PLUS ONE MATHEMATICS

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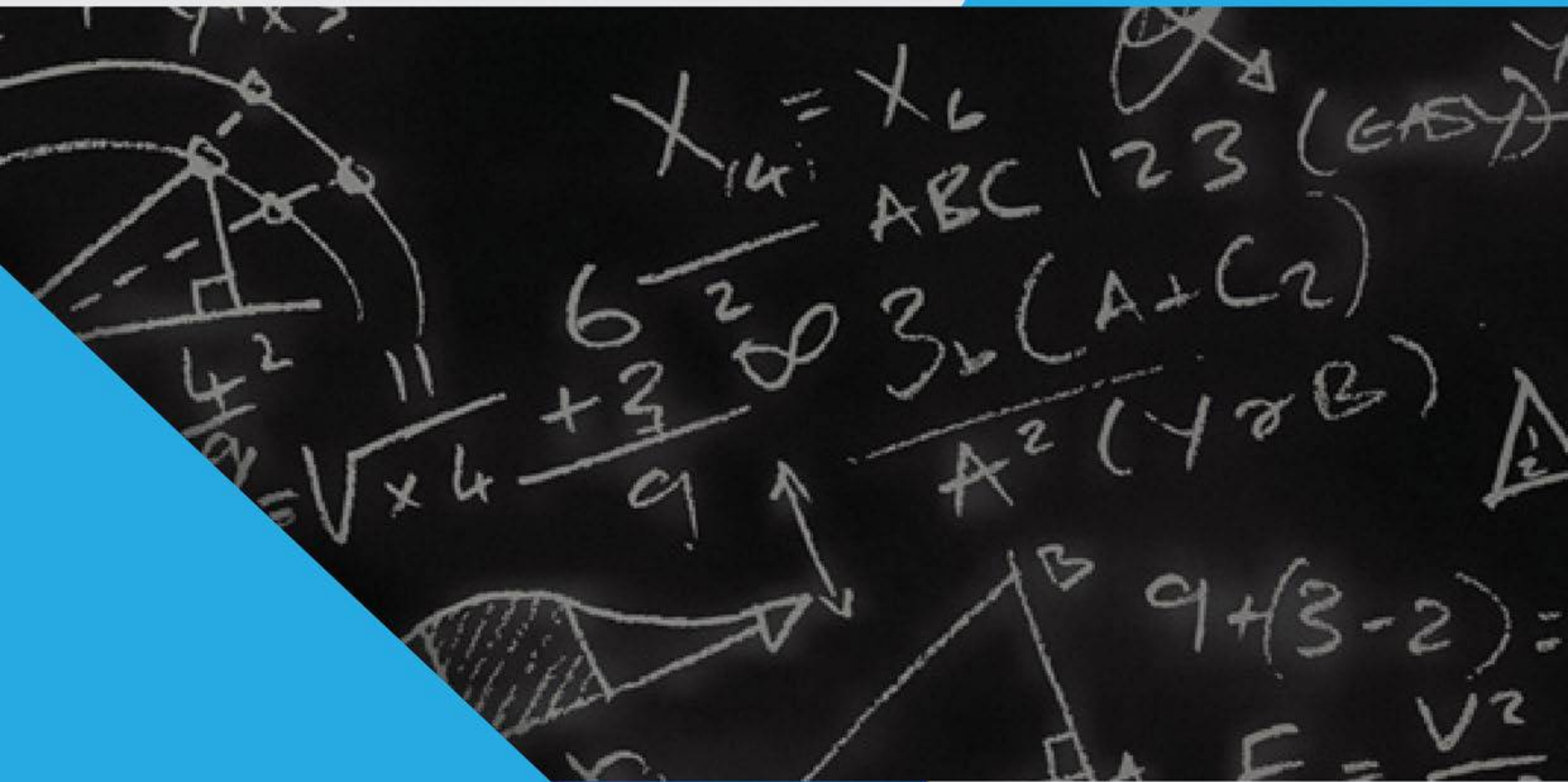
STUDY MATERIAL



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MALAPPURAM

HIGHER SECONDARY NATIONAL SERVICE SCHEME

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SETS

KEY NOTES

Sets

- A set is a well defined collection of objects
  Eg : Collection of even integers
  Collection of all natural numbers less than 100
- Objects or members in a set are called its elements
  N : Set of all natural numbers
  Z : Set of all integers
  Q : Set of all rational numbers
  R : Set of all real numbers
  Z^+ : Set of all positive integers
  R^+ : Set of all positive real numbers

Representation of sets

- Two methods

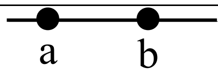
Roster form/Tabular form	Set builder form/Rule method
Described by listing the elements separated by commas, enclosed with the braces Eg : If A is the set of all vowels in English Alphabet , $A = \{a, e, i, o, u\}$	In this method we write down a property or rule which represents all the elements of the set. Eg : $A = \{x : x$ is vowels in English alphabet$\}$

- Subsets
  The set A is said to be a subset of a set B if every element of A is also an element of B, we write it as $A \subset B$
  $A \subset B$ if $x \in A \Longrightarrow x \in B$
  Eg : $\{a, b\}$ is a subset of $\{a, b, c, d\}$
- ϕ is a subset of every set, $\phi \subset A$
- Every set is a subset of itself, $A \subset A$
- If $A \subset B$ and $A \neq B$, then A is called the proper subset of B and B is called the superset of A.
  Eg : $\{1, 2\}$ is a proper subset of $\{1, 2, 3\}$
- In a finite set having 'n' elements ,
  Number of subsets : 2^n
  Number of proper subsets : $2^n - 1$

Properties of subsets

- If $A \subset B$ and $B \subset C$, then $A \subset C$
- $A = B \Leftrightarrow A \subset B$ and $B \subset A$

Intervals as subsets of R

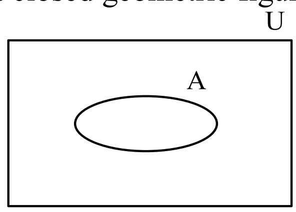
Interval	(a, b)	$[a, b]$	$[a, b)$	$(a, b]$
Set builder form	$\{x: a < x < b\}$	$\{x: a \leq x \leq b\}$	$\{x: a \leq x < b\}$	$\{x: a < x \leq b\}$
Representation on real line				
	a, b not included	a, b included	a included b not included	a not included b included

Eg : $\{x: x \in R, -4 < x \leq 6\}$ represents $(-4, 6]$

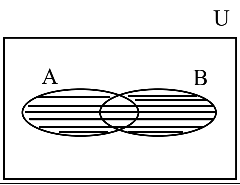
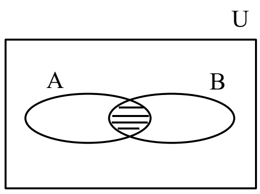
- $(0, \infty) \rightarrow$ Set of positive real numbers
- $[0, \infty) \rightarrow$ Set of non negative real numbers
- $(-\infty, 0) \rightarrow$ Set of negative real numbers
- $(-\infty, \infty) \rightarrow$ Set of real numbers

VENN DIAGRAMS

The diagrammatic representation of sets is called Venn diagram. In Venn diagram, the universal set is represented by a rectangular region and its subset is represented by circle or a closed geometric figure inside the rectangular region.



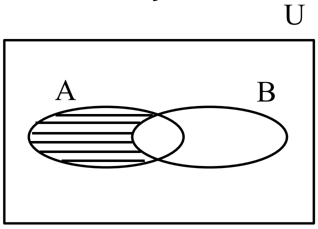
Operations on sets

Union of sets :	Intersection of Sets :
Union of sets : The union of two sets A and B , denoted by $A \cup B$, is the set of all those elements which are either in A or in B or both in A and B; $A \cup B = \{x: x \in A \text{ or } x \in B\}$ 	**Intersection of Sets :** The intersection of two sets, A and B denoted by $A \cap B$, is the set of all those elements which are common to both A and B. $A \cap B = \{x: x \in A \text{ and } x \in B\}$ 

Eg : Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ $A \cup B = \{1,2,3,4,5\}$ **Properties** • $A \cup B = B \cup A$ • $(A \cup B) \cup C = A \cup (B \cup C)$ • $A \cup \phi = A$ • $A \cup U = U$ • $A \cup A = A$ • If $A \subset B$, then $A \cup B = B$	Eg : Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ $A \cap B = \{3\}$ **Properties** • $A \cap B = B \cap A$ • $(A \cap B) \cap C = A \cap (B \cap C)$ • $A \cap \phi = \phi$ • $A \cap U = A$ • $A \cap A = A$ • $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ • $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ • If $A \subset B$, then $A \cap B = A$

3. **Difference of sets :** The difference of two sets A and B is the set consisting of all elements of A, which are not in set B.

   $A - B = \{x : x \in A \text{ and } x \notin B\}$



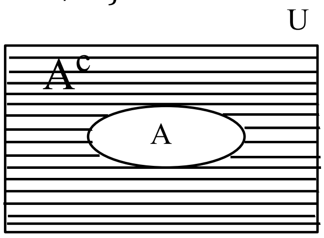
   Eg : Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$

   $A - B = \{1, 2\}$

   - $(A - B) \cup (A \cap B) = A$

4. **Complement of sets :** The set of all elements belongs to U and does not belong to A is the complement of set A denoted by A' or A^c

   $A' = \{x : x \in U \text{ and } x \notin A\} = U - A$



   Eg : If $U = \{a, b, c, d, e, f, g\}$

   $A = \{a, b, c\}$, then $A' = \{d, e, f, g\}$

Properties

- $A' \subset U$
- $A \cup A' = U$
- $\phi' = U$
- $U' = \phi$

- $A \cap A' = \phi$
- $A - B = A \cap B'$
- $(A')' = A$
- $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Practical problems on Union and Intersection of Two sets

- If A and B are any two finite sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- If A and B are finite and disjoints, then $n(A \cup B) = n(A) + n(B)$
- $n(A \cap B') = n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B)' = n(U) - n(A \cup B)$

QUESTIONS AND ANSWERS

1. Write the following in Roster form

(a) $\{x: x \text{ is an integer}, 0 \leq x \leq 6\}$

(b) $\{x: x \text{ is an prime number}, < 10\}$

(c) $\{x: x \text{ is an integer}, x^2 \leq 4\}$

(d) $\{x: x \text{ is a natural number and divisor of } 12\}$

(e) $\{x: x \in R\ , x^2 + x - 2 = 0\}$

Ans :

(a) $\{0,1,2,3,4,5,6\}$

(b) $\{2,3,5,7\}$

(c) $\{-2,-1,0,1,2\}$

(d) $\{1,2,3,4,6,12\}$

(e) $x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x + 2 = 0, x - 1 = 0$

$x = -2, x = 1$

$\therefore \{-2,1\}$

2. Let $A = \{2,3,4,5\}$ and $B = \{4,5,6,7\}$

(a) Write $A \cup B$

(b) Write $A \cap B$

(c) Write $A - B$

(d) Write $B - A$

(e) Verify that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

Ans :

$A = \{2,3,4,5\}$ and $B = \{4,5,6,7\}$

(a) $A \cup B = \{2,3,4,5,6,7\}$

(b) $A \cap B = \{4,5\}$

(c) $A - B = \{2,3\}$

(d) $B - A = \{6,7\}$

(e) $(A \cup B) - (A \cap B) = \{2,3,4,5,6,7\} - \{4,5\}$

$= \{2,3,6,7\}$

$(A - B) \cup (B - A) = \{2,3\} \cup \{6,7\}$

$= \{2,3,6,7\}$

$\therefore (A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

3. If $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,4,6,8\}$, $B = \{2,3,5,7\}$. Find

(a) A' and B'

(b) $A \cup B$

(c) $A \cap B$

(d) Verify that $(A \cup B)' = A' \cap B'$

(e) Verify that $(A \cap B)' = A' \cup B'$

Ans :

$U = \{1,2,3,4,5,6,7,8,9\}$,

$A = \{2,4,6,8\}$,

$B = \{2,3,5,7\}$

(a) $A' = U - A = \{1,3,5,7,9\}$,

$B' = U - B = \{1,4,6,8,9\}$

(b) $A \cup B = \{2,3,4,5,6,7,8\}$

(c) $A \cap B = \{2\}$

(d) $A \cup B = \{2,3,4,5,6,7,8\}$

$\therefore (A \cup B)' = U - (A \cup B) = \{1,9\}$

$A' = \{1,3,5,7,9\}$,

$B' = \{1,4,6,8,9\}$

$\therefore A' \cap B' = \{1,9\}$

$\Longrightarrow (A \cup B)' = A' \cap B'$

(e) $A \cap B = \{2\}$

$\therefore (A \cap B)' = \{1,3,4,5,6,7,8,9\}$

$A' = \{1,3,5,7,9\}$,

$B' = \{1,4,6,8,9\}$

$\therefore A' \cup B' = \{1,3,4,5,6,7,8,9\}$

$\Longrightarrow (A \cap B)' = A' \cup B'$

4. Let $A = \{1,2,3\}$.Write all the subsets of A

Ans :

$A = \{1,2,3\}$

Subsets : $\{1,2,3\}, \{1,2\}, \{1,3\}, \{2,3\},$
$\{1\}, \{2\}, \{3\}, \emptyset\}$

5. Choose the correct answer from the bracket

(a) Which one of the following is equal to $\{x : x \in R, 2 < x \leq 4\}$

(i) {2, 3, 4} (ii) {3, 4}
(iii) [2, 4] (iv) (2, 4]

(b) The set builder form of (6, 12) is

(i) $\{x : x \in R, 6 < x \leq 12\}$
(ii) $\{x : x \in R, 6 \leq x \leq 12\}$
(iii) $\{x : x \in R, 6 < x < 12\}$
(iv) $\{x : x \in R, 6 \leq x < 12\}$

(c) If A and B are two sets such that $A \subset B$ then $A \cup B$ is

(i) A (ii) B
(iii) Null set (iv) $\{\phi\}$

Ans :

(a) (iv) or (2, 4]
(b) (iii) or $\{x : x \in R, 6 < x < 12\}$
(c) (ii) or B

6. Fill in the blanks :

(a) If $A = \{x : x$ is a natural number, $x < 5$ and $x > 7\}$.Then $n(A)$ is ....

