HIGHER SECONDARY MATHEMATICS TEACHERS ASSOCIATION

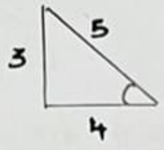
Mathematics Test Series-01		
Sets Relations & Functions Trigonometric Functions Principles of Mathematical Induction		Class : XI Time : 2 Hr. Max. Score : 60 Cool off time : 15min

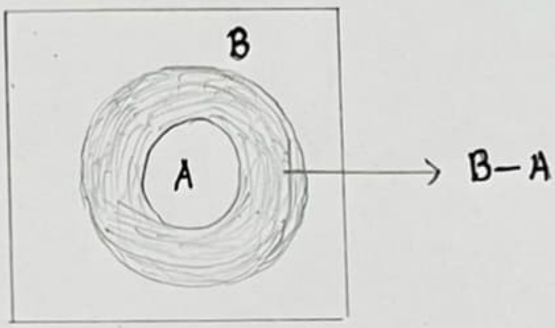
ANSWER KEY

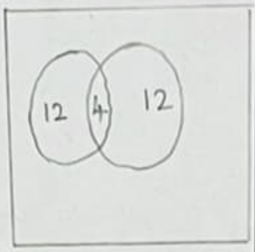
UNIT - I (3 Marks)

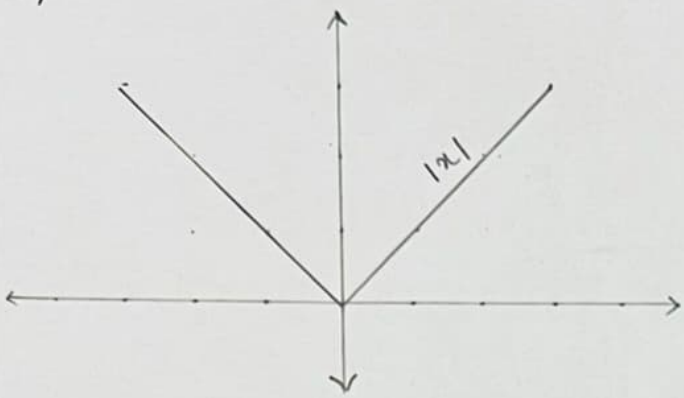
Answer any 6 Questions from 1–8

1.	a) $A = \{M, A, T, H, E, I, C, S\}$	½	3
	$B = \{S, T, A, I, C\}$	½	
	b) $A - B = \{M, H, E\}$	1	
	c) $A \cap B = \{A, T, I, C, S\}$	1	
2.	a) $n(P(A)) = 2^n = 2^3 = 8$	1	3
	b) $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\} \{2,3\}, \{1,3\}$ $\{1,2,3\}\}$	2	
3.	$f(x) = \dfrac{x^2+3x+5}{x^2-5x+4} = \dfrac{x^2+3x+5}{(x-1)(x-4)}$		
	$f(x)$ is defined when $(x-1)(x-4) \neq 0$	1	3
	ie when $x \neq 1$ & $x \neq 4$	1	
	$\therefore$ Domain $= R - \{1, 4\}$	1	
4.	$f(x) = x^2 + 5x \qquad g(x) = 2x+1$		
	$f+g = f(x)+g(x) = x^2+5x+2x+1 = x^2+7x+1$	1	
	$f \cdot g = f(x) \cdot g(x) = (x^2+5x)(2x+1)$ $= 2x^3 + x^2 + 10x^2 + 5x = 2x^3 + 11x^2 + 5x$	1	3
	$f/g = \dfrac{f(x)}{g(x)} = \dfrac{x^2+5x}{2x+1}$	1	

5.	a) (ii) $-\sin x$	1	
	b) $\tan\left(\frac{\pi}{4}+x\right) = \dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}$	½	
	$\tan\left(\frac{\pi}{4}-x\right) = \dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}$	½	3
	$\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \dfrac{1+\tan x / 1-\tan x}{1-\tan x / 1+\tan x}$	½	
	$= \dfrac{1+\tan x}{1-\tan x} \times \dfrac{1+\tan x}{1-\tan x}$	½	
	$= \left[\dfrac{1+\tan x}{1-\tan x}\right]^2$		
6.	a) $\frac{2\pi}{3}$ radian $= \frac{2\pi}{3} \times \frac{180}{\pi} = 120^\circ$	1	
	b) $\sin x = \frac{3}{5}$, II quadrant		3
	$\cos x = \frac{-4}{5}$ 	1	
	$\tan x = \dfrac{3/5}{4/5} = -3/4$	1	
7.	a) $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ or $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$	2	
	$\therefore$ Principal solutions are $\frac{\pi}{6}$ & $\frac{5\pi}{6}$		3
	b) General solution of $\sin x = ½$ is	1	
	$x = n\pi + (-1)^n \pi/6$ where $n \in Z$		

