MATHEMATICS TEACHER'S ASSOCIATION MALAPPURAM (MAM)

Mathematics Test Series. IV May 2022

Class. XI
Max. Score - 60

Time - 2 hrs
Cool off time - 15 min

(Limits and Derivatives, Mathematical Reasoning, Statistics, Probability)

Unit. I. Answer any six, each question carries 3 marks

1 (a) $3^2+1=10$	1	1		
(b) $\lim_{x\to 0}\frac{\sin 3x}{x} = \lim_{x\to 0}\frac{3\sin 3x}{3x}$	1			
$= 3\lim_{3x\to 0}\frac{\sin 3x}{3x}$	½	2		(3)
$= 3\times 1 = 3$	½			
2. (a) na^{n-1}	1	1		
(b) $\lim_{x\to 2}\frac{x^3-8}{x-2} = \lim_{x\to 2}\frac{x^3-2^3}{x-2}$	½			
$= 3\times 2^{3-1} = 3\times 2^2$	½+½	2		(3)
$= 3\times 4 = 12$	½			
3. (a) nx^{n-1}	1	1		
(b) $f'(x) = 2x$	1			(3)
$f'(5) = 2\times 5 = 10$	1	2		
4. $\frac{d}{dx}(f(x)g(x)) = f(x)\cdot\frac{d}{dx}(g(x)) + g(x)\cdot\frac{d}{dx}(f(x))$	1			
$\therefore \frac{d}{dx}(x^2\sin x) = x^2\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x^2)$	1			
$= x^2\cos x + \sin x \times 2x$	1	3		(3)
$= x^2\cos x + 2x\sin x$.				
5 (a) and	1	1		
(b) Component statements are:				(3)
p : All living things have two legs	2	2		
q : All living things have two eyes.				

6 (a) Mean $\bar{x} = \frac{\sum x_i}{n}$ — ½

$= \frac{2+4+6+8+10}{5} = 6$ — ½ — 1

(b) Mean Deviation about Mean,

$MD_{(\bar{x})} = \frac{\sum |x_i - \bar{x}|}{n}$ — ½

$= \frac{|2-6|+|4-6|+|6-6|+|8-6|+|10-6|}{5}$ — ½ — 2

$= \frac{4+2+0+2+4}{5} = \frac{12}{5} = 2.4$ — 1 — (3)

7 (a) $S = \{HH, HT, TH, TT\}$ — 1 — 1

(b) A : Exactly one head, B : Atleast one head

$A = \{HT, TH\}$ $B = \{HH, HT, TH\}$ — ½+½

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$ — ½ — 2

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$ — ½ — (3)

8 (a) (i) 6^2 — 1 — 1

(b) A : Getting a doublet $= \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$ — 1

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$ — 1 — 2 — (3)

UNIT-II Answer any six, each question carries 4 marks.

9 (a) $\frac{d}{dx}\left(\frac{1}{x^3}+x^3\right) = \frac{d}{dx}\left(x^{-3}+x^3\right) = -3x^{-3-1} + 3x^{3-1}$ — ½

$= -3x^{-4} + 3x^2 = 3\left(\frac{-1}{x^4}+x^2\right)$ — ½ — 1

(b) $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} = \lim_{x\to 0} \frac{(x+1)^{1/2} - 1^{1/2}}{(x+1)-1}$ — 1

Putting $x+1=y$, $= \lim_{y\to 1} \frac{y^{1/2} - 1^{1/2}}{y-1}$ — 1 — 3 — (4)

$= \frac{1}{2} \times 1^{1/2-1} = 1/2$ — 1

10. (a) $\lim_{x\to 1} f(x) = \lim_{x\to 1} 3(x+1)$ — ½

$= 3(1+1) = 3\times 2 = 6$ — ½ — 1

(b) $LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} (2x+3)$ — ½

$= 2\times 0 + 3 = 3$ — ½

$RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} 3(x+1)$ — ½

$= 3(0+1) = 3$ — ½ — 3 — (4)

$LHL = RHL = 3$. — ½

$\therefore \lim_{x\to 0} f(x) = 3$ — ½

11 (a) 1 — 1 — 1

(b) 1 — 1 — 1

(c) $\lim_{x\to 0} \frac{e^{4x}-1}{3x} = \lim_{x\to 0} \frac{4(e^{4x}-1)}{4(3x)}$ — ½