(b) If U is the universal set and A is any set then $U \cap A$ = ...

Ans :

(a) Here , $A = \{ \}$
$\therefore n(A) = 0$

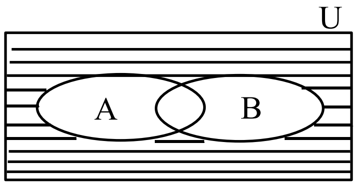
(b) $U \cap A = A$

7. Draw Venn diagram which represents

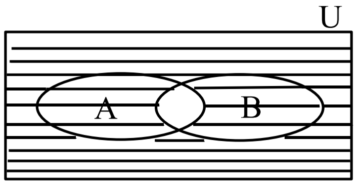
(i) $(A \cup B)'$
(ii) $A' \cup B'$

Ans :

(i)



(ii)



8. If $A = \{x : x$ is a letter in the word "MATHEMATICS"$\}$,
$B = \{y : y$ is a letter in the word "STATISTICS"$\}$. Then find

(i) $A - B$
(ii) $B - A$
(iii) $A \cap B$

Ans :

$A = \{M, A, T, H, E, I, C, S\}$,
$B = \{S, T, A, I, C\}$

(i) $A - B = \{M, H, E\}$
(ii) $B - A = \{ \}$
(iii) $A \cap B = \{A, T, I, C, S\}$

9. Let $A = \{x : x \in N, 1 \leq x \leq 5\}$,
$B = \{2,5,6,9\}$ and $C = \{1,4,5,8,9,10\}$

(a) Find the number of elements of A

(b) Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Ans :

$A = \{1,2,3,4,5\}$, $B = \{2,5,6,9\}$,
$C = \{1,4,5,8,9,10\}$

(a) $n(A) = 5$

(b) $B \cup C = \{1,2,4,5,6,8,9,10\}$

$A \cap (B \cup C) = \{1,2,4,5\}$ .....(1)

$A \cap B = \{2,5\}$

$A \cap C = \{1,4,5\}$

$(A \cap B) \cup (A \cap C) = \{1,2,4,5\}$..(2)

From (1) and (2)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

10. If $U = \{1,2,3,4,5,6,7,8\}$, $A = \{2,4,6,8\}$ and $B = \{2, 4, 8\}$

(a) Write A' and B'

(b) For the above sets A and B, prove that $(A \cup B)' = A' \cap B'$

(c) Check whether $(A \cap B)' = A' \cup B'$

Ans :

(a) $A' = U - A = \{1,3,5,7\}$,

$B' = U - B = \{1,3,5,6,7\}$

(b) $A \cup B = \{2, 4, 6, 8\}$

$(A \cup B)' = U - (A \cup B) = \{1,3,5,7\}$

Here, $A' \cap B' = \{1,3,5,7\}$

$\therefore (A \cup B)' = A' \cap B'$

(c) $A \cap B = \{2,4, 8\}$

$(A \cap B)' = U - (A \cap B)$

$= \{1,3,5,6,7\}$

Here, $A' \cup B' = \{1,3,5,6,7\}$

$\therefore (A \cup B)' = A' \cap B'$

11. If X and Y are two sets such that $n(X) = 17, n(Y) = 23$ and $n(X \cup Y) = 38$, then find $n(X \cap Y)$

Ans :

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$38 = 17 + 23 - n(X \cap Y)$$

$$n(X \cap Y) = 40 - 38 = 2$$

12. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 students were taking coffee. 100 were taking both tea and coffee.

(a) How many take tea or Coffee

(b) How many take neither Coffee nor Tea

(c) How many take Coffee only

Ans :

$$n(U) = 600$$

$$n(T) = 150$$

$$n(C) = 225$$

(a) $n(Tea\ or\ Coffee) = n(T \cup C)$

$= n(T) + n(C) - n(T \cap C)$

$= 150 + 225 - 100$

$= 275$

(b) $P(Neither\ tea\ nor\ coffee) = n(T' \cap C')$

$= n(T \cup C)'$

$= 600 - 275$

$= 325$

(c) $n(Coffee\ o$

RELATIONS AND FUNCTIONS

2

KEY NOTES

Cartesian Products :

$$A \times B = \{(a, b): a \in A, b \in B\}$$

If $n(A) = p$ and $n(B) = q$, $n(A \times B) = pq$

Relations

- R is a relation from A to B if $R \subseteq A \times B$
- Representation → Roster form
  → Set builder form
  → Arrow diagram (Diagrammatic representation)
- A relation on A is a subset of $A \times A$
- If $n(A) = p$, $n(B) = q$, then total number of relations from A to B is 2^{pq}
- If $R : A \rightarrow B$
  - Co-domain = B
  - Domain = $\{x \in A : (x, y) \in R\}$ = Set of all first elements
  - Range = $\{y \in B: (x, y) \in R\}$ = Set of all second elements
  - Range of $R \subseteq$ codomain of R
  - If $(x, y) \in R$, y is the image of x

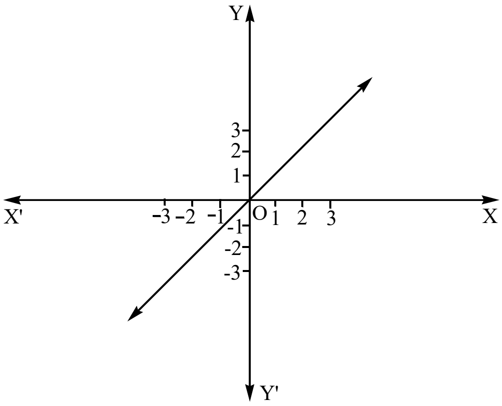
Functions

- A function $f : A \rightarrow B$ is a relation in which each element of A has one and only one image in B
- $f : A \rightarrow B$, where $f(x) = y$
- A is the domain
- B is the codomain of f
- Range of f is the set of all images of x under f
- A real function has the set of real numbers or one of its subsets both as its domain and range

Graphs of some Real Functions

I. Identity functions

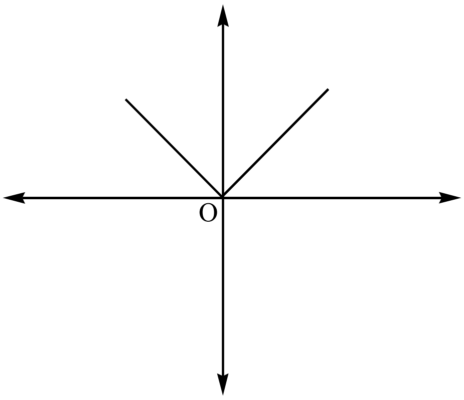
$f : R \rightarrow R$, $f(x) = x$ and $x \in R$



Domain = R, Range = R

II. Modulus function

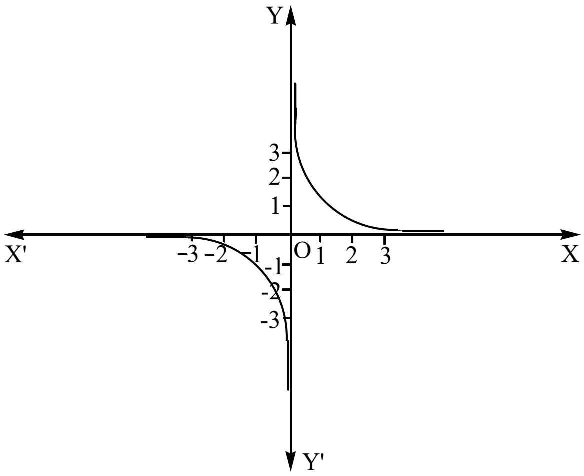
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Domain = R, Range = $[0, \infty)$

III. Reciprocal function

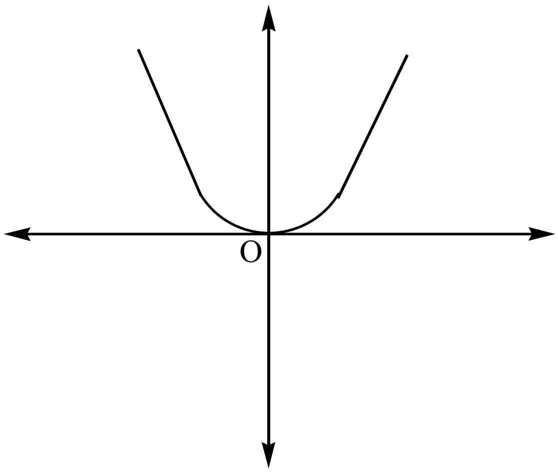
$f(x) = \frac{1}{x}$, $x \in R - \{0\}$



Domain = $R - \{0\}$, Range = $R - \{0\}$

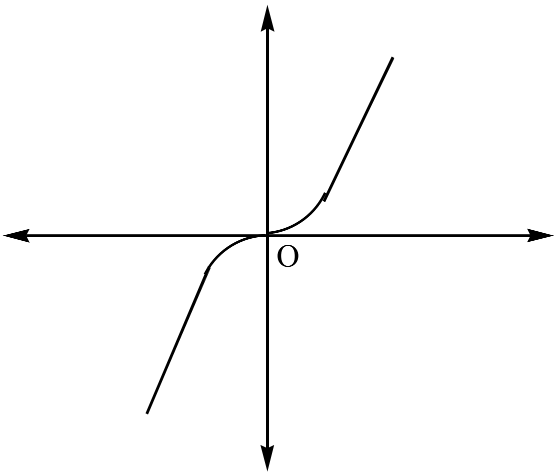
IV.

$f : R \rightarrow R$, $f(x) = x^2$



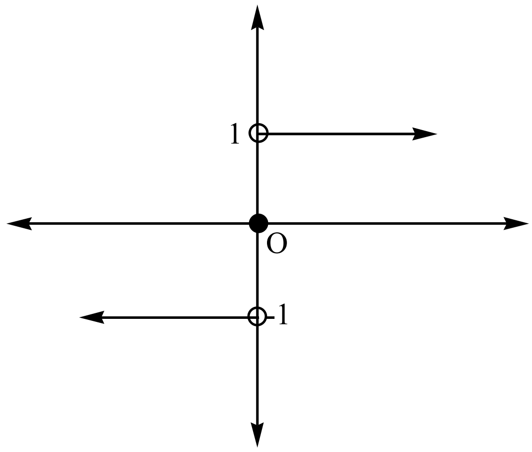
Domain = R, Range = $[0, \infty)$

V $: R \to R,\ f(x) = x^3$



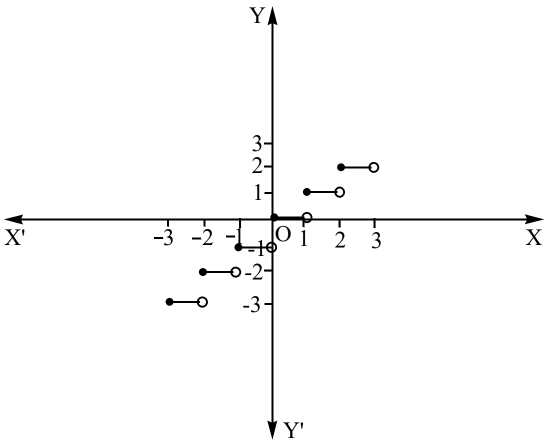
VI. Signum function

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$



VII. Greatest integer function

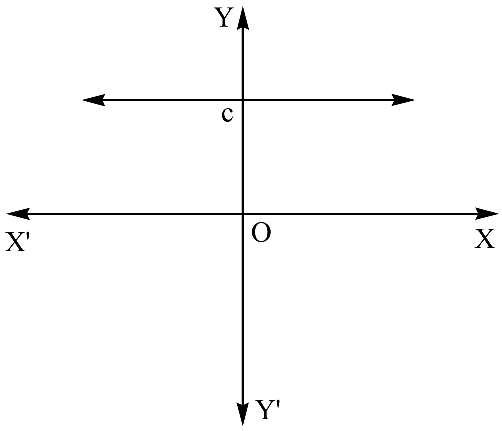
$f(x) = [x]$, x be a real number then $[x]$ is the greatest integer $\leq x$



VIII. Constant function

$f : R \to R,\ f(x) = c$

Where c is a constant



Algebra of functions

$f : X \to R$ and $g : X \to R$

- $(f + g)(x) = f(x) + g(x), x \in X$
- $(f - g)(x) = f(x) - g(x), x \in X$
- $(f \cdot g)(x) = f(x) \cdot g(x)$
- $(kf)(x) = k(f(x))$, k is a constant
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

1. Checking whether a Given Relation is a Function

$R : A \to B$ is a function, (R in roster form) if

(a) Set of all first elements in $R = A$. i.e., Domain of $R = A$

(b) No first element is repeated in R

2. Check whether a given graph is that of a Function or Not

Draw vertical lines through the points in the domain, if it intersects the graph at more than one point, then it is not a function

QUESTIONS AND ANSWERS

1. Let $= \{1,2,3,4,5,6\}$. Define a relation R from A to A by $R = \{(x, y): y = x + 1\}$

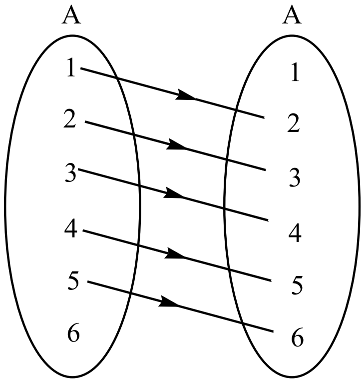
(a) Depict the relation using an arrow diagram

(b) Write down the domain, codomain and range of R.

