MATHEMATICS TEACHERS ASSOCIATION MALAPPURAM.

Mathematics Test Series- II - May 2022.

[Complex numbers and Quadratic Equations, Linear Inequalities, Permutations and combinations & Binomial Theorem]

Answer Key.

Unit-I

No.	Answer	Mark		Total
1.	a) $i^3 = -i$	1	1	
	b) $i^9 + i^{19} = i^{8+1} + i^{16+3}$	½		3
	$= 1\times i + 1\times i^3$	½	2	
	$= i + -i$	½		
	$= 0 + 0i$	½		
2.	a) ii) $3-4i$.	1	1	
	b) Multiplicative inverse of z is $z^{-1} = \frac{\bar{z}}{\lvert z\rvert^2}$.	½		
	$z = 3+4i$, $\lvert z\rvert^2 = 3^2+4^2 = 25$, $\bar{z} = 3-4i$.	½+½	2	3.
	$\therefore z^{-1} = \frac{3-4i}{25} = \frac{3}{25} - \frac{4}{25}i$	½		
	or $z^{-1} = \frac{1}{z} = \frac{1}{3+4i} = \frac{3+4i}{(3+4i)(3-4i)}$ $\frac{3-4i}{3^2+4^2} = \frac{3-4i}{25}$	½+½ +½+½		
3.	a) iv) $3x-1 \geq 5$, $x \in \mathbb{R}$	1	1	
	b) Let Ravi obtain x marks in the third test. Then			
	average $= \frac{70+75+x}{3}$	½		3.
	Given that average is atleast 60.			
	$\Rightarrow \frac{70+75+x}{3} \geq 60$	½	2	
	$\Rightarrow 145+x \geq 180$	½		
	$\Rightarrow x \geq 180-145 = 35$	½		
4.	Let x, $x+2$ be the consecutive even positive integers.	½		
	Given that $x>5$ and $x+x+2<23$.	1		
	$\Rightarrow x>5$ and $2x+2<23$			
	$\Rightarrow x>5$ and $2x<21$		3	3
	$\Rightarrow x>5$ and $x<10.5$	½		
	$\Rightarrow 5<x<10.5$.			
	$\Rightarrow$ the required integers are			
	6, 8 or, 8, 10. or 10, 12.	1		

5.	a) iii) 64	1	1	3
	b) $n=6,\ r=3.$	$\frac{1}{2}$		
	$\therefore$ number of 3 digit numbers $= 6P_3$	1	2	
	$= 6\times5\times4 = 120$	$\frac{1}{2}$		
6.	a) 114	1	1	3
	b) $nC_8 = nC_9 \Rightarrow n = 8+9 = 17$	1	2	
	$\therefore nC_{17} = 17C_{17} = 1$	1		
7.	$(x+\frac{1}{x})^6$			
	$= x^6 + 6C_1 x^5 (\frac{1}{x}) + 6C_2 x^4 (\frac{1}{x})^2 + 6C_3 x^3 (\frac{1}{x})^3 + 6C_4 x^2 (\frac{1}{x})^4 + 6C_5 x (\frac{1}{x})^5 + 6C_6 (\frac{1}{x})^6$	1		
	$= x^6 + 6.x^4 + 15.x^4 \times \frac{1}{x^2} + 20x^3 \cdot \frac{1}{x^3} + 15x^2 \times \frac{1}{x^4} + 6x \times \frac{1}{x^5} + 1 \times \frac{1}{x^6}.$	1	3	3
	$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$	1		
8.	$a = x,\ b = \frac{2}{x^2},\ n = 6$	$\frac{1}{2}$		
	$T_{r+1} = nC_r\, a^{n-r} b^r.$	$\frac{1}{2}$		
	$= 6C_r\, x^{n-r} (\frac{2}{x^2})^r.$			
	$= 6C_r\, x^{6-3r}\, 2^r$	$\frac{1}{2}$		
	The middle term is the fourth term.	$\frac{1}{2}$	3	3
	$\therefore T_4 = 6C_3\, x^{6-3\times3} \cdot 2^3$	$\frac{1}{2}$		
	$= \frac{6\times5\times4}{1\times2\times3} \times x^{-3} \cdot 2^3$			
	$= \frac{20\times8}{x^3} = \frac{160}{x^3}$	$\frac{1}{2}$		
9.	**Unit II**			
	a) $z = \frac{1+3i}{1-2i}$			
	$= \frac{(1+3i)(1+2i)}{(1-2i)(1+2i)}$	1		
	$= \frac{1+2i+3i+6i^2}{1^2-(2i)^2}$	$\frac{1}{2}$	2	
	$= \frac{1+5i-6}{1+4} = \frac{-5+5i}{5} = -1+i$	$\frac{1}{2}$		

b) Let $-1+i = r(\cos\theta + i\sin\theta)$ — ½

$r = \sqrt{(-1)^2+1^2} = \sqrt{2}$. — ½

$r\cos\theta = -1, \quad r\sin\theta = 1$ — ½ — 2 — 4

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}, \quad \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}$. — ½