8.	Let $P(n): 7^n - 3^n$ divisible by 4		
	a) Consider $P(1) = 7^1 - 3^1 = 4$ is divisible by 4	1	
	$\therefore$ The given statement is true for $n=1$		
	b) Assume $P(k)$ be true for some natural no: k		
	ie $P(k): 7^k - 3^k$ is divisible by 4	½	3
	$P(k+1): 7^{k+1} - 3^{k+1}$	½	
	$= 7^k.7 - 3^k.3$	½	
	$= 7^k.7 - 7.3^k + 7.3^k - 3^k.3$		
	$= 7(7^k - 3^k) + (7-3)3^k$	½	
	$= 7(7^k - 3^k) + (4)3^k$, is divisible by 4		
	$\therefore P(k+1)$ is true. Thus $P(k+1)$ is true whenever $P(k)$ is true.		
	Hence by PMI $P(n)$ is true for all $n \in N$		
	UNIT II (4 Marks)		
	Answer any 6 Questions from 9–16		
9.	a) $A \cup B = B$	1	
	b) $A \cap B = A$	1	
	c)		4
		2	

10.	a) M : Teachers who teach Mathematics		
	P : Teachers who teach Physics		
	$n(M \cup P) = 20 \quad n(M) = 12 \quad n(P) = 12$		
	$n(M \cup P) = n(M) + n(P) - n(M \cap P)$	1	
	$\therefore n(M \cap P) = n(M) + n(P) - n(M \cup P)$	½	4
	$= 12 + 12 - 20 = 24 - 20$	½	
	$\therefore n(M \cap P) = 4$		
			
	b) $n(P\ \text{only}) = 12 - 4 = 8$	2	
11.	a) $A = \{2, 3\}$	1	
	$B = \{-3, 3\}$	1	
	b) $A - B = \{2\}$	½	
	$B - A = \{-3\}$	½	
	$(A - B) \cup (B - A) = \{2, -3\}$		4
	$A \cup B = \{-3, 2, 3\}$	½	
	$A \cap B = \{3\}$	½	
	$(A \cup B) - (A \cap B) = \{-3, 2\}$		
	$\therefore (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$		
12.	a) (ii) 64	1	
	b) i) $R = \{(1,3), (2,6), (3,9), (4,12)\}$	1	
	ii) Domain $= \{1, 2, 3, 4\}$	1	4
	Range $= \{3, 6, 9, 12\}$	1	

13.	a) 	2	4
	b) Domain = R Range = $[0, \infty)$	1+1	
14	a) $\cos 15^\circ = \cos(45^\circ - 30^\circ)$ $= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$ $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$		
	b) $\cos(x+y) + \cos(x-y) = 2\cos x \cos y$ $\cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right) = 2\cos\frac{\pi}{4}\cos x$ $= 2 \times \frac{1}{\sqrt{2}} \cos x$ $= \sqrt{2}\cos x$		
15.	a) $\sin \frac{31\pi}{3} = \sin 1860^\circ = \sin(5 \times 360^\circ + 60^\circ)$ $= \sin 60^\circ = \sqrt{3}/2$	1	
	or $\sin \frac{31\pi}{3} = \sin\left(\frac{30\pi + \pi}{3}\right) = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3}$ $= \sqrt{3}/2$		
	b) We have, $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$	1	4
	LHS $\Rightarrow \left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]^2 +$ $\left[2\cos\left(\frac{x-y}{2}\right)\sin\left(\frac{x+y}{2}\right)\right]^2$	½	

$$= \left[4\cos^2\left(\frac{x+y}{2}\right)\cos^2\left(\frac{x-y}{2}\right)\right] + \left[4\cos^2\left(\frac{x-y}{2}\right)\sin^2\left(\frac{x+y}{2}\right)\right]$$ ½

½

$$= 4\cos^2\left(\frac{x-y}{2}\right) \times 1$$ $\boxed{\sin^2 x + \cos^2 x = 1}$ ½

$$= 4\cos^2\left(\frac{x-y}{2}\right)$$

$= RHS$

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16. $P(n) : 1 + 3 + 3^2 + \cdots + 3^{n-1} = 3^n - 1$