(c) Is R a function? Why ?

Ans :

(a) $R = \{(1,2), (2,3), (3,4), (4,5), (5,6)\}$



(b) Domain $= \{1,2,3,4,5\}$

Range $= \{2,3,4,5,6\}$

Codomain $= \{1,2,3,4,5,6\}$

(c) R is not a function,

$\because 6 \notin$ Domain

2. (a) A=$\{2,3\}$, B=$\{1,3,5\}$, then the number of relations from A to B is

(i) 2 (ii) 64

(iii) 32 (iv) 62

(b) R is a relation defined on the set $A = \{1,2,3, \ldots, 14\}$ by $R = \{(x, y): 3x - y = 0, x, y \in A\}$ Write the domain , codomain and the range.

(c) Let $f(x) = x^2$, $g(x) = 2x + 1$ be two real functions. Find

(i) $(f \cdot g)(x)$

(ii) $(f + g)(x)$

Ans :

(a) Number of relations from A to B

$= 2^{pq} = 2^{2\times3} = 2^6 = 64$

$[n(A) = p = 2, n(B) = q = 3]$

(b) $R = \{(x, y): 3x - y = 0, x, y \in A\}$

$R = \{(x, y): y = 3x, x, y \in A\}$

$R = \{(1,3), (2,6), (3,9), (4,12)\}$

Domain $= \{1,2,3,4\}$

Range $= \{3, 6, 9, 12\}$

Codomain $= \{1,2,3, \ldots ,14\}$

(c) $f(x) = x^2, g(x) = 2x + 1$

(i) $(fg)(x) = f(x). g(x)$

$= x^2(2x + 1)$

$= 2x^3 + x^2$

(ii) $(f + g)(x) = f(x) + g(x)$

$= x^2 + (2x + 1)$

$= x^2 + 2x + 1$

$= (x + 1)^2$

3. (a) Draw the graph of $f(x) = |x|$.What is its domain?

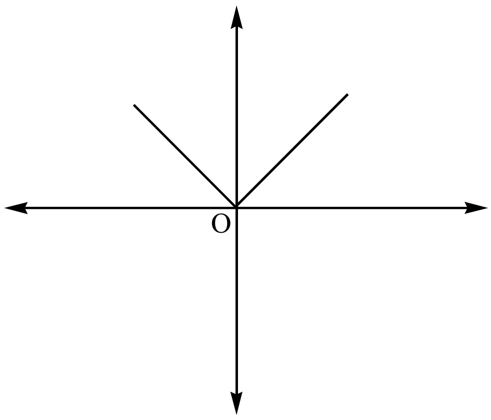
(b) Find the domain of the function

$$f(x) = \frac{x^2+3x+5}{x^2-5x+4}$$

(c) Let $A = \{1,2,3,4\}$, $B = \{1,5,9,11,15,16\}$ and $f = \{(1,5), (2,9), (3,1), (4,5), (2,1)\}$. State with reason whether f is a function or Not

Ans :

(a)



Domain = R

(b) $x^2 - 5x + 4 = 0$

$\implies (x-4)(x-1) = 0$

$\implies x = 4$ and $x = 1$

$\because f(x)$ is a rational function $\left(\frac{p(x)}{q(x)}\right)$,

$q(x) \neq 0$

$\implies x^2 - 5x + 4 \neq 0$

$\therefore$ Domain = $R - \{1, 4\}$

(c) f is not a function from A to B because the element 2 has two images 9 and 1(i.e., the first element 2 is repeated twice in f)

4. (a) Find the domain and range of the function $f(x) = \sqrt{9 - x^2}$

(b) Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined on the set of non-negative real numbers.

Find (i) $(f - g)(x)$

(ii) $\left(\frac{f}{g}\right)(x)$

Ans :

(a) $f(x) = \sqrt{9 - x^2}$

$\implies 9 - x^2 \geq 0$

$\implies x^2 \leq 9$

$\implies -3 \leq x \leq 3$

$\therefore$ Domain = $[-3, 3]$

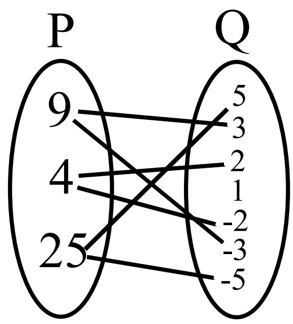
Range = $[0, 3]$

(b) $f(x) = \sqrt{x}$; $g(x) = x$

(i) $(f - g)(x) = f(x) - g(x) = \sqrt{x} - x$

(ii) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}, x \neq 0$

5. The figure shows a relation between the sets P and Q.



(a) Write this relation in set builder form

(b) Write this relation in roster form

(c) Write its domain and range

Ans : Here the relation R is "x is the square of y"

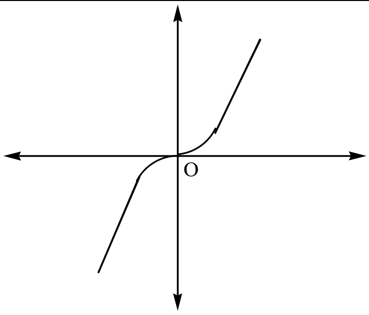
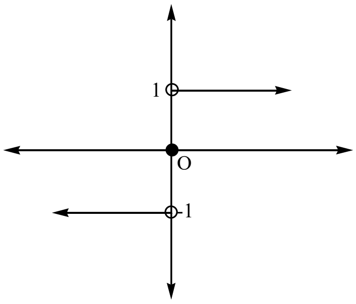
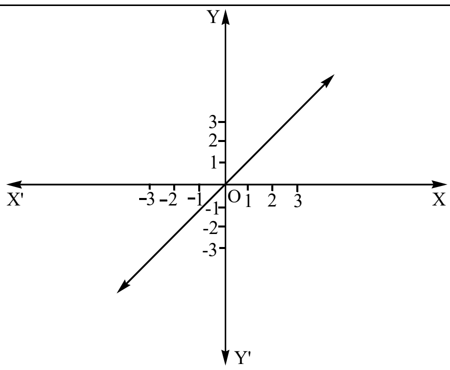
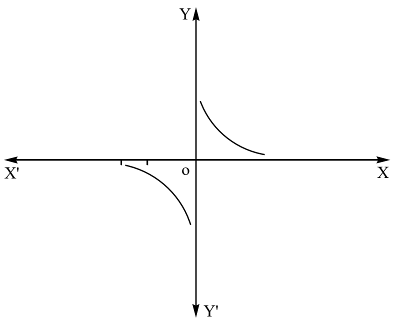
(a) $R = \{(x, y): x$ is the square of $y, x \in P, y \in Q\}$

(b) $R = \{(9,3), (9,-3), (4,2), (4,-2), (25,5), (25,-5)\}$

(c) Domain = $\{4, 9, 25\}$

Range = $\{-2, 2, 3, -3, 5, -5\}$

6. Match the following :

	Column A		Column B
(a)		(i)	$f: R \rightarrow R$ given by $f(x) = \frac{1}{x}, x \neq 0$
(b)		(ii)	$f: R \rightarrow R$ given by $f(x) = x^3, x \epsilon R$
(c)		(iii)	$f: R \rightarrow R$ given by $f(x) = x, x \epsilon R$
(d)		(iv)	$f: R \rightarrow R$ given by $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0, \\ -1, & x < 0 \end{cases}$
		(v)	$f: R \rightarrow R$ $f(x) = \lvert x \rvert, x \epsilon R$

Ans :

(a) → (ii); (b) → (iv); (c) → (iii); (d) → (i)

PRACTICE PROBLEMS

1. (a) Let $A = \{1,2,3,4,6\}$. Let R be the relation defined by $\{(a, b): a, b \in A,$ b is exactly divisible by $a\}$. Write

(i) R in roster form

(ii) Domain

(iii) Range

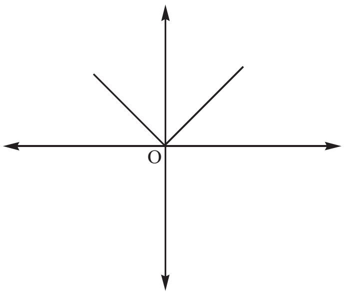
(b) How many relations are there from A to A, where $A = \{-1,1\}$

(c) Draw the graph of the function $f(x) = (x-1)^2$

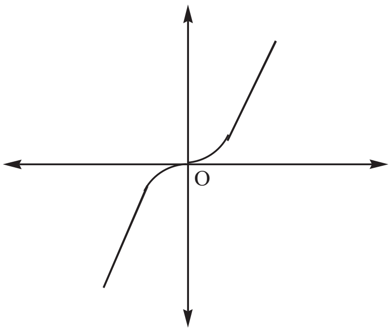
2.

(a) Which of the following graphs does not represent a function? [Hint : Vertical line test]

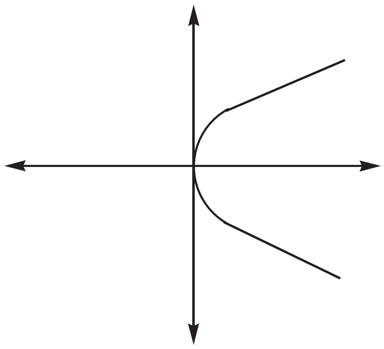
(b) Identify the function $f(x) = \frac{1}{x}$ from the graphs.



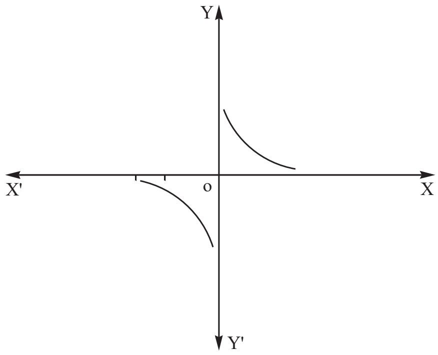
(i)



(ii)



(iii)



(iv)

3. Let $A = \{1,2,3,4\}$, $B = \{1,5,9,11,15,16\}$, $f = \{(1,5),(2,9),(3,1),(4,5),(2,11)\}$ Is f a function from A to B? Justify your answer.

4. (a) Find the domain of

(i) $f(x) = \frac{x^2+2x+3}{x^2-8x+12}$

(ii) $f(x) = -|x|$

(b) $R = \{(x, x+5): x \in \{0,1,2,3,4,5\}$, determine the domain and range of R.

(c) $f, g : R \rightarrow R$, $f(x) = x + 1$, $g(x) = 2x - 3$.