$\therefore -1+i = \sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$

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10. a) $x+iy = \frac{a+ib}{a-ib} = \frac{(a+ib)(a+ib)}{(a-ib)(a+ib)} = \frac{(a+ib)^2}{a^2+b^2}$. — ½

$x-iy = \frac{(a-ib)^2}{a^2+b^2}$ — ½ — 2

$x^2+y^2 = (x+iy)(x-iy)$ — ½

$= \frac{(a+ib)^2}{a^2+b^2} \times \frac{(a-ib)^2}{a^2+b^2} = \frac{[(a+ib)(a-ib)]^2}{(a^2+b^2)^2}$ — ½

$= \frac{(a^2+b^2)^2}{(a^2+b^2)^2} = \underline{\underline{1}}$. — 4

b) $\sqrt{5}x^2 + x + \sqrt{5} = 0$

$a = \sqrt{5}, \; b = 1, \; c = \sqrt{5}$ — ½

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ — ½ — 2

$= \frac{-1 \pm \sqrt{1-4\times\sqrt{5}\times\sqrt{5}}}{2\sqrt{5}} = \frac{-1\pm\sqrt{1-20}}{2\sqrt{5}} = \frac{-1\pm\sqrt{-19}}{2\sqrt{5}}$ — ½

$= \frac{-1 \pm i\sqrt{19}}{2\sqrt{5}}$. — ½

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11. a) $3(3x-4) \geq 2(x+1)$ — ½

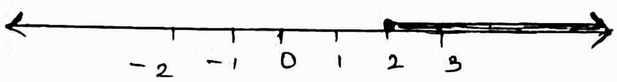
$\Rightarrow 9x - 12 \geq 2x+2$ — 1

$\Rightarrow 9x - 2x \geq 2+12$ — ½ — 3

$\Rightarrow 7x \geq 14$ — ½

$\Rightarrow x \geq \frac{14}{7} = 2 \Rightarrow x \geq 2$. — ½ — 4

b)



— 1 — 1

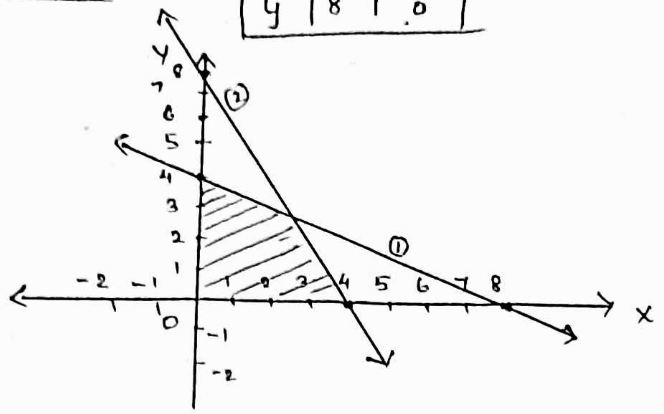
12. $x + 2y = 8$ ①

x	0	8
y	4	0

$2x + y = 8$ ②

x	0	4
y	8	0

1 4 4



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13. a) There are nine letters with no repetition.

1) The number of arrangements if all letters are used at a time

$= {}^9P_9 = 9!$ 1 2

2) The number of arrangements which starts with C and end in Y

$= {}^7P_7 = 7!$ 1 4

b) Girls can be selected in ${}^4C_3$ ways. ½

Boys can be selected in ${}^7C_4$ ways. ½ 2

Required number of ways $= {}^4C_3 \times {}^7C_4$ ½

$= 4 \times \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = 140.$ ½

14. a) number of chords $= {}^{21}C_2$ ½ 1

$= \frac{21 \times 20}{1 \times 2} = 210$ ½

b) Given that $\frac{{}^{2n}C_3}{{}^nC_3} = \frac{12}{1}$ ½

$\Rightarrow {}^{2n}C_3 = 12 \times {}^nC_3$

$\frac{2n(2n-1)(2n-2)}{1 \times 2 \times 3} = 12 \cdot \frac{n(n-1)(n-2)}{1 \times 2 \times 3}$ ½ 4

