IIT-JEE 2010 Physics Paper II

PART III - Physics

SECTION - I
Single Correct Choice Type

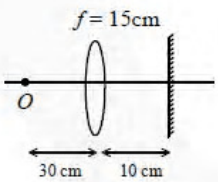
This Section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C), and (D) out of which **ONLY ONE is correct**.

39. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is
(A) 5 grams (B) 10 grams (C) 20 grams (D) 40 grams

39. **(B)** Fundamental frequency of pipe is given by $f_2 = v / 4l_2$ ($v = 320\ ms^{-1}$)
Second harmonic of string is given by $f_1 = \sqrt{(T/\mu)} / l_1$
Both are same $f_1 = f_2$ $\Rightarrow$ $m = Tl_1 / 2500 = 10$ grm

40. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is
(A) Virtual and at a distance of 16 cm from the mirror
(B) Real and at a distance of 16 cm from the mirror
(C) Virtual and at a distance of 20 cm from the mirror
(D) Real and at a distance of 20 cm from the mirror

40. **(B)**



Using lens formula $\{(1/v) + (1/u)\} = 1/f$
$\{(1/v) + (1/30) = 1/15$ $\Rightarrow$ $v = 30$ cm
So, object for plane mirror is 20 cm from it towards right.
$\Rightarrow$ Image formed = 20 cm left from plane mirror.
This behaves as virtual object for lens at a distance of 10 cm left.
using lens formula,
$(1/v') + (1/u) = 1/f$
$\Rightarrow$ $(1/v') + (1/-10) = 1/15$ $\Rightarrow$ $v' = 6$ cm
So, image formed is real & at a distance 16 cm from mirror.

41. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is:
(A) 0.02 mm (B) 0.05 mm (C) 0.1 mm (D) 0.2 mm

41. **(D)** 20 VSD = 16 MSD
$\therefore$ 1 VSD = (16 / 20) MSD = (4 / 5) MSD
LC = 1 VSD – 1 MSD = (1 / 5) = 0.2 mm

42. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\{(81\pi / 7) \times 10^5\}\ Vm^{-1}$. When the field is switch off, the drop is observed to fall with terminal velocity $2 \times 10^{-3}\ ms^{-1}$. Given $g = 9.8\ ms^{-2}$, viscosity of the air $= 1.8 \times 10^{-5}\ Ns\ m^{-2}$ and the density of oil $= 900\ kg\ m^{-3}$, the magnitude of q is

(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C (C) 4.8×10^{-19} C (D) 8.0×10^{-19} C

42. **(D)** $v_T = (2/9\eta)\, r^2 (\rho_s - \rho_l) g$

$2 \times 10^{-3} = \{2/(9 \times 1.8 \times 10^{-5})\, r^2 (900) \times 9.8$ (since $\rho_l = \rho_{air} \approx 0$)

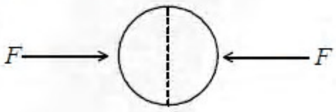
$\Rightarrow r \approx 4.28 \times 10^{-6}$ m & $qE + (4/3)\pi r^3 \times \rho_{air} \times g = (4/3)\pi r^3 \times \rho_{oil}\, g$

$\Rightarrow qE = (4/3)\pi r^3 g (\rho_{oil})$

$\Rightarrow q = [\{(4/3) \times \pi \times (4.28 \times 10^{-6})^3 \times 9.8 \times 900 \times 7\}/(81\pi \times 10^5)]$

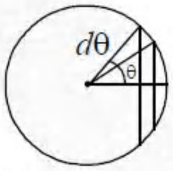
$= 7.968 \times 10^{-19}$ C $\approx 8 \times 10^{-19}$ C

43. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to



(A) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2 R$ (C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

43. **(A)**

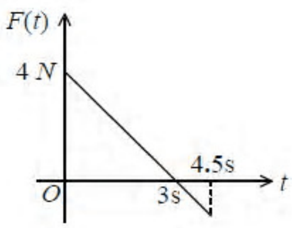


$E_{\text{on surface}} = \{K(4\pi R^2\sigma)/2R^2\}$

$$F = \int_0^{\pi/2} (\sigma 2\pi R \sin\theta\, R d\theta)\ \frac{k 4\pi R^2 \sigma}{2R^2}.\cos\theta$$

$$= \sigma^2 R^2 \frac{\pi}{\varepsilon_0} \int_0^{\pi/2} \sin\theta \cos\theta\, d\theta = \frac{\sigma^2 R^2 \pi}{2\varepsilon_0}$$ or dimensionally only option (A) is correct.

44. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent froce $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



(A) 4.50 J (B) 7.50 J (C) 5.06 J (D) 14.06 J

44. **(C)** From graph $F = \{(-4/3)t + 4\}$ $\Rightarrow$ $a = \{(-2t/3) + 2\}$

V at 4.5 s = $\int_0^{4.5}\left(\frac{-2t}{3}+2\right)dt$ = 2.25 ms^{-1}

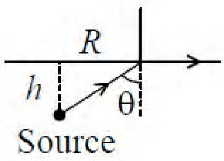
K. E. = (1 / 2) mv^2 = 5.06 J

SECTION - II (Integer Type)

This Section contains **5 questions.** The answer to each question is a **single-digit integer,** ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

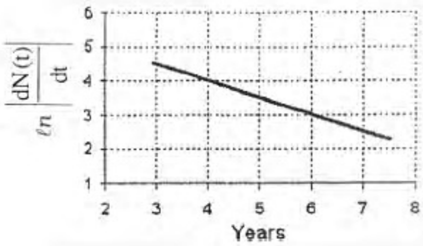
45. A large glass slab (μ = 5 / 3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

45. **[6]**



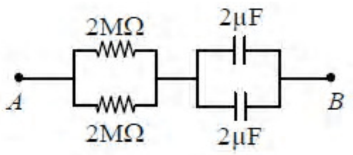
From Snell's law : (5 / 3) . sin i = 1 × sin 90°
⇒ (5 / 3) × {R / √($R^2 + h^2$) = 1
⇒ R = 6 cm

46. To determine the half life of a radioactive element, a student plots a graph of $ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is.

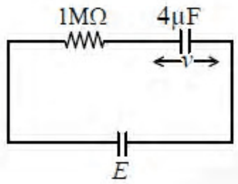


46. **[8]** ln A = – λt + C
⇒ ln [($dN(t)$ / dt] = – λt + C
∴ λ = {(4 – 3) / (6 – 4)} = 1 / 2 (from graph)
We have, $t_{1/2}$ = ln2 / λ = 1.396
⇒ $N = N_0 / 2^3 = N_0 / 8$
∴ P = 8

47. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V? [Take : ln 5 = 1.6, ln 3 = 1.1]



47. **[2]**



$E = 10$ v, $V = 4$ v during changing of capacitor

$Q = CE\,(1 - e^{-t/CR})$

$\therefore$ Potential difference across capacitor $V = Q / C = E\,(1 - e^{-t/CR})$

$\Rightarrow$ $V / E = (1 - e^{-t/CR})$

$\therefore$ $t = CR\,[\,ln5 - ln3] = 2$

48. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from (25 / 3) m to (50 / 7) m in 30 seconds. What is the speed of the object in km per hour?

48. **[3]** From mirror formula : $(1 / 10) = \{(1 / u_1) + (3 / 25)\}$

$\Rightarrow u_1 = -50$m (using cartesian system)

and $(1 / 10) = \{(1 / u_2) + (7 / 50)\}$ $\Rightarrow u_2 = -25$m

$\therefore$ Speed of object $= \{(50 - 25) / 30\} = 25 / 30\ \text{ms}^{-1} = 3$ kmh

49. A diatomic ideal gas is compressed adiabatically to (1 / 32) of its initial volume. In the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is

49. **[4]** Far adiabatic process $TV^{\gamma-1}$ = constant

$\Rightarrow$ $T_i\,(V_i)^{2/5} = a\,.\,T_i\,(V_i / 32)^{2/5}$

$\Rightarrow$ $a = (32)^{2/5} = 4$

SECTION - III

Link Comprehension Type

This Section contains **2 paragraphs**. Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these question has fours choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for question 50 to 52

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

50. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is

(A) $2\pi rT$ (B) $2\pi RT$ (C) $\frac{2\pi r^2 T}{R}$ (D) $\frac{2\pi R^2 T}{r}$

50. **(C)** $F_{vertical} = 2\pi rT \text{ Sin } \theta = 2\pi rT \cdot \frac{r}{R} = \frac{2\pi r^2 T}{R}$

51. If $r = 5 \times 10^{-4}$ m, $r = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m (C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

51. **(A)** $\frac{2\pi r^2 T}{R} = \frac{4}{3}\pi R^3 \rho g$ $\therefore R^4 = \frac{3}{2}\frac{r^2 T}{\rho g} = 4.125 \times 10^{-12} \Rightarrow R \approx 1.4 \times 10^{-3}$ m

52. After the drop detaches, its surface energy is

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J (C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

52. **(B)** Surface energy $= 4\pi R^2 T = 2.7 \times 10^{-6}$ J

Paragraph for question 53 to 55

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

53. A diatomic molecule has moment of inertia i. By Bohr's quantization condition its rotational energy in the n^{th} level ($n = 0$ is not allowed) is

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$ (C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

53. **(D)** $I\omega = \frac{nh}{2\pi}$ $\therefore$ Rotational energy, $E_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{nh}{2\pi}\left(\frac{nh}{2\pi I}\right) = \frac{n^2 h^2}{8\pi^2 I}$

54. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is closes to $\frac{4}{\pi} \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to (Take $h = 2\pi \times 10^{-34}$ J s)

(A) 2.76×10^{-46} kg m^2 (B) 1.87×10^{-46} kg m^2
(C) 4.67×10^{-47} kg m^2 (D) 1.17×10^{-47} kg m^2

54. **(B)** $\frac{h^2}{8\pi^2 I}(4-1) = hf$ $\therefore I = \frac{3h}{8\pi^2 f} = \frac{3}{16} \times 10^{-45} = 1.87 \times 10^{-46}$ kg m^2

55. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. $= \frac{5}{3} \times 10^{-27}$ kg, is close to

(A) 2.4×10^{-10} m (B) 1.9×10^{-10}m (C) 1.3×10^{-10} m (D) 4.4×10^{-11} m

55. (C) COM w.r.t. C atom $= \left(\frac{16}{16+12}\right) \cdot d$ where, d is the separation

$$\therefore I_{COM} = (16\text{ a.m.u}) \times \left(\frac{12d}{28}\right)^2 + (12\text{ a.m.u}) \times \left(\frac{16d}{28}\right)^2$$

On solving, $d = 0.128 \times 10^{-9}$ m $= 1.3 \times 10^{-10}$ m

SECTION - IV

Matrix Type

This Section contains **2 questions.** Each question has four statements (A, B, C and D) given in **Column I** and five statements (P, Q, R, S and T) in **Column I I**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column I I**. For example, if for a given question, statement *B* matching with the statements given in *q* and *r*, then for that particular question, against statement *B*, darken the bubbles corresponding to *q* and *r* in the *ORS*.

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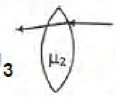
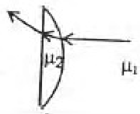
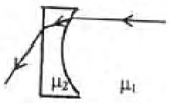
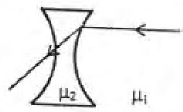
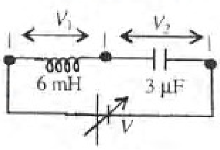
56. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as show in figures in **Column I I**. A ray traversing these media is also shown in the figures. In **Column I** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in **Column I I**.

Column I	Column I I
(A) $\mu_1 < \mu_2$	(P) 
(B) $\mu_1 > \mu_2$	(Q) 
(C) $\mu_2 = \mu_3$	(R) 
(D) $\mu_2 > \mu_3$	(S) 
	(T) 

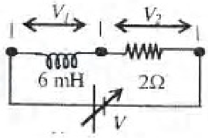
56. (A) → (P,R); (B) → (Q,S,T); (C) → (P,R,T); (D) → (Q,S)

If ray bends towards normal after refraction, then $\mu_{2nd\ medium} > \mu_{1st\ medium}$ & vice –versa.

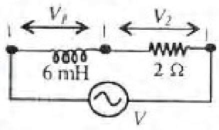
57. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column I I**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in **Column I**. Match the two

Column I	Column I I
(A) $I \neq 0$, V_1 is proportional to I	(P) 

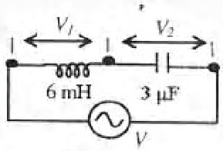
(B) $I \neq 0$, $V_2 > V_1$

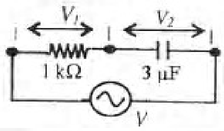
(Q) 

(C) $V_1 = 0$, $V_2 = V$

(R) 

(D) $I \neq 0$, V_2 is proportional to I

(S) 

(T) 

57. (A) → (R,S,T); (B) → (Q, R,S,T); (C) → (P,Q); (D) → (Q,R,S,T)
- (P) No steady state. Actually, if steady state assumed, then $V_1 = 0$, $V_2 = V$.
- (Q) At steady state, $V_1 = 0$, $V_2 = V$ & $I = V/2$ $\Rightarrow V_2 = V = 2I$
- (R) $X_L = (6 \times 10^{-3}) \times (2\pi \times 50) = 0.6\,\pi = 1.88\,\Omega$
  $\therefore I = V/\sqrt{(2^2 + X_L^2)}$ & $V_1 = IX_L$, $V_2 = IX_C$
- (S) $X_L = 1.88\,\Omega$, $X_C = 1/((2\pi \times 50)(3 \times 10^{-6})) = 1061\,\Omega$
  $\therefore I = V/|X_L - X_C|$ & $V_1 = I X_L$, $V_2 = IX_C$
- (T) $X_C = 1061\,\Omega$, $I = V/\sqrt{(1000^2 + X_C^2)}$ & $V_1 = 1000\,I$, $V_2 = IX_C = 1061\,I$.