Find $f + g$, $f - g$, fg and $\frac{f}{g}$

3

TRIGONOMETRIC FUNCTIONS

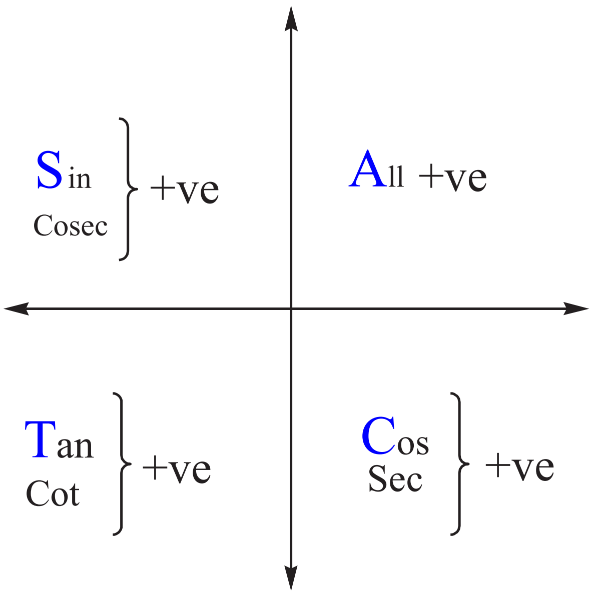
KEY NOTES

$\tan x = \frac{\sin x}{\cos x}$ $\qquad$ $\sec x = \frac{1}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$ $\qquad$ $\mathrm{cosec}\, x = \frac{1}{\sin x}$

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\sec^2 x - \tan^2 x = 1$ $\qquad$ $\mathrm{cosec}^2 x - \cot^2 x = 1$

Function	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan x	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined



$$sin\,(x+y) = sin\,x\,cos\,y + cos\,x\,sin\,y$$
$$sin\,(x-y) = sin\,x\,cos\,y - cos\,x\,sin\,y$$
$$cos\,(x+y) = cos\,x\,cos\,y - sin\,x\,sin\,y$$
$$cos\,(x-y) = cos\,x\,cos\,y + sin\,x\,sin\,y$$

$$tan\,(x+y) = \frac{tan\,x + tan\,y}{1 - tan\,x\,tan\,y}$$
$$tan\,(x-y) = \frac{tan\,x - tan\,y}{1 + tan\,x\,tan\,y}$$

$$sin\,2x = 2\,sin\,x\,cos\,x$$
$$sin\,2x = \frac{2tan\,x}{1+tan^2 x}$$

$$cos\,2x = cos^2x - sin^2x$$
$$cos\,2x = 1 - 2sin^2x$$
$$cos\,2x = 2cos^2x - 1$$
$$cos\,2x = \frac{1-tan^2 x}{1+tan^2 x}$$

$$tan\,2x = \frac{2tan\,x}{1-tan^2 x}$$
$$sin\,3x = 3\,sin\,x - 4sin^3\,x$$
$$cos\,3x = 4cos^3\,x - 3\,cos\,x$$
$$tan\,3x = \frac{3tan\,x - tan^3\,x}{1-3tan^2 x}$$

$$sin\,(A+B) + sin\,(A-B) = 2sin\,A\,cos\,B$$
$$sin\,(A+B) - sin\,(A-B) = 2cos\,A\,sin\,B$$
$$cos\,(A+B) + cos\,(A-B) = 2cos\,A\,cos\,B$$
$$cos\,(A+B) - cos\,(A-B) = -2sin\,A\,sin\,B$$

$$sin\,C + sin\,D = 2\,sin\left(\frac{C+D}{2}\right)\,cos\left(\frac{C-D}{2}\right)$$
$$sin\,C - sin\,D = 2\,cos\left(\frac{C+D}{2}\right)\,sin\left(\frac{C-D}{2}\right)$$
$$cos\,C + cos\,D = 2\,cos\left(\frac{C+D}{2}\right)\,cos\left(\frac{C-D}{2}\right)$$
$$cos\,C - cos\,D = -\,2\,sin\left(\frac{C+D}{2}\right)\,sin\left(\frac{C-D}{2}\right)$$

1. $\cos x = -\frac{3}{5}$, x lies in the third quadrant. Find the values of other five trigonometric functions.

Ans :

$\sin x = \frac{-4}{5}$ $\tan x = \frac{4}{3}$

$\text{cosec}\, x = \frac{-5}{4}$ $\cot x = \frac{3}{4}$

$\sec x = \frac{-5}{3}$

2. (a) $\sin 765^0 =. ....$

(b) Prove that: $\frac{Cos7x+Cos5x}{Sin7x-Sin5x} = cotx$

Ans :

(a) $\sin 765$

$= 1 - 2sin^2(2x)$
$= 1 - 2(2\sin x \cos x)^2$
$= 1 - 8\sin^2 x\, cos^2 x$

7. **(a)** $\tan\frac{\pi}{4} = \ldots\ldots\ldots$

(b) Prove that $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}$

Ans :

(a) $\tan\frac{\pi}{4} = 1$

(b) $\frac{\sin(x+y)}{\sin(x-y)}$

$= \frac{\sin x\cos y+\cos x\sin y}{\sin x\cos y-\cos x\sin y}$

Dividing numerator and denominator by $\cos x \cos y$

$= \frac{\tan x+\tan y}{\tan x-\tan y}$

8. **(a)** $\cos(x+y) + \cos(x-y) = \ldots$

(b) Prove that

$$\cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right) = \sqrt{2}\cos x$$

Ans :

(a) $\cos(x+y) + \cos(x-y) = 2\cos x\cos y$

(b) $\cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right)$

$= 2\cos\frac{\pi}{4}\,\cos x$

$= 2 \times \frac{1}{\sqrt{2}}\cos x$

$= \sqrt{2}\cos x$

9. **(a)** $1 + tan^{\,2}x = \ldots\ldots.$

(b) Prove that $sin\,3x = 3\,sin\,x - 4sin^3\,x$

Ans :

(a) $1 + tan^{\,2}x = sec^2x$

(b) $\sin 3x = \sin(2x + x)$

$= sin\,2x\,cos\,x + cos\,2x\,sin\,x$
$= 2\sin x\cos x\cos x + (1 - 2sin^2x)\sin x$
$= 2\sin x\cos^2 x + \sin x - 2sin^3x$
$= 2\sin x\,(1 - sin^2x) + \sin x - 2sin^3x$
$= 2\sin x - 2sin^3x + \sin x - 2sin^3x$
$= 3\,sin\,x - 4sin^3\,x$

10. **(a)** $sin(-x) = \ldots\ldots.$

(i) $sin\,x$ (ii) $-sin\,x$

(iii) $cos\,x$ (iv) $-cos\,x$

(b) Prove that

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+tan\,x}{1-tan\,x}\right)^2$$

Ans :

(a) (ii) or $sin\,(-x) = -sin\,x$

(b) $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \frac{\left(\frac{1+tan\,x}{1-tan\,x}\right)}{\left(\frac{1-tan\,x}{1+tan\,x}\right)}$

$= \left(\frac{1+tan\,x}{1-tan\,x}\right) \times \left(\frac{1+tan\,x}{1-tan\,x}\right)$

$= \left(\frac{1+tan\,x}{1-tan\,x}\right)^2$

PRACTICE PROBLEMS

1. Find the value of $\frac{\sin x+\cos x}{\sin x-\cos x}$ if $\tan x = \frac{3}{4}$, x lies in the 1st quadrant
2. Prove that $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x} = \tan 4x$
3. Prove that $sin^2\frac{\pi}{6} + cos^2\frac{\pi}{3} - tan^2\frac{\pi}{4} = -\frac{1}{2}$
4. Find the value of tan 15°
5. Prove that $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4cos^2\left(\frac{x+y}{2}\right)$
6. Prove that $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)} = cot^2x$

4

PRINCIPLE OF MATHEMATICAL INDUCTION

KEY NOTES

Working Rule

- Denote the given statement by $P(n)$ and show that $P(1)$ is true
- Assume that $P(k)$ is true for some natural number k
- Show that $P(k+1)$ is true , whenever $P(k)$ is true
- Hence, by the principle of Mathematical induction, $P(n)$ is true for all natural number n.

QUESTIONS AND ANSWERS

1. Consider the statement

$$P(n): 1+3+3^2+3^3+\cdots+3^{n-1} = \frac{3^n-1}{2}$$

(a) Show that $P(1)$ is true

(b) Prove by principle of mathematical induction, that $P(n)$ is true for all $n \in N$

Ans :

$$P(n): 1+3+3^2+\cdots.+3^{n-1} = \frac{3^n-1}{2}$$

(a) When $n=1$

$$P(1): \ 1 = \frac{3^1-1}{2} = \frac{2}{2} = 1$$

$\Rightarrow p(1)$ is true

(b) Assume that P (k) is true

$$P(k): 1+3+3^2+\cdots+3^{k-1} = \frac{3^k-1}{2}$$

Now,

$$p(k+1): 1+3+3^2+\cdots+3^{k-1}+3^k$$

$$= \frac{3^k-1}{2}+3^k$$

$$= \frac{3^k-1+2\times3^k}{2} = \frac{3\times3^k-1}{2}$$

$$= \frac{3^{k+1}-1}{2}$$

$\Rightarrow p(k+1)$ is true

$\therefore P(n)$ is true for all $n \in N$

2. A statement p (n) for a natural number n is given by

$$P(n): \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}\dots\dots\dots+\frac{1}{2^n} = 1-\frac{1}{2^n}$$

(a) Prove that P(1) is true.

(b) verify by the method of mathematical induction $P(n)$ is true for all $n \in N$.

Ans : Let $P(n): \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}\dots\dots\dots+\frac{1}{2^n} = 1-\frac{1}{2^n}$

(a) For $n=1$,

$$P(1): \frac{1}{2} = 1-\frac{1}{2^1} = 1-\frac{1}{2} = \frac{1}{2}$$

$\therefore p(1)$ is true

(b) Assume that $p(k)$ is true for some natural number k.

$$p(k): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^k}$$

$$= 1-\frac{1}{2^k}$$

Now,

$$p(k+1): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^k}+\frac{1}{2^{k+1}}$$

$$= 1-\frac{1}{2^k}+\frac{1}{2^{k+1}}$$

$$= 1-\frac{1}{2^k}+\frac{1}{2\times2^k}$$

$= 1 - \frac{1}{2^k}\left(1 - \frac{1}{2}\right)$

$= 1 - \frac{1}{2^k} \times \frac{1}{2} = 1 - \frac{1}{2^{k+1}}$

$\Rightarrow$ p(k + 1) is true

$\therefore P(n)$ is true for all natural numbers

3. Prove by principle of mathematical induction that

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

Ans : Let

$P(n): 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$

When $n = 1$,

$P(1) : 1 = \frac{1(1+1)}{2}$

$= \frac{1(2)}{2}$

$= 1$

$\Rightarrow$ P(1) is true

Assume that $P(k)$ is true

$P(k): 1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$

Now,

$P(k + 1): 1 + 2 + 3 + \cdots \ldots \ldots + k + (k + 1)$

$= \frac{k(k+1)}{2} + (k + 1)$

$= (k + 1)\left(\frac{k}{2} + 1\right)$

$= \frac{(k+1)(k+1+1)}{2}$

$\Rightarrow$ P$(k + 1)$ is true

$\therefore P(n)$ is true for all natural numbers.

4. Given that

$P(n) = 1^3 + 2^3 + 3^3 + \cdots . + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

(a) Check whether $P(1)$ is true.

(b) If $P(k)$ is true, prove that $P(k + 1)$ is also true.

(c) Is the statement $P(n)$ is true for all $n \in N$? Justify your answer

Ans :

(a) $P(n): 1^3 + 2^3 + 3^3 + \cdots . + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$P(1) : 1^3 = \left[\frac{1(1+1)}{2}\right]^2$

$1 = \left[\frac{1(2)}{2}\right]^2 = 1$

$\Rightarrow$ p(1) is true

(b) Assume that $P(k)$ is true

$P(k): 1^3 + 2^3 + 3^3 + \cdots . + k^3 = \left[\frac{k(k+1)}{2}\right]^2$

Now,

$P(k + 1): 1^3 + 2^3 + 3^3 + \cdots . + k^3 + (k + 1)^3$

$= \left[\frac{k(k+1)}{2}\right]^2 + (k + 1)^3$

$= (k + 1)^2\left[\frac{k^2}{4} + (k + 1)\right]$

$= (k + 1)^2\left[\frac{k^2+4k+4}{4}\right]$

$= \frac{(k+1)^2(k+2)^2}{4}$

$= \left[\frac{(k+1)(k+2)}{2}\right]^2$

$\Rightarrow$ P$(k + 1)$ is true

(c) Yes, By the principle of mathematical induction, $P(n)$ is true for all $n \in N$.

5. Prove by the principle of mathematical induction that

$$2 + 2^2 + 2^3 + \cdots . + 2^n = 2(2^n - 1)$$

Ans :

Let

$P(n): 2 + 2^2 + 2^3 + \cdots . + 2^n$
$= 2(2^n - 1)$

For $n = 1$,

$P(1):\ 2 = 2(2^1 - 1)$
$= 2(2 - 1)$
$= 2$
$\Rightarrow$ p(1) is true

(b) Assume that $P(k)$ is true

$P(k): 2 + 2^2 + 2^3 + \cdots . + 2^k$
$= 2(2^k - 1)$

Now,

$P(k+1): 2 + 2^2 + 2^3 + \cdots . + 2^k + 2^{k+1}$
$= 2(2^k - 1) + 2^{k+1}$
$= 2(2^k - 1) + 2^k \cdot 2$
$= 2[2^k - 1 + 2^k]$
$= 2[2 \cdot 2^k - 1]$
$= 2[2^{k+1} - 1]$
$\Rightarrow$ p(k + 1) is true

Hence, given $P(n)$ is true for all $n \in N$

6. Prove that $1.2 + 2.3 + 3.4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

Ans :

Let

$P(n): 1.2 + 2.3 + 3.4 + \cdots + n(n+1)$
$= \frac{n(n+1)(n+2)}{3}$

For $n = 1$,

$P(1): 2 = \frac{1(1+1)(1+2)}{3}$
$= \frac{1\times2\times3}{3}$
$= 2$
$\Rightarrow$ p(1) is true

Assume that $P(k)$ is true

$P(k): 1.2 + 2.3 + 3.4 + \cdots + k(k+1)$
$= \frac{k(k+1)(k+2)}{3}$

Now,

$P(k+1): 1.2 + 2.3 + 3.4 + \cdots + k(k+1) + (k+1)(k+2)$
$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$
$= (k+1)(k+2)\left[\frac{k}{3} + 1\right]$
$= \frac{(k+1)(k+2)(k+3)}{3}$
$\Rightarrow$ p(k + 1) is true

Hence, given $P(n)$ is true for all $n \in N$

PRACTICE PROBLEMS

1. Prove by Principle of mathematical induction that
$1 + 3 + 5 + \cdots \ldots . + (2n - 1) = n^2$

2. Given
$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

(a) Show that $P(1)$ is true.