$\Rightarrow 2 \times (2n-1) \times 2 \times (n-1) = 12(n-1)(n-2)$ ½

$\Rightarrow 4(2n-1) = 12(n-2)$ ½ 3

$\Rightarrow 8n - 4 = 12n - 24$ ½

$\Rightarrow 24 - 4 = 12n - 8n$

$\Rightarrow 4n = 20 \Rightarrow n = \frac{20}{4} = 5$ ½

15.	a) $(a+b)^4 = a^4 + 4C_1 a^3 b + 4C_2 a^2 b^2 + 4C_3 ab^3 + 4C_4 b^4$	1/2		
	$\therefore (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	1/2	2	
	$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$	1/2		
	$\therefore (a+b)^4 - (a-b)^4 = 8a^3b + 8ab^3$	1/2		4
	$= 8ab(a^2+b^2)$			
	b) $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$			
	$= 8\sqrt{3}\sqrt{2}\,[(\sqrt{3})^2 + (\sqrt{2})^2]$	1	2	
	$= 8 \times \sqrt{6} \times (3+2)$	1/2		
	$= 40\sqrt{6}$.	1/2		
16.	a) $a = x,\ b = \frac{1}{x},\ n = 10$	1/2		
	$T_{r+1} = nC_r\, a^{n-r} b^r$	1/2	2	
	$= 10C_r\, x^{10-r} (\frac{1}{x})^r$	1/2		
	$= 10C_r\, x^{10-2r}$.	1/2		4
	b) For a term independent of x, $10-2r=0$	1	2	
	$\Rightarrow 10 = 2r \Rightarrow r = 5$.	1/2		
	$\therefore$ term independent of x, $T_6 = 10C_5$.	1/2		
	UNIT – III			
17.	a) Let $1+i\sqrt{3} = r(\cos\theta + i\sin\theta)$	1		
	$r\cos\theta = 1,\ r\sin\theta = \sqrt{3}$, $1+i\sqrt{3}$ lies in 1st quadrant.	1/2		
	$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$.	1/2	3	
	$\cos\theta = \frac{1}{2},\ \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$	1/2		
	or $\tan\theta = \sqrt{3} \Rightarrow \theta = \pi/3$			
	$\therefore 1+i\sqrt{3} = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$	1/2		
	b) $z = -7-24i$ Let $\sqrt{z} = x+iy$	1/2		
	$\Rightarrow z = (x+iy)^2$.			
	$\Rightarrow -7-24i = x^2 - y^2 + i \times 2xy$.			
	$\Rightarrow x^2 - y^2 = -7$ —— (1)	1/2		
	$2xy = -24$ —— (2)			
	Now $(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$			
	$= (-7)^2 + (-24)^2 = 625$	1/2		
	$\therefore x^2 + y^2 = 25$ —— (3)			

$(3)+(1) \Rightarrow 2x^2 = 18 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$ ½

$(3)-(1) \Rightarrow 2y^2 = 32 \Rightarrow y^2 = 16 \Rightarrow y = \pm 4$ ½ 3 6

$(2) \rightarrow 2xy = -24$, a negative number.

$\therefore x = 3 \Rightarrow y = -4$ and $x = -3 \Rightarrow y = 4$. ½

$\therefore$ Square roots are $3-4i$ and $-3+4i$

or

b) Let $z = -7-24i$

$x = -7,\ y = -24 < 0$. ½

$|z| = \sqrt{x^2+y^2} = \sqrt{49+576} = 25$ ½

Now, $A = \sqrt{\frac{|z|+x}{2}} = \sqrt{\frac{25+(-7)}{2}} = \sqrt{\frac{18}{2}} = 3$ ½

$B = \sqrt{\frac{|z|-x}{2}} = \sqrt{\frac{25--7}{2}} = \sqrt{\frac{32}{2}} = 4$. ½ 3.

$\therefore \sqrt{z} = \sqrt{-7-24i} = \pm(A-iB)$ since $y<0$. ½

$= \pm(3-4i)$. ½

$\therefore$ Square roots are $3-4i$ and $-3+4i$.