$= \lim_{x\to 0} \frac{4\times(e^{4x}-1)}{3\times 4x}$ — ½ — 2 — (4)

$= \frac{4}{3} \lim_{4x\to 0} \frac{e^{4x}-1}{4x}$ — ½

$= \frac{4}{3}\times 1 = \frac{4}{3}$ — ½

12 | $f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad (1)$$

$$= \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} \quad (1/2)$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h} \quad (1)$$

$$= \lim_{h\to 0} \frac{\cos(x+h/2)\sin h/2}{h/2} \quad (1/2)$$

$$= \lim_{h\to 0} \cos(x+h/2) \times \lim_{h/2\to 0}\left(\frac{\sin h/2}{h/2}\right) \quad (1/2)$$

$$= \cos(x+0) \times 1 = \cos x \quad (1/2)$$

$\therefore \frac{d}{dx}(\sin x) = \cos x$

4 (H)

13 (a) If xy is not odd, then x is not odd or y is not odd. (1)

1

(b) P: x and y are odd, q: xy is odd (1/2)

Assume that q is not true $\Rightarrow xy$ is not odd (1/2)

$\Rightarrow xy$ is even $\Rightarrow x$ is even or y is even (1)

$\Rightarrow x$ is not odd or y is not odd (1/2)

$\Rightarrow$ P is not true. (1/2)

$\therefore$ By contrapositive method, the statement is true

3 (H)

14

x_i	f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
5	7	25	35	175
10	4	100	40	400
15	6	225	90	1350
20	3	400	60	1200
25	5	625	125	3125
	$N=25$		350	6250

(1 1/2)

Mean, $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{350}{25} = 14$ — 1

Variance, $\sigma^2 = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2$ — ½

$= \frac{6250}{25} - 14^2 = 250 - 196 = 54$ — ½ — 4 (4)

S.D, $\sigma = \sqrt{54} = 7.35$ — ½

15 (a) $P(A') = 1 - P(A)$ — ½

$= 1 - 0.5 = 0.5$ — ½ — 1

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ — 1

$0.8 = 0.5 + 0.6 - P(A \cap B)$ — ½

$\therefore P(A \cap B) = 0.3$ — ½ — 2 (4)

(c) $P(A' \cup B') = P(A \cap B)'$ — ½

$= 1 - P(A \cap B)$ — 1

$= 1 - 0.3 = 0.7$ — ½

16 (a) $P(\text{all balls are black}) = \frac{{}^6C_3}{{}^{17}C_3} = \frac{20}{680}$ — ½

$= \frac{1}{34} = 0.029$ — ½ — 1

(b) $P(\text{Exactly 2 yellow balls}) = \frac{{}^6C_2 \times {}^{11}C_1}{{}^{17}C_3}$ — ½

$= \frac{15 \times 11}{680} = 0.243$ — ½ — 1

(c) $P(\text{Atleast 2 yellow balls})$

$= P(\text{2 yellow balls}) + P(\text{3 yellow balls})$

$= \frac{{}^6C_2 \times {}^{11}C_1}{{}^{17}C_3} + \frac{{}^6C_3}{{}^{17}C_3}$

$= 0.243 + 0.029 = 0.272$

UNIT-III. Answer any three questions, each carries 6 marks

17 (a) $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \cdots + \frac{x^2}{2} + x + 1$

$f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \cdots + \frac{2x}{2} + 1 + 0$ — 1

$= x^{99} + x^{98} + \cdots + x + 1$ — ½ — 3 — (3)

$f'(0) = 0 + 0 + \cdots + 0 + 1 = 1 \Rightarrow 100 f'(0) = 100 \times 1 = 100$ — ½ + ½

$f'(1) = 1^{99} + 1^{98} + \cdots + 1^2 + 1 = 1 + 1 + 1 + \cdots + 1 = 100$ — ½

$f'(1) = 100 = 100 \times f'(0)$

(b) $y = \frac{4 + 5\sin x}{3 + 7\cos x} = \frac{u}{v}$

$u = 4 + 5\sin x$

$\frac{du}{dx} = 0 + 5\cos x = 5\cos x$ — ½

$v = 3 + 7\cos x$

$\frac{dv}{dx} = 0 + 7(-\sin x) = -7\sin x$ — ½

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$ — ½

$= \frac{(3 + 7\cos x)5\cos x - (4 + 5\sin x)(-7\sin x)}{(3 + 7\cos x)^2}$ — ½ — (6)