(b) Prove by principle of mathematical induction, that P(n) is true for all $n \in N$

3. Prove by the principle of mathematical induction that
$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(r^n - 1)}{r-1}$

4. Prove that $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{(n+1)}$ by PMI.

5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY NOTES

- Complex number :A number of the form $a+ib, a, b \in R, i=\sqrt{-1}$
- If $z=a+ib$, then $Re(z)=a, Im(z)=b$
- If $z=a+ib$, then modulus of z , $|z|=\sqrt{a^2+b^2}$ and conjugate of z, $\bar{z}=a-ib$
- $z_1=a+ib$ and $z_2=c+id$ then ,
  - $z_1+z_2=(a+c)+i(b+d)$
  - $z_1-z_2=(a-c)+i(b-d)$
  - $z_1z_2=(ac-bd)+i(ad+bc)$

    OR

    Multiply term by term and write in the form of $a+ib$
  - $\frac{z_1}{z_2}=\frac{a+ib}{c+id}, z_2 \neq 0$

    To find this , multiply numerator and denominator by the conjugate of denominator , then simplify and write in the form of $a+ib$
- Multiplicative inverse of z $(z \neq 0)$ is $\frac{1}{z}$
- $z\bar{z}=|z|^2$

  OR

  $\frac{1}{z}=\frac{\bar{z}}{|z|^2}$
- $i^{4k}=1$, $i^{4k+1}=i$, $i^{4k+2}=-1$, $i^{4k+3}=-i$,$k \in Z$
- $(z_1 \pm z_2)^2=z_1^2 \pm 2z_1z_2+z_2^2$
- $(z_1 \pm z_2)^3=z_1^3 \pm 3z_1^2z_2+3z_1z_2^2 \pm z_2^3$
- $z_1^2-z_2^2=(z_1+z_2)(z_1-z_2)$
- $|z_1z_2|=|z_1||z_2|$
- $\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}$, $|z_2| \neq 0$
- $\overline{z_1z_2}=\bar{z}_1\bar{z}_2$
- $\overline{z_1 \pm z_2}=\bar{z}_1 \pm \bar{z}_2$
- $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$

QUESTIONS AND ANSWERS

1. Express the following in the form of $a + ib$

(a) $i^9 + i^{19}$

Ans :

$$i^9 + i^{19} = i^{8+1} + i^{16+3}$$
$$= i^{4\times 2+1} + i^{4\times 4+3}$$
$$= i^1 + i^3$$
$$= i - i$$
$$= 0$$

(b) i^{-35}

$$i^{-35} = i^{-36+1}$$
$$= i^{4\times -9+1}$$
$$= i^1$$
$$= i$$
$$= 0 + i$$

(c) $3(7 + i7) + i(7 + i7)$

$$3(7 + i7) + i(7 + i7) \Rightarrow$$
$$= 21 + 21i + 7i + 7i^2$$
$$= 21 + 28i + 7 \times -1$$
$$= 21 - 7 + 28i$$
$$= 14 + 28i$$

(d) $z = \frac{5+i}{2+3i}$

$$z = \frac{5+i}{2+3i}$$
$$= \frac{5+i}{2+3i} \times \frac{2-3i}{2-3i}$$
$$= \frac{10-15i+2i-3i^2}{2^2-(3i)^2}$$
$$= \frac{10-13i+3}{4-9\times -1}$$
$$= \frac{13-13i}{13}$$
$$= 1 - i$$

(e) $(1 - i)^2$

$$(1 - i)^2 = 1^2 - 2i + i^2$$
$$= 1 - 2i - 1$$
$$= -2i$$
$$= 0 - 2i$$

(f) $(1 + 2i)(3 - i)$

$$= 3 - i + 6i - 2i^2$$
$$= 3 + 5i - 2 \times -1$$
$$= 5 + 5i$$

2. (a) $(a + ib)(a - ib)$ = …….

(b) Find the multiplicative inverse of $z = 3 + 4i$

Ans :

(a) $a^2 + b^2$

(b) Multiplicative inverse of z,

$$\frac{1}{z} = \frac{1}{(3+4i)} = \frac{3-4i}{(3+4i)(3-4i)}$$
$$= \frac{3-4i}{9-(4i)^2}$$
$$= \frac{3-4i}{9+16}$$
$$= \frac{3}{25} - \frac{4}{25}i$$

OR

$$\frac{1}{z} = \frac{\bar{z}}{|z|^2}$$
$$= \frac{3-4i}{3^2+4^2}$$
$$= \frac{3-4i}{25}$$
$$= \frac{3}{25} - \frac{4}{25}i$$

3. (a) Express $(1 - i)^4$ in the form of $a + ib$

(b) Find the conjugate of $\frac{1+i}{1-i}$

Ans :

(a) $(1 - i)^4 = ((1 - i)^2)^2$

$$= (1 - 2i + i^2)^2$$
$$= (-2i)^2$$
$$= 4i^2$$
$$= -4$$
$$= -4 + i0$$

(b) $z = \frac{1+i}{1-i}$

$$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$
$$= \frac{1+2i+i^2}{1^2-i^2}$$

$= \frac{1+2i-1}{1-(-1)}$

$= \frac{2i}{2}$

$= i$

$\bar{z} = 0 - i$

OR

$\bar{z} = \frac{1-i}{1+i}$

$= \frac{1-i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{1-2i+i^2}{1^2-i^2}$

$= \frac{1-2i-1}{1-(-1)}$

$= \frac{-2i}{2}$

$= -i$

$\bar{z} = 0 - i$

4. (a) Express $z = \frac{1+2i}{1-3i}$ in the form of $a + ib$

   (b) Hence find $|z|$

Ans :

(a) $z = \frac{1+2i}{1-3i}$

$= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$

$= \frac{1+3i+2i+6i^2}{1-9i^2}$

$= \frac{1+5i-6}{1+9}$

$= \frac{-5+5i}{10}$

$= -\frac{5}{10} + \frac{5}{10}i$

$= -\frac{1}{2} + \frac{1}{2}i$

(b) $|z| = \left|-\frac{1}{2} + \frac{1}{2}i\right|$

$= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{1}{4}}$

$= \frac{1}{\sqrt{2}}$

5. (a) Express $(1+2i)^3$ in the form of $a + ib$

   (b) If $x + iy = \frac{a+ib}{a-ib}$, Prove that $x^2 + y^2 = 1$

Ans :

(a) $z = (1+2i)^3$

$= 1^3 + 3 \times 1^2 \times 2i + 3 \times 1 \times (2i)^2 + (2i)^3$

$= 1 + 6i - 12 + 8 \times -i$

$= -11 - 2i$

(b) $x + iy = \frac{a+ib}{a-ib}$

$\Longrightarrow |x+iy| = \frac{|a+ib|}{|a-ib|}$

$\sqrt{x^2+y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}} = 1$

$\therefore x^2 + y^2 = 1$

OR

$x + iy = \frac{a+ib}{a-ib}$, $x - iy = \frac{a-ib}{a+ib}$

$x^2 + y^2 = (x+iy)(x-iy)$

$= \frac{a+ib}{a-ib} \times \frac{a-ib}{a+ib}$

$= 1$

6. $|3+i| = \ldots..$

   (i) $\sqrt{8}$ (ii) $\sqrt{10}$

   (iii) $\sqrt{12}$ (iv) $\sqrt{17}$

Ans : (ii) or $\sqrt{10}$

[$|3+i| = \sqrt{3^2+1^2} = \sqrt{10}$]

PRACTICE PROBLEMS

1. Express in the form of $a + ib$
   (a) $\frac{2-i}{(1-i)(1+2i)}$
   (b) $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$
   (c) $\frac{5+i}{2+3i}$
   (d) $\left(\frac{1}{3} + 3i\right)^3$
2. Write the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$
3. Find the multiplicative inverse of
   (a) $4 - 3i$
   (b) $\sqrt{5} + 3i$
   (c) $-i$
4. Write the roots of $x^2 + x + 1 = 0$, in $a + ib$ form
5. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

LINEAR INEQUALITIES

6

KEY NOTES

- Two real numbers or two algebraic expressions related by the symbol $<, >, \leq, \geq, \neq$ form an inequality.
- An inequality is said to be linear, if the variable(s) occur in first degree only and there is no terms involving the product of the variables.
  Eg : $ax + b \leq 0$, $ax + by + c > 10$, $ax \leq 4$
- A linear inequality which have only two variables is called linear inequality in two variables.
  Eg: $3x + 11y \leq 0, 4y + 2x > 0$
- The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.

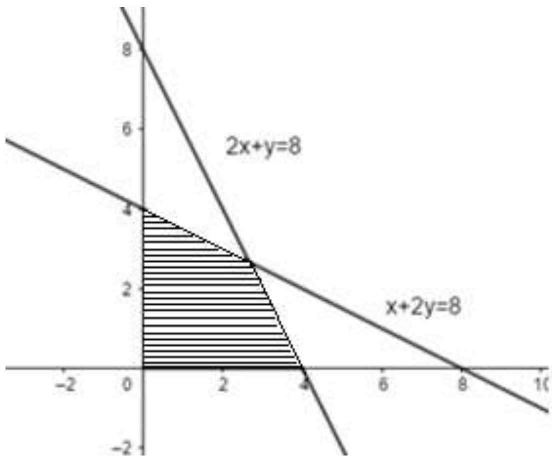
QUESTIONS AND ANSWERS

1. Solve the following system of linear inequalities graphically.
   $x + 2y \leq 8$
   $2x + y \leq 8$
   $x \geq 0, y \geq 0$

Ans :

$x + 2y = 8$		
x	8	0
y	0	4

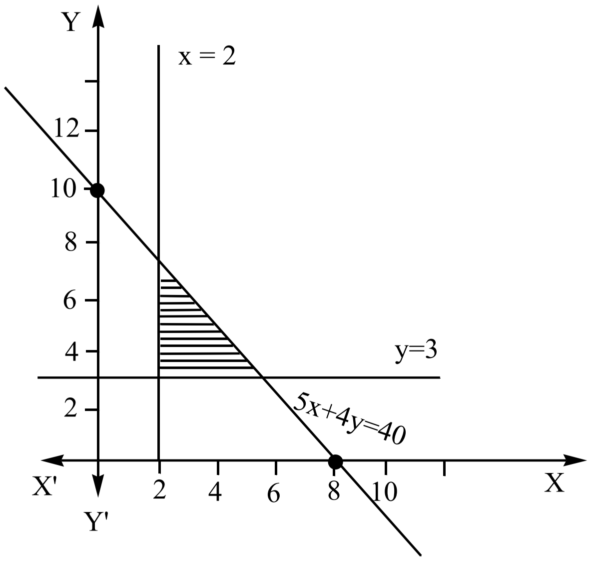
$2x + y = 8$		
x	4	0
y	0	8



The shaded region in the figure gives the solution of the system of linear inequalities.