a) $P(1) : \quad 1 = \frac{3^1 - 1}{2}$ 1

$1 = 1$, $P(1)$ is true

b) Assume that $P(k)$ is true for some k

$P(k) : 1 + 3 + 3^2 + \cdots + 3^{k-1} = \frac{3^k - 1}{2}$ 1

$P(k+1) : 1 + 3 + 3^2 + \cdots + 3^{k-1} + 3^k = \frac{3^k - 1}{2} + 3^k$ ½ 4

$$= \frac{3^k - 1 + 2 \cdot 3^k}{2}$$ ½

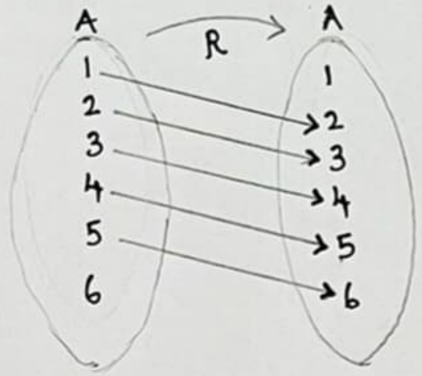
$$= \frac{3 \cdot 3^k - 1}{2}$$ 1

$$= \frac{3^{k+1} - 1}{2}$$

$\therefore P(k+1)$ is true

Thus $P(k+1)$ is true whenever $P(k)$ is true.

Hence by PMI, $P(n)$ is true for all natural number 'n'.

	UNIT III (6 Marks)		
	Answer any 3 questions from 17 – 21		
17.	$A \cap B = \{2\}$		
	$(A \cap B)' = U - (A \cap B) = \{1, 3, 4, 5, 6, 7, 8, 9\}$	1	
	$A' = U - A = \{1, 3, 5, 7, 9\}$	½	
	$B' = U - B = \{1, 4, 6, 8, 9\}$	½	6
	$A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$	1	
	$(A \cap B)' = A' \cup B'$, Hence verified		
	$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$	1	
	$(A \cup B)' = U - A \cup B = \{1, 9\}$	1	
	$A' \cap B' = \{1, 9\}$	1	
	$(A \cup B)' = A' \cap B'$, Hence verified		
18.	$R = \{(x, y) : y = x + 1\}$		
	$R = \{(1,2)\ (2,3)\ (3,4)\ (4,5)\ (5,6)\}$		
	a) 	2	6
	b) Domain $= \{1, 2, 3, 4, 5\}$	1	
	Co Domain $= \{1, 2, 3, 4, 5, 6\}$	1	
	Range $= \{2, 3, 4, 5, 6\}$	1	
	c) NO, 'R' is not a function since the element '6' has no image	1	

19.	a) We have $3x = x+2x$		
	$\therefore \tan 3x = \tan(x+2x)$		
	i.e. $\tan 3x = \dfrac{\tan x + \tan 2x}{1-\tan x \tan 2x}$	1	
	i.e, $\tan 3x(1-\tan x \tan 2x) = \tan x + \tan 2x$	½	
	i.e, $\tan 3x - \tan x \tan 2x \tan 3x = \tan x + \tan 2x$	½	
	$\therefore \tan x \tan 2x \tan 3x = \tan 3x - \tan 2x - \tan x$		
	b) $\sin 765^\circ = \sin(2\times 360^\circ + 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$	1	
	c) $\dfrac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = \dfrac{2\cos\left[\frac{7x+5x}{2}\right]\cos\left[\frac{7x-5x}{2}\right]}{2\cos\left[\frac{7x+5x}{2}\right]\sin\left[\frac{7x-5x}{2}\right]}$	1 1	
	$= \dfrac{\cos 6x \cos x}{\cos 6x \sin x} = \cot x$	1	
20.	a) $\sin 2x + \cos x = 0$		
	$\Rightarrow 2\sin x \cos x + \cos x = 0$	1	
	$\Rightarrow \cos x(2\sin x + 1) = 0$		
	$\Rightarrow \cos x = 0$ or $2\sin x + 1 = 0$	½	
	$\cos x = 0$ or $\sin x = -\frac{1}{2} = \sin(\pi + \pi/6) = \sin 7\pi/6$	½	
	$\Rightarrow x = (2n+1)\pi/2$ or $x = n\pi + (-1)^n \frac{7\pi}{6}, n \in \mathbb{Z}$	1	
	b) $\dfrac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x} =$		
	$= \dfrac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$	1	

	$= \dfrac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$	1	
	$= \dfrac{\cos 3x\,\cancel{(2\cos x+1)}}{\sin 3x\,\cancel{(2\cos x+1)}}$	1	
	$= \cot 3x \quad = RHS$		
21.	a) $x+1=3 \Rightarrow x=2$	½	
	$y-2=1 \Rightarrow y=3$	½	
	b) $A \times A = \{(-1,-1),(-1,1),(1,-1),(1,1)\}$	1	
	c) i) $R = \{(x,y): x$ is the square of $y, x \in P\ y \in Q\}$	1	6
	ii) $R = \{(9,3),(9,-3),(4,2),(4,-2),(25,5),(25,-5)\}$	1	
	iii) Domain $= \{4,9,25\}$	1	
	Range $= \{-2,2,3,-3,5,-5\}$	1	