18. (1) $2x+y=4$

x	0	2
y	4	0

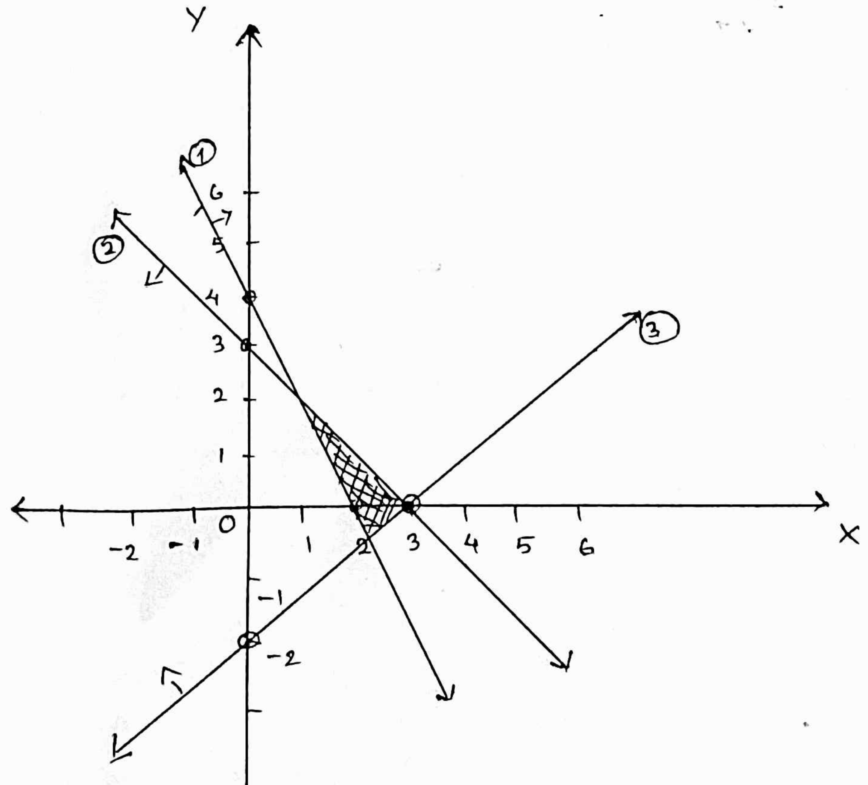
(2) $x+y=3$

x	0	3
y	3	0

(3) $2x-3y=6$

x	0	3
y	-2	0

1½



4½ 6

19.	a) Vowels are IEEEE	$\frac{1}{2}$		
	Total number of letters of the word when all the vowels occur together $= 7+1 = 8$.	$\frac{1}{2}$	3	
	∴ required number of arrangements $= \frac{8!}{3! \times 2!} \times \frac{5!}{4!}$	1+1		6
	b) Vowels are I, O, U, E.	$\frac{1}{2}$		
	Consonants are N, V, L, T.	$\frac{1}{2}$	3	
	∴ required number of words $= 4C_3 \times 4C_2 \times 5!$	1		
	$= 4 \times 6 \times 5! = 24 \times 5! = 2880$	1		
20.	a) $3 \times nP_4 = 5 \times (n-1)P_4$			
	$\Rightarrow 3n(n-1)(n-2)(n-3) = 5(n-1)(n-2)(n-3)(n-4)$	1		
	$\Rightarrow 3n = 5(n-4)$		2	
	$\Rightarrow 3n = 5n - 20$	$\frac{1}{2}$		
	$\Rightarrow 20 = 2n \Rightarrow n = 10$.	$\frac{1}{2}$		
	b) $52C_4$	1		6
	i) $13C_4 + 13C_4 + 13C_4 + 13C_4 = 4 \times 13C_4$.	1		
	ii) $13C_1 \times 13C_1 \times 13C_1 \times 13C_1$	1	4	
	iii) $26C_2 \times 26C_2$	1		
21	a) 10	1	1	
	b) $a = x,\ b = 2y,\ n = 9$.			
	$T_{r+1} = nC_r\, a^{n-r} b^r$	$\frac{1}{2}$		
	$= 9C_r\, x^{9-r} (2y)^r$	$\frac{1}{2}$		
	$= 9C_r\, 2^r x^{9-r} y^r$		2	
	For coeff. of x^6y^3, $9 - r = 6 \Rightarrow r = 3$.	$\frac{1}{2}$		
	∴ coeff. of $x^6y^3 = 9C_3\, 2^3 = 672$.	$\frac{1}{2}$		
	c) $a = 2,\ b = a,\ n = 50$	$\frac{1}{2}$		6
	$T_{r+1} = 50C_r\, 2^{50-r} a^r$.	$\frac{1}{2}$		
	$T_{17} = T_{18} \Rightarrow 50C_{16}\, 2^{50-16} a^{16} = 50C_{17}\, 2^{50-17} a^{17}$		3	
	$\Rightarrow 50C_{16}\, 2^{34} a^{16} = 50C_{17}\, 2^{33} a^{17}$	1		
	$\Rightarrow 50C_{16} \times 2 = 50C_{17} \times a$.	$\frac{1}{2}$		
	$\Rightarrow a = \frac{50C_{16} \times 2}{50C_{17}}$	$\frac{1}{2}$		