2. Solve the following system of inequalities graphically.
   $5x + 4y \leq 40$
   $x \geq 2$, $y \geq 3$

$5x + 4y = 40$		
x	8	0
y	0	10



3. Solve the following system of inequalities graphically

$4x + 3y \leq 60$

$y \geq 2x$

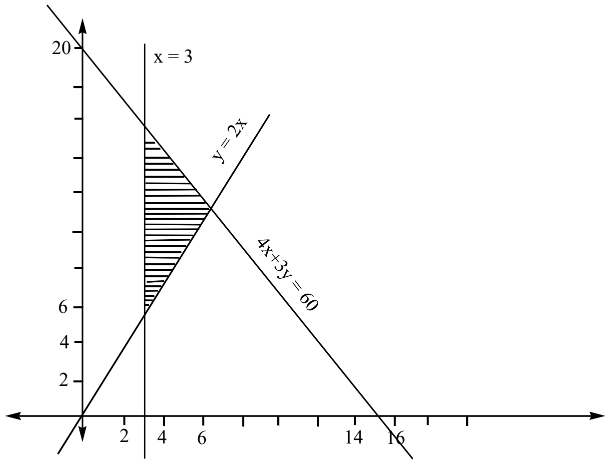
$x \geq 3$

$x, y \geq 0$

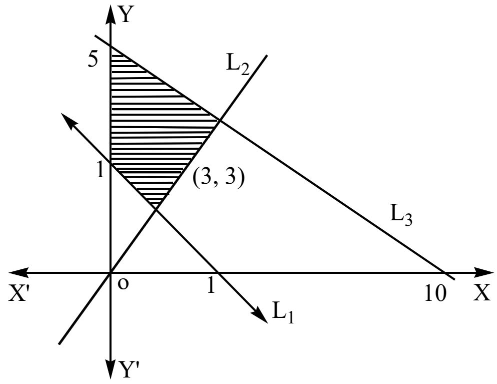
Ans :

$4x + 3y = 60$		
x	0	15
y	20	0

$y = 2x$			
x	0	1	2
y	0	2	4



4. Shaded region in the graph shows solution of a system of linear inequalities. Find the inequalities.



Ans:

L_1 passes through (0,1) and (1, 0)

Equation of L_1 is,

$\frac{x}{1} + \frac{y}{1} = 1$ (intercept form)

$\Rightarrow x + y = 1$

L_2 passes through (0,0) and (3, 3)

Equation of L_2 is,

$\Rightarrow \quad y = x$

L_3 passes through (0,5) and (10, 0)

Equation of L_3 is,

$\frac{x}{10} + \frac{y}{5} = 1$

$\Rightarrow 5x + 10y = 50$

$\Rightarrow x + 2y = 10$

$\therefore$ the inequalities are

$x + y \geq 1$

$y \geq x$

$x + 2y \leq 10$

$x, y \geq 0$

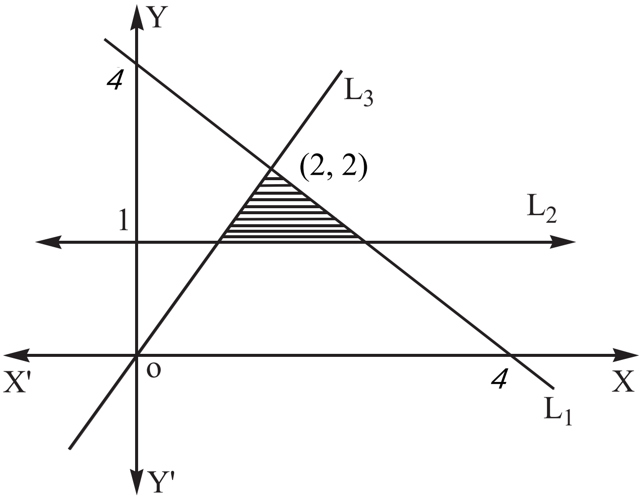
PRACTICE PROBLEMS

1. Solve the inequalities graphically
   $2x + 3y \leq 12$
   $x \geq 1, y \geq 2$

2. Solve graphically
   $2x + y \geq 4$
   $x + y \leq 3$
   $2x - 3y \leq 6$
   $x \geq 0, y \geq 0$

3. Solve the following inequalities graphically
   $x + y \leq 9$
   $y > x$
   $x \geq 0$

4. The graphical solution of a system of linear inequalities is shown in the figure.



(a) Find the equation of the lines L_1, L_2, L_3
(b) Find the inequalities representing the solution region.

7

PERMUTATIONS AND COMBINATIONS

KEY NOTES

Fundamental principle of counting

If an event can occur in $\boldsymbol{m}$ different ways, following which another event can occur in $\boldsymbol{n}$ different ways, then both together can be happened in $m \times n$ ways

If an event can occur in $\boldsymbol{m}$ different ways, following which another event can occur in $\boldsymbol{n}$ different ways, following which a third event can occur in $\boldsymbol{p}$ different ways, then the total number of occurrence of the events in the given order is $\boldsymbol{m} \times \boldsymbol{n} \times \boldsymbol{p}$.

Factorial Notation

The notation $n!$ represents the product of first n natural numbers,

ie, $1 \times 2 \times 3 \times \ldots \times (n-1) \times n = n!$

We read this symbol as 'n factorial'.

$$0! = 1$$

$$n! = n(n-1)!$$

$$= n(n-1)(n-2)! \text{ [Provided } (n \geq 2)]$$

$$= n(n-1)(n-2)(n-3)! \text{ [Provided } (n \geq 3)]$$

Combinations:

Combination is selection of a number things taken all or some at time.

The number of combinations of $'n'$ objects taken 'r' at a time is denoted and defined as

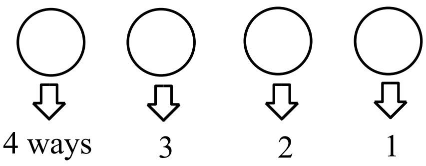
$$^nC_r = \frac{n!}{r!(n-r)!}, 0 \leq r \leq n$$

- $^nC_0 = 1$
- $^nC_n = 1$
- $^nC_1 = n$
- $^nC_r = {^nC_{n-r}}$
- $^nC_a = {^nC_b} \Longrightarrow a + b = n$
- $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

QUESTIONS AND ANSWERS

1. Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is not allowed.

Ans :



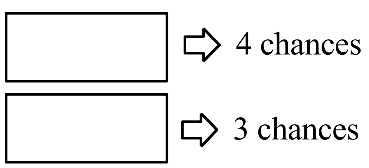
Total Words(without repetition)

$= 4 \times 3 \times 2 \times 1$

$= 24$

2. Given 4 flags of different colors, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

Ans :

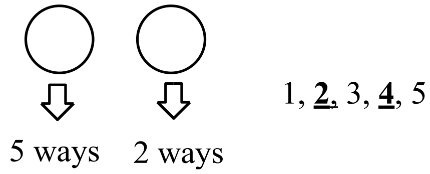


Total number of Signals $= 4 \times 3$

$= 12$

3. How many 2 digit even numbers can be formed from the digits 1, 2, 3, 4, 5 if the digits can be repeated?

Ans :



Total 2 digit numbers $= 5 \times 2$

$= 10$

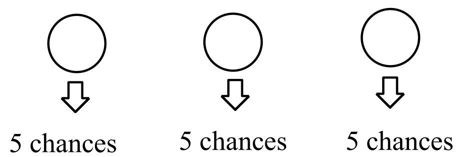
4. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that

(a) repetition of the digits is allowed?

(b) repetition of the digits is not allowed?

Ans :

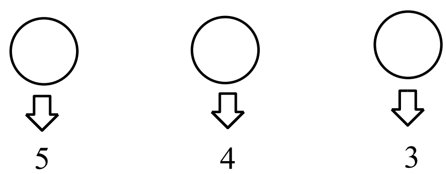
(a) 1, 2, 3, 4, 5



Total 3 digit numbers $= 5 \times 5 \times 5$

$= 125$

(b)



Total 3 digit numbers $= 5 \times 4 \times 3$

$= 60$

5. How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once?

Ans :

$0 \rightarrow 9$, total 10 digits

6	7	◯	◯	◯
⇩	⇩	⇩	⇩	⇩
1 way	1 way	8	7	6

Total numbers starting with 67

$= 1 \times 1 \times 8 \times 7 \times 6$

$= 336$

6. (a) If ${}^{n}C_8 = {}^{n}C_9$, find ${}^{n}C_{17}$

(b) In how many ways can 3 boys and 2 girls be selected from 5 boys and 6 girls.

Ans :

(a) ${}^{n}C_8 = {}^{n}C_9 \Longrightarrow n = 8 + 9 = 17$

${}^{n}C_{17} = {}^{17}C_{17} = 1$

(b) 3 boys from 5 boys, selection

$= {}^{5}C_3$

2 girls from 6 girls ,

Selection $= {}^{6}C_2$

Total Selection of 3 boys and 2

girls $= {}^{5}C_3 \times {}^{6}C_2$

$= \frac{5\times4\times3}{1\times2\times3} \times \frac{6\times5}{1\times2}$

$= 150$

7.

(a) Find n if ${}^{2n}C_3 : {}^{n}C_3 = 12 : 1$

(b) How many chords can be drawn through 21 points on a circle?

Ans :

$$\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$$

$$\frac{2n(2n-1)(2n-2)/_{1\times2\times3}}{n(n-1)(n-2)/_{1\times2\times3}} = \frac{12}{1}$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{12}{1}$$

$$\frac{4(2n-1)}{n-2} = \frac{12}{1}$$

$$8n - 4 = 12n - 24$$

$$4n = 20$$

$$n = 5$$

(b) The number of chords $= {}^{21}C_2$

$= \frac{21\times20}{1\times2} = 210$

8. **(a)** Find the value of 'r' such that ${}^{10}C_r$ is maximum

(b) What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these

(i) four cards are of the same suit,

(ii) four cards belong to four different suits,

(iii) are face cards,

(iv) two are red cards and two are black cards

Ans :

(a) $r = 5$

(b) Total selections $= {}^{52}C_4$

(i) 4 Diamonds or 4 Hearts or 4 spades or 4 clubs

Total selection $= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times {}^{13}C_4$

$= 2860$

(ii) 1 Diamond and 1 Heart and 1 spade and 1 club

Total selection $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$= \left({}^{13}C_1\right)^4$

$= 13^4$

(iii) Total face cards $= 12$

$\therefore$ Total selection of 4 face cards $= {}^{12}C_4$

$= 495$

(iv) Total Red $= 26$, Total Black $= 26$

$\therefore$ 2 Red and 2 Black , total selection $= {}^{26}C_2 \times {}^{26}C_2$

$= 105625$

9. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Ans :

1 Ace and 4 Non Ace, total selection

$$= {}^{4}C_1 \times {}^{48}C_4$$
$$= 778320$$

10. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Ans :

Black: 5 , Red: 6

2 Black and 3 Red, Total selection

$$= {}^{5}C_2 \times {}^{6}C_3$$
$$= 10 \times 20$$
$$= 200$$

PRACTICE PROBLEMS

1. (a) How many 3-digit numbers are there with no digit repeated?
   (b) In how many ways can 3 vowels and 2 consonants be selected from the letters of the word INVOLUTE

2. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each color.

3. A group consists of 4 girls and 7 boys. In how many ways can of 5 members be selected if the team has team
   (i) no girl?
   (ii) at least one boy and one girl?
   (iii) at least 3 girls?

BINOMIAL THEOREM

8

KEY NOTES

Binomial theorem for any positive integer n

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \cdots + {}^nC_n b^n$$

- There are $(n+1)$ terms in the expansion of $(a+b)^n$

QUESTIONS AND ANSWERS

1. (a) Number of terms in the expansion of $(x+y)^{10}$ is ....

(b) Expand $\left(x+\frac{1}{x}\right)^6$ using Binomial theorem.

Ans :

(a) Number of terms $= 11$

(b) $\left(x+\frac{1}{x}\right)^6$

$$= {}^6C_0 x^6 + {}^6C_1 x^5\left(\frac{1}{x}\right) + {}^6C_2 x^4\left(\frac{1}{x}\right)^2 + {}^6C_3 x^3.\left(\frac{1}{x}\right)^3 + {}^6C_4 x^2.\left(\frac{1}{x}\right)^4 + {}^6C_5 x.\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

2. Write the expansion of $\left(x^2+\frac{3}{x}\right)^4$

Ans :

$$\left(x^2+\frac{3}{x}\right)^4 = {}^4C_0 (x^2)^4 + {}^4C_1 (x^2)^3\left(\frac{3}{x}\right) + {}^4C_2 (x^2)^2\left(\frac{3}{x}\right)^2 + {}^4C_3 x^2\left(\frac{3}{x}\right)^3 + {}^4C_4\left(\frac{3}{x}\right)^4$$

$$= x^8 + 4x^6 \times \frac{3}{x} + 6x^4\frac{9}{x^2} + 4x^2\frac{27}{x^3} + \frac{81}{x^4}$$

$$= x^8 + 12x^5 + 54x^2 + \frac{108}{x} + \frac{81}{x^4}$$

3. (a) Find $(a+b)^4 - (a-b)^4$

(b) Hence evaluate : $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$

Ans :

(a)

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ …….. (1)

$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$ …….. (2)

(1) – (2) $\Rightarrow$

$$(a+b)^4 - (a-b)^4 = 8a^3b + 8ab^3$$

$$= 8ab(a^2+b^2)$$

(b) $(a+b)^4 - (a-b)^4 = 8ab(a^2+b^2)$

Put $= \sqrt{3}$, $b = \sqrt{2}$

$$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8 \times \sqrt{3} \times \sqrt{2}\left(\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right)$$

$$= 8\sqrt{6}\,(3+2)$$

$$= 40\sqrt{6}$$

4. Using binomial theorem, Prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

Ans :

We have

$$(1+a)^n = {}^nC_0 + {}^nC_1a + {}^nC_2a^2 + \cdots + {}^nC_na^n$$

For $a = 5$, we get

$$(1+5)^n = {}^nC_0 + {}^nC_15 + {}^nC_25^2 + \cdots + {}^nC_n5^n$$

i.e., $6^n = 1 + 5n + 5^2.\,{}^nC_2 + 5^3.\,{}^nC_3 + \cdots + 5^n$

i.e., $6^n - 5n = 1 + 5^2({}^nC_2 + 5.\,{}^nC_3 + \cdots + 5^{n-2})$

or $6^n - 5n = 1 + 25({}^nC_2 + 5.\,{}^nC_3 + \cdots + 5^{n-2})$

or $6^n - 5n = 25k + 1$, where $k = {}^nC_2 + 5.\,{}^nC_3 + \cdots + 5^{n-2}$

This shows that when divided by 25, $6^n - 5n$ leaves remainder 1

5. **(a)** Which is larger $(1.1)^{10000}$ or 1000 ?

(b) Evaluate $(101)^4$ using Binomial theorem

Ans :

(a) $(1.1)^{10000} = (1+0.1)^{10000}$

$$= {}^{10000}C_0 + {}^{10000}C_1 \times 0.1 + (\text{Positive terms})$$

$= 1 + 10000 \times 0.1 + (\text{Positive terms})$

$= 1001 + (\text{positive terms})$

i.e., $(1.1)^{10000} > 1000$

(b) $(101)^4 = (100 + 1)^4$

$= (100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + 1$

$= 100000000 + 4000000 + 60000 + 400 + 1$

$= 104060401$

6. (a) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = \cdots$

(b) Expand $(x + 2)^6$

Ans :

(a) 2^n

(b) $(x + 2)^6 = {}^6C_0x^6 + {}^6C_1x^5.2 + {}^6C_2x^4.2^2 + {}^6C_3x^3.2^3 + {}^6C_4x^2.2^4 + {}^6C_5x.2^5 + {}^6C_6.2^6$

$= x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

PRACTICE PROBLEMS

1. Expand $(1 - 2x)^5$
2. Expand $\left(\frac{2}{x} - \frac{x}{2}\right)^5$ using binomial theorem
3. Find $(x + 1)^6 + (x - 1)^6$. Hence or otherwise evaluate $\left(\sqrt{2} + 1\right)^6 + \left(\sqrt{2} - 1\right)^6$
4. Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.
5. Which is larger $(1.01)^{1000000}$ or 10,000 ?
6. Evaluate $(102)^5$ using binomial theorem

9

SEQUENCES AND SERIES

KEY NOTES

$$a^m \times a^n = a^{m+n}, \frac{a^m}{a^n} = a^{m-n}, (a^m)^n = a^{mn}$$

Geometric progression (G.P)

A sequence $a_1, a_2, a_3, \ldots$ is called a geometric progression if $\frac{a_{k+1}}{a_k} = r$, a constant.

i.e., $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots = r$, r is called common ratio.

Eg : (a) 2, 6, 18, 54, ………; $r = 3$

(b) $1, -5, 25, -125, \ldots\ldots$; $r = -5$

(c) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots$; $r = \frac{1}{2}$

A geometric progression with first term a and common ratio r is given by $a, ar, ar^2, \ldots$.
If a, b, c are in G.P, then $b^2 = ac$

Important formulae

n^{th} term of a G.P is $a_n = ar^{n-1}$

Sum of n terms of a G.P is $S_n = \frac{a(r^n-1)}{r-1}$ if $r > 1$

$S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$

Geometric Mean (GM)

The geometric mean between two positive numbers a and b is $\sqrt{ab}$

QUESTIONS AND ANSWERS

1. Find the 20th and n^{th} terms of the G.P is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$

Ans :

Here, $a = \frac{5}{2}$, $r = \frac{5/4}{5/2} = \frac{1}{2}$

$a_{20} = ar^{19} = \frac{5}{2}.\left(\frac{1}{2}\right)^{19} = \frac{5}{2^{20}}$

$a_n = ar^{n-1} = \frac{5}{2}.\left(\frac{1}{2}\right)^{n-1} = \frac{5}{2^n}$

2. Find the 12th term of a G.P whose 8th term is 192 and common ratio is 2

Ans :

$a_8 = ar^7 = 192$

$a_{12} = ar^{11} = ar^7 \times r^4$

$= 192 \times 2^4$

$= 192 \times 16$

$= 3072$

3. How many terms of the G.P $3, \frac{3}{2}, \frac{3}{4}, \ldots$ are needed to give the sum $\frac{3069}{512}$?

Ans :

Let n be the number of terms.

Given that $a = 3, r = \frac{1}{2}, S_n = \frac{3069}{512}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\frac{3069}{512} = \frac{3(1-\frac{1}{2^n})}{1-\frac{1}{2}}$$

$$\frac{3069}{512} = \frac{3(1-\frac{1}{2^n})}{\frac{1}{2}}$$

$$\frac{3069}{512} = 6\left(1-\frac{1}{2^n}\right)$$

$$\left(1-\frac{1}{2^n}\right) = \frac{3069}{512\times 6} = \frac{3069}{3072}$$

$$\frac{1}{2^n} = 1 - \frac{3069}{3072} = \frac{3}{3072} = \frac{1}{1024}$$

$$\Longrightarrow 2^n = 1024 = 2^{10}$$

$$\Longrightarrow n = 10$$

4. The 5th , 8th and 11th terms of a G.P are p, q and s respectively. Show that p, q and s are in G.P.

Ans :

Let a and r be the first term and common ratio respectively, then

$$a_5 = p \Longrightarrow ar^4 = p$$
$$a_8 = q \Longrightarrow ar^7 = q$$
$$a_{11} = s \Longrightarrow ar^{10} = s$$

If a, b, c are in G.P, then $b^2 = ac$

Now , $q^2 = (ar^7)^2 = a^2r^{14}$

$ps = ar^4.ar^{10} = a^2r^{14}$

i.e., $q^2 = ps \therefore p, q$ and s are in G.P

5. The sum of first three terms of a G.P is $\frac{13}{12}$ and their product is -1. Find the common ratio and the terms

Ans :

Let $\frac{a}{r}$, a, ar be the three terms ,

then $\frac{a}{r} \times a \times ar = -1$

$$\Longrightarrow a^3 = -1$$
$$\Longrightarrow a = -1$$

Also, $\frac{a}{r} + a + ar = \frac{13}{12}$

i.e., $a\left(\frac{1}{r} + 1 + r\right) = \frac{13}{12}$

$i.e., \frac{1}{r} + 1 + r = \frac{-13}{12}$

Or

$$\frac{1+r+r^2}{r} = \frac{-13}{12}$$

$$\Longrightarrow 12 + 12r + 12r^2 + 13r = 0$$

$$\Longrightarrow 12r^2 + 25r + 12 = 0$$

$$r = \frac{-25\pm\sqrt{625-4\times12\times12}}{2\times12} = \frac{-25\pm\sqrt{49}}{24} = \frac{-3}{4} \text{ or } \frac{-4}{3}$$

When $r = \frac{-3}{4}$, the G.P is $\frac{4}{3}, -1, \frac{3}{4}$

When $r = \frac{-4}{3}$, the G.P is $\frac{3}{4}, -1, \frac{4}{3}$

6. Find the sum of the sequence, 8, 88, 888, …. to n terms

Ans :

$S = 8 + 88 + 888 + \cdots ..\ n$ terms

$= 8(1 + 11 + 111 + \cdots ..\ n$ terms

$= \frac{8}{9}(9 + 99 + 999 + \cdots ..\ n$ terms

$= \frac{8}{9}(10-1) + (100-1) + \cdots ..\ n$ terms

$= \frac{8}{9}(10 + 100 + 1000 + \cdots - (1 + 1 + \cdots + 1)n$ terms

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{9} - n\right]$$

7. (a) Find the $G.M$ of 4 and 16

(b) Insert two numbers between 3 and 81 so that the resulting sequence is a G.P

Ans :

(a) G.M of 4 and 16 $= \sqrt{4\times16} = \sqrt{64} = 8$

(b) Let $3, a_2, a_3, 81$ are in G.P

Then $a_4 = 81$

$\Longrightarrow ar^3 = 81$

$\Rightarrow 3 \times r^3 = 81$
$\Rightarrow r^3 = 27$
$\Rightarrow r = 3$
$\therefore a_2 = 3 \times 3 = 9$
$a_3 = 9 \times 3 = 27$
$\therefore$ we can insert 9 and 27 between 3 and 81 so that the resulting sequence is a G.P

8. The number of bacteria in a certain culture, doubles in every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour 4^{th} hour and n^{th} hour. ?

Ans :

Here $a_1 = 30$ and $r = 2$
At the end of 2^{nd} hour,
number of bacteria $= a_3 = ar^2$
$= 30 \times r^2 = 120$

At the end of 4^{th} hour,
number of bacteria $= a_5 = ar^4$
$= 30 \times r^4 = 480$
At the end of n^{th} hour,
number of bacteria $= a_{n+1} = ar^n$
$= 30 \times 2^n$

PRACTICE PROBLEMS

1. In a G.P, the third term is 24 and 6^{th} term is 192. Find the 10^{th} term

2. Find the sum of n terms of the G.P $1, \frac{2}{3}, \frac{4}{9}, \ldots$..

3. Which term of the G.P $\sqrt{3}, 3, 3\sqrt{3}, \ldots$. is 729 ?

4. The n^{th} term of the sequence 5, $-5/2$, $5/4, \ldots$...is $\frac{5}{1024}$. Find n

5. The sum of first 3 terms of a G.P is 16 and the sum of next three terms is 128. Find the first term, common ratio and sum of n terms.

6. The sum of first three terms of a G.P is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms

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STRAIGHT LINES

KEY NOTES

- **Slope of line joining 2 given points**
  If (x_1, y_1) and (x_2, y_2) are any two points on a line , then the slope of the line is given by
  $$M = \frac{y_2-y_1}{x_2-x_1} = \frac{y_1-y_2}{x_1-x_2}$$
- **Conditions for parallelism and perpendicularity**
  Consider two straight lines with slopes m_1 and m_2
  - If the lines are parallel, then $m_1 = m_2$
  - If the line are perpendicular, then $m_1 m_2 = -1$

    We can also write it as $m_2 = \frac{-1}{m_1}$
- **Equation of a straight line in intercept form**
  Equation of any line with x-intercept $'a'$ and y-intercept $'b'$ is given by
  $$\frac{x}{a} + \frac{y}{b} = 1$$
- **General equation of a straight line**
  General equation of any straight line is $Ax + By + C = 0$, where A, B, C are real constants with A and B cannot be zero simultaneously.

  Here, **Slope** $\boldsymbol{m = \frac{-A}{B}}$

  $\boldsymbol{x}$**-intercept** $= -\frac{C}{A}$

  $\boldsymbol{y}$**-intercept** $= -\frac{C}{B}$

QUESTIONS AND ANSWERS

1. Find the slope of a straight line passing through the points (3,-2) and (-1,4)

Ans :

Let $(x_1, y_1) = (3, -2)$

$(x_2, y_2) = (-1, 4)$

Slope $(m) = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{4-(-2)}{-1-3}$

$= \frac{6}{-4}$

$= -\frac{3}{2}$

2. Line passing through the points (-2,6) and (4,8) is perpendicular to the line through (8,12) and (x,24). Find x.

Ans :

Let the given points are A(-2,6), B(4,8), C(8,12) and D$(x, 24)$

Slope of $AB, m_1 = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{8-6}{4-(-2)}$

$= \frac{2}{6} = \frac{1}{3}$

Slope of $CD, m_2 = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{24-12}{x-8}$

$= \frac{12}{x-8}$

Given $AB \perp$ CD

$\Rightarrow \quad m_1 m_2 = -1$

$\Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1$

$\Rightarrow \quad 4 = -(x-8)$

$\Rightarrow \quad x = 8 - 4 = 4$

3. Find the equation of the line which makes intercepts -3 and 2 on the x and y axes respectively.

Ans :

Given $a = -3$ and $b = 2$

$\therefore$ Equation is $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \quad \frac{x}{-3} + \frac{y}{2} = 1$

$\Rightarrow \quad -2x + 3y = 6$

$\Rightarrow \quad 2x - 3y + 6 = 0$

4. Equation of a line is $3x - 4y + 10 = 0$ Find its

(i) Slope

(ii) x and y intercepts

Ans :

Given $3x - 4y + 10 = 0$

Here $A = 3, B = -4, C = 10$

(i) Slope $m = \frac{-A}{B} = \frac{-3}{-4} = \frac{3}{4}$

(ii) x-intercept $= \frac{-C}{A} = \frac{-10}{3}$

y-intercept $= \frac{-C}{B} = \frac{-10}{-4} = \frac{10}{4} = \frac{5}{2}$

5. Reduce the equation of straight line $3x - 4y + 12 = 0$ into intercept form. Hence write its x and y intercepts.

Ans :

Given line is $3x - 4y + 12 = 0$

$3x - 4y = -12$

$\Rightarrow \quad \frac{3x}{-12} - \frac{4y}{-12} = 1$

$\Rightarrow \quad \frac{x}{-4} + \frac{y}{3} = 1$, which is the intercept form $\frac{x}{a} + \frac{y}{b} = 1$

x-intercept $= -4$

y-intercept $= 3$

6. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2,3)

Ans :

Let x-intercept $= y$-intercept $= a$

$\therefore$ equation of the line in intercept form is $\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \; x + y = a \quad ......(1)$

Since (1) passes through (2,3), we get

$2+3 = a$

$\therefore a = 5$

From (1) ,

$x + y = 5$

7. Which among the following straight line is parallel to the line $2x - 3y + 5 = 0$

(a) $3x + 2y + 5 = 0$

(b) $2x - 3y + 7 = 0$

(c) $2x + 3y + 5 = 0$

(d) $3x - 2y + 7 = 0$

Ans :

(b) or $2x - 3y + 7 = 0$

PRACTICE PROBLEMS

1. Find the slope of the straight line passing through the points (2, -1) and (4,5)

2. A line passes through (x_1, y_1) and (h, k). If the slope of the line is m, show that $k - y_1 = m(h - x_1)$.

3. Without using Pythagorus theorem show that the points A(4,4), B(3,5) and C(-1,-1) are the vertices of a right angled triangle. [Hint : Find slopes of AB, BC and AC]

   OR

   Consider a triangle ABC with vertices A(4,4), B(3,5) and C(-1,-1)
   (i) Find the slopes of the sides AB, BC and AC
   (ii) Is ΔABC a right angled triangle? Justify

4. Find the equation of the straight line whose x and y intercepts are 2 and -5 respectively.

5. Equation of a line is $3x - 2y - 6 = 0$. Find its
   (i) Slope
   (ii) x and y intercepts

6. Reduce the equation $2x + 5y - 10 = 0$ of straight line into intercept form. Hence write its x and y intercepts

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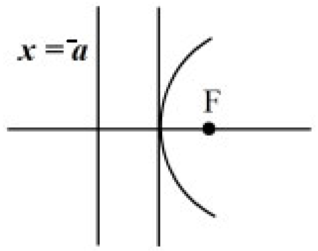
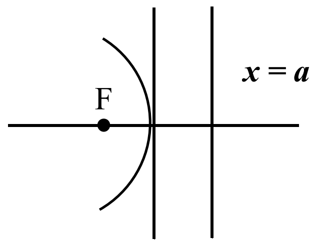
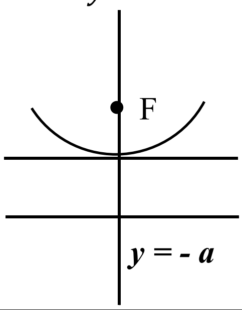
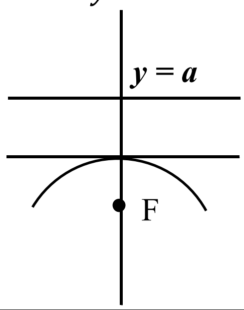
CONIC SECTIONS

KEY NOTES

1. **Circle :** A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.
   The fixed point is called the centre and the distance from the centre to a point on the circle is called the radius
   Equation of a circle with centre at (h, k) and radius r is
   $$(x-h)^2+(y-k)^2=r^2$$
2. **Parabola :** A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point on the plane.
   The fixed point is called the focus and the fixed line is called the directrix.
   Standard equation of Parabola

<table>
<tr>
<td>$y^2=4ax$
Focus $(a,0)$
Directrix $x=-a$
Length of Latus rectum $=4a$
Axis of Parabola – x axis

</td>
<td>$y^2=-4ax$
Focus $(-a,0)$
Directrix $x=a$
Length of Latus rectum $=4a$
Axis of Parabola – x axis

</td>
</tr>
<tr>
<td>$x^2=4ay$
Focus $(0,a)$
Directrix $y=-a$
Length of Latus rectum $=4a$
Axis of Parabola – y axis

</td>
<td>$x^2=-4ay$
Focus $(0,-a)$
Directrix $y=a$
Length of Latus rectum $=4a$
Axis of Parabola – y axis

</td>
</tr>
</table>

3. **Ellipse :** An ellipse is the set of all points in a plane , the sum of whose distances from two fixed points in the plane is a constant

The two fixed points are called the foci of the ellipse

Standard equations	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1\ (a>b)$
Relation between a, b and c	$a^2=b^2+c^2$	$a^2=b^2+c^2$
Foci	$(\pm c, 0)$	$(0, \pm c)$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Length of major Axis	$2a$	$2a$
Length of Minor axis	$2b$	$2b$
Eccentricity	$e=\frac{c}{a}$	$e=\frac{c}{a}$
Latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

QUESTIONS AND ANSWERS

1. Find the equation of the circle with centre (-2, 3) and radius 4

Ans :

Here, $h=-2, k=3, r=4$

Equation of a circle is

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-(-2))^2+(y-3)^2=4^2$$
$$(x+2)^2+(y-3)^2=16$$
$$(x^2+4x+4)+(y^2-6y+9)=16$$
$$x^2+y^2+4x-6y-3=0$$

2. Find the centre and radius of the circle

$$x^2+y^2-8x+10y-12=0$$

Ans :

Given equation is

$$x^2+y^2-8x+10y-12=0$$
$$x^2-8x+16+y^2+10y+25$$
$$=12+16+25$$
$$(x-4)^2+(y+5)^2=53$$
$$(x-4)^2+(y-(-5))^2=\left(\sqrt{53}\right)^2$$

Comparing with

$$(x-h)^2+(y-k)^2=r^2$$
$$h=4, k=-5, r=\sqrt{53}$$

centre (h, k) ie., $(4, -5)$

Radius $=\sqrt{53}$

3. Find the coordinates of the focus , Axis, the equation of the directrix and latus rectum of the parabola $y^2 = 10x$

Ans :

The given equation involves y^2, so the axis of symmetry is along x axis.

The coefficient of x is positive so the parabola opens to the right. Comparing the given equation with $y^2 = 4ax$, we have $4a = 10, a = \frac{5}{2}$

$\therefore$ focus $(a, 0)$ i.e., $\left(\frac{5}{2}, 0\right)$

Equation of directrix. $x = -a$; $x = -\frac{5}{2}$

Length of latus rectum, $4a = 4 \times \frac{5}{2} = 10$

4. Find the equation of the parabola with vertex at (0,0) and focus at (0,2)

Ans :

Since the vertex is at (0,0) and the focus is at (0,2) which lies on y axis, the y axis is the axis of the parabola.Since the focus is at (0,2) the parabola opens upwards.

Focus $(0, a)$ is (0,2)

$\therefore a = 2$

Equation of the parabola is $x^2 = 4ay$

$x^2 = 4 \times 2y$

$x^2 = 8y$

5. Find the coordinates of the foci, the vertices , length of major axis and minor axis and eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Ans :

Since denominator of x^2 is larger than the denominator of y^2, the major axis is along x axis

Comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ we get $a = 5, b = 3$

$c^2 = a^2 - b^2$

$= 25 - 9 = 16$

$c = 4$

$\therefore$ coordinates of foci $(\pm c, 0)$ is $(\pm 4,0)$

Vertices $(\pm a, 0)$ is $(\pm 5,0)$

Length of major Axis = $2a = 10$

Length of Minor axis $= 2b = 6$

Eccentricity $e = \frac{c}{a} = \frac{4}{5}$

Latus rectum $= \frac{2b^2}{a} = \frac{18}{5}$

6. Find the coordinates of the foci, the vertices , length of major axis and minor axis and eccentricity of the ellipse $9x^2 + 4y^2 = 36$

Ans :

Given equation of ellipse can be written as $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Since denominator of y^2 is larger than the denominator of x^2, the major axis is along y axis

Comparing the given equation with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ we get $a = 3, b = 2$

$c^2 = a^2 - b^2 = 9 - 4 = 5$

$\therefore c = \sqrt{5}$

Coordinates of foci $(0, \pm c)$ is $(0, \pm\sqrt{5})$

Vertices $(0, \pm a)$ is $(0, \pm 3)$

Length of major Axis = $2a = 6$

Length of Minor axis $= 2b = 4$

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

7. Find the equation of the ellipse whose vertices are $(\pm 13,0)$ and foci are $(\pm 5,0)$

Ans :

Since the vertices are on x axis , the equation is of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Given vertices $(\pm a, 0)$ is $(\pm 13,0)$

$\therefore a = 13$

Foci $(\pm c, 0)$ is $(\pm 5,0)$ $\therefore c = 5$

$b^2 = a^2 - c^2 = 13^2 - 5^2 = 144$

$\therefore b = 12$

$\therefore$ Equation is $\frac{x^2}{13^2} + \frac{y^2}{12^2} = 1$

i.e., $\frac{x^2}{169} + \frac{y^2}{144} = 1$

PRACTICE PROBLEMS

1. Find the equation of the circle with centre (2,4) and radius is 5

2. Find the centre and radius of the circle

   $$x^2 + y^2 - 8x + 12y - 3 = 0$$

3. Find the coordinates of focus, axis of parabola , equation of directrix, length of latus rectum of the parabola $x^2 = -16y$

4. Find the equation of the parabola with focus (6,0) directrix $x = -6$

5. Find the coordinates of the foci, the vertices , length of major axis and minor axis ,eccentricity and latus rectum of the ellipse $\frac{x^2}{100} + \frac{y^2}{400} = 1$

6. Find the equation of the ellipse with length of major axis 20 and foci are $(0, \pm 5)$

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INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

KEY NOTES

Coordinate axes and coordinate planes in 3d space

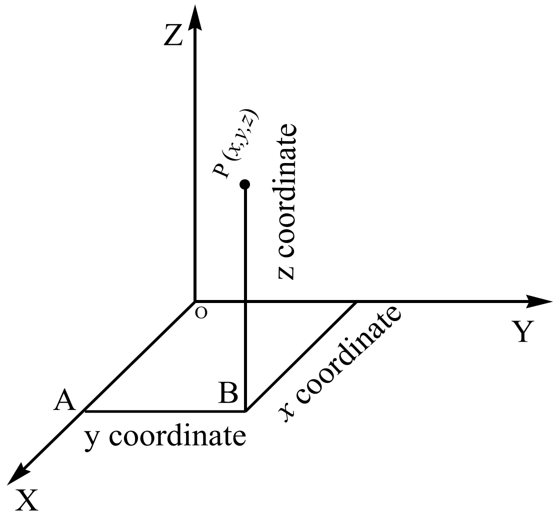
Three mutually perpendicular planes (xy, yz, xz) divide the entire space into 8 equal parts called octants.[Name as I,II,III,IV,V,VI,VII,VIII]

Coordinate of a point in Space

Coordinate of origin – (0,0,0)

Any point on
x-axis – $(x, 0,0)$
y-axis – $(0, y, 0)$
z-axis – $(0,0, z)$

Any point on
xy-plane - $(x, y, 0)$
yz-plane - $(0, y, z)$
xz-plane - $(x, 0, z)$



octant		I	II	III	IV	V	VI	VII	VIII
sign	x	+	–	–	+	+	–	–	+
	y	+	+	–	–	+	+	–	–
	z	+	+	+	+	–	–	–	–
Eg :		(3,2,1)	(-1,2,4)	(-2,-3,1)	(4,-1,2)	(2,1,-3)	(-1,3,-1)	(-2,-1,-3)	(3,-2,-1)

Distance between two points

- Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be two points in space. Then distance between A and B is given by $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- The distance of the point $P(x, y, z)$ from the origin , $OP = \sqrt{x^2 + y^2 + z^2}$
- Distance of a point from

x-axis = $\sqrt{y^2 + z^2}$
y-axis = $\sqrt{x^2 + z^2}$
z-axis = $\sqrt{x^2 + y^2}$

Eg : Find the distance between the points (-1,1,1), (1,2,3)

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{\left(1-(-1)\right)^2 + (2-1)^2 + (3-1)^2}$$
$$= \sqrt{(2)^2 + 1^2 + 2^2}$$
$$= \sqrt{4+1+4}$$
$$= \sqrt{9}$$
$$= 3$$

QUESTIONS AND ANSWERS

1. Fill in the blanks :
   (a) The x-axis and z-axis taken together determine a plane known as …….
   (b) The coordinates of points in the XY plane are of the form ……….
   (c) If P is a point on YZ plane then its x coordinates is ……….

Ans :
(a) XZ-plane
(b) $(x, y, 0)$
(c) Zero

2. Which of the following points lies in the sixth octant
   (i) $(-4,2,-5)$ (ii) $-4,-2,-5)$
   (iii) $(4,-2,-5)$ (iv) $(4,2,5)$

Ans : (i) or $(-4,2,-5)$

3. Show that the points $P(-2,3,5)$, $Q(1,2,3)$ and $R(7,0,-1)$ are collinear.

Ans :
We know that points are said to be collinear if they lie on a line

$$PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$$
$$= \sqrt{9+1+4}$$
$$= \sqrt{14}$$
$$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= 2\sqrt{14}$$
$$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$

Thus $PQ + QR = PR$. Hence P, Q and R are collinear

4. Verify that the points $(0,7,10), (-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle

Ans :

Let $P(0,7,10), Q(-1,6,6)$ and $R(-4,9,6)$be the points

$$PQ = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$$
$$= \sqrt{1+1+16}$$
$$= \sqrt{18}$$
$$QR = \sqrt{(-4-(-1))^2 + (9-6)^2 + (6-6)^2}$$
$$= \sqrt{9+9+0}$$
$$= \sqrt{18}$$
$$PR = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2}$$
$$= \sqrt{16+4+16}$$
$$= \sqrt{36}$$

Here , $PQ^2 + QR^2 = 18 + 18 = 36 = PR^2$, ΔPQR is a right angled triangle

5. Find the equation of set of points P such that $PA^2 + PB^2 = 2K^2$, where A and B are the points $(3,4,5)$ and $(-1,3,-7)$ respectively.

Ans :

Let the coordinates of point P be (x, y, z)

$$PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$$
$$PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$$

By the given condition, $PA^2 + PB^2 = 2K^2$

We have,

$$(x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = 2K^