$= \frac{15\cos x + 35\cos^2 x + 28\sin x + 35\sin^2 x}{(3 + 7\cos x)^2}$ — ½ — 3

$= \frac{15\cos x + 28\sin x + 35(\cos^2 x + \sin^2 x)}{(3 + 7\cos x)^2}$

$= \frac{15\cos x + 28\sin x + 35 \times 1}{(3 + 7\cos x)^2}$ — ½

$= \frac{15\cos x + 28\sin x + 35}{(3 + 7\cos x)^2}$

18 (a) "It is not true that $\sqrt{2}$ is irrational"

OR

"It is false that $\sqrt{2}$ is irrational" — 1 — 1

(b) "If a number is divisible by 3, then it is divisible by 9" — 1 — 1

(c) Assume that $\sqrt{2}$ is rational

Then $\sqrt{2} = \frac{a}{b}$, where a and b are integers, $b \neq 0$, a and b have no common factors — 1

Squaring, $2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2 \rightarrow (1)$ — ½

$\Rightarrow$ 2 divides $a^2 \Rightarrow$ 2 divides a — ½ — (6)

$\therefore$ Take $a = 2k$ — ½

Then $(1) \Rightarrow (2k)^2 = 2b^2$ — ½ — 4

ie $4k^2 = 2b^2$.

ie $b^2 = 2k^2 \Rightarrow$ 2 divides b^2 — ½

$\Rightarrow$ 2 divides b

$\therefore$ 2 is a common factor of a and b — ½

This contradicts the assumption that $\sqrt{2}$ is ~~irra~~ rational.

Hence, $\sqrt{2}$ is irrational

19 (a)

Class	f_i	x_i	$f_i x_i$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135
70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
	N=50		3100		10050

3

(i) Mean $\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{3100}{50} = \underline{62}$. 1

(ii) Variance, $\sigma^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{N}$ 2

$= \frac{10050}{50} = 201$

(6)

OR

class	f_i	x_i	$f_i x_i$	x_i^2	$f_i x_i^2$
30-40	3	35	105	1225	3675
40-50	7	45	315	2025	14175
50-60	12	55	660	3025	36300
60-70	15	65	975	4225	63375
70-80	8	75	600	5625	45000
80-90	3	85	255	7225	21675
90-100	2	95	190	9025	18050
	N=50		3100		202250

3 3

Mean $= \frac{\Sigma f_i x_i}{N} = \frac{3100}{50} = 62$. 1 1

(6)

Variance, $\sigma^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$ ½

$= \frac{202250}{50} - (62)^2$ ½

$= 4045 - 3844$ 2

$= \underline{201}$ ½

½

20 (a) $P(\text{Card drawn is Ace}) = \frac{\text{No: of Ace cards}}{\text{Total no: of cards}}$ — 1

$= \frac{4}{52} = \frac{1}{13}$ — 1 — 2

(b) (i) $P(\text{Committee with one man}) = P(\text{1 man \& 1 Woman})$

$= \frac{{}^2C_1 \times {}^2C_1}{{}^4C_2} = \frac{2\times 2}{6} = \frac{2}{3}$ — 2 — 4 — (6)

(ii) $P(\text{Committee with two men}) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$ — 2

21 (a) Let A : Opted NCC

B : Opted NSS.

Then, $P(A) = \frac{n(A)}{n(S)} = \frac{30}{60}$, $P(B) = \frac{32}{60}$ — ½

$P(A \cap B) = \frac{24}{60}$ — 2

$P(\text{NCC or NSS}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$ — ½

$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{38}{60} = \frac{19}{30}$ — 1

(b) $P(\text{Neither NCC nor NSS}) = P(A' \cap B')$ — ½ — (6)

$= P(A \cup B)' = 1 - P(A \cup B)$ — 1 — 2

$= 1 - \frac{19}{30} = \frac{11}{30}$ — ½

(c) $P(\text{Exactly one of NCC or NSS})$

$= P((A \cap B') \cup (A' \cap B))$ — ½

$= P(A \cup B) - P(A \cap B)$ — ½ — 2

$= \frac{38}{60} - \frac{24}{60} = \frac{14}{60} = \frac{7}{30}$ — 1

OR $P(A \cap B' \cup A' \cap B) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= \frac{30}{60} - \frac{24}{60} + \frac{32}{60} - \frac{24}{60} = \frac{14}{60} = \frac{7}{30}$