r=\frac{29\pm\sqrt{(29)^2-4.10.10}}{2.10}$

$r=\frac{5}{2}$ and $r=\frac{2}{5}$

If $r=\frac{5}{2}$, then three terms are $\frac{2}{5},1,\frac{5}{2}$.

If $r=\frac{2}{5}$, then three terms are $\frac{5}{2},1,\frac{2}{5}$

15. a) y=0 **(1)**

b) i) Equation reduced into slope intercept form,

$y=-\frac{3}{2}x+6$ **(1)**

then slope = $-\frac{3}{2}$

ii) Equation reduced into intercept form **(2)**

$\frac{x}{4}+\frac{y}{6}=1$

x-intercept = 4

y-intercept = 6

16. Here a = 4 and b =3 **(4)**

$c^2=16-9=7$

$c=\sqrt{7}$

Eccentricity , e = $\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{9}{2}$

17. i) P(a diamond) = $\frac{13}{52}=\frac{1}{4}$ **(1)**

ii) P(not an ace) = $\frac{48}{52}=\frac{12}{13}$ **(2)**

iii) P(black card) = $\frac{26}{52}=\frac{1}{2}$ **(1)**

Answer any 3 questions. Each carries 6 scores.

18. a) $\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$ **(3)**

$$\frac{\tan(\frac{\pi}{4}+x)}{\tan(\frac{\pi}{4}-x)}=\frac{\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}$$

$$=\frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}$$

$$=\left(\frac{1+\tan x}{1-\tan x}\right)x\left(\frac{1+\tan x}{1-\tan x}\right)$$

$$=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

b) $\cos x+\cos y=2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$ **(3)**

$$\cos 3x+\cos x-\cos 2x=2\cos(\frac{3x+x}{2})\cos(\frac{3x-x}{2})-\cos 2x$$

$$=2\cos 2x\cos x-\cos 2x$$

$$=\cos 2x(2\cos x-1)$$

$\cos 2x=0$ or $2\cos x-1=0$

$2x=(2n+1)\frac{\pi}{2}$ $\quad$ $2\cos x=1$

$x=(2n+1)\frac{\pi}{4},n\in Z$ $\quad$ $\cos x=\frac{1}{2}$

$x=2n\pi\pm\frac{\pi}{3},n\in Z$

19. a) $i^4=1$ **(1)**

b) $Z^{-1}=\frac{a}{a^2+b^2}+i\frac{-b}{a^2+b^2}$ **(2)**

$$=\frac{1}{1^2+(-1)^2}+i\frac{--1}{1^2+(-1)^2}$$

$$=\frac{1}{2}+i\frac{1}{2}$$

c) $Z=r(\cos\theta+i\sin\theta), r=\sqrt{x^2+y^2}, \tan\theta=\frac{y}{x}$

$r=\sqrt{1^2+1^2}=\sqrt{2}$ **(3)**

$\tan\theta=\frac{-1}{1}$,then $\theta=\frac{-\pi}{4}$

Polar form is $Z=\sqrt{2}(\cos\frac{-\pi}{4}+i\sin\frac{-\pi}{4})$

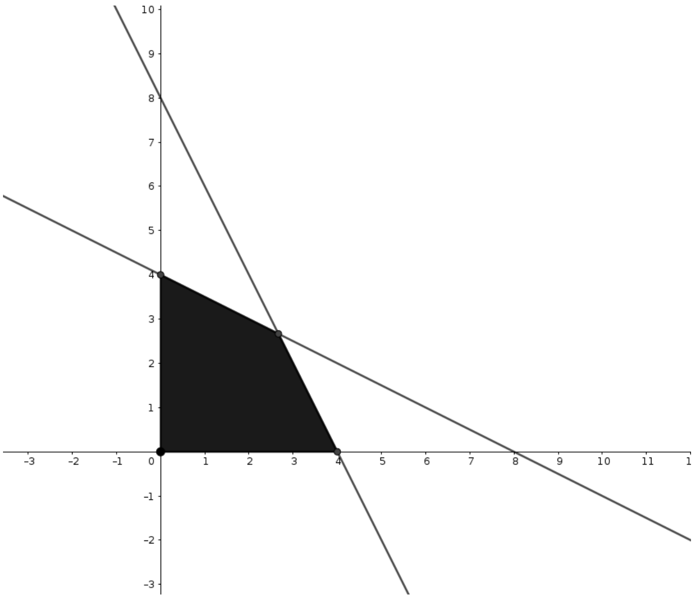
20. x + 2y = 8

x	y
0	4
8	0

(6)

2x + y = 8

x	y
0	8
4	0



21. a) $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ **(3)**

$=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$

$=\lim_{h\to 0}\frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$

$=\lim_{h\to 0}\frac{2\cos\left(\frac{h+2x}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$=\lim_{h\to 0}\cos\left(\frac{h+2x}{2}\right)\lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$

$=\cos\frac{2x}{2}=\cos x$

b) $f'(x)=\frac{d}{dx}(5\sin x)-\frac{d}{dx}(6\cos x)+\frac{d}{dx}(7)$ **(3)**

$=5\frac{d}{dx}(\sin x)-6\frac{d}{dx}(\cos x)+\frac{d}{dx}(7)$

$=5\cos x+6\sin x$

22.

class	f_i	x_i	$x_i f_i$	x_i^2	$x_i^2 f_i$
0 - 10	5	5	25	25	125
10 - 20	8	15	120	225	1800
20 - 30	15	25	375	625	9375
30 - 40	16	35	560	1225	19600
40 - 50	6	45	270	2025	12150
	N = 50		1350		43050

i) Mean, $\bar{X}=\frac{\sum_{i=1}^{n}x_i f_i}{N}=\frac{1350}{50}=27$ **(2)**

ii) Variance, $\sigma^2=\frac{\sum_{i=1}^{n}x_i^2 f_i}{N}-(\bar{X})^2$ **(4)**

$=\frac{43050}{50}-27^2=132$

S.D = $\sqrt{(132)}=11.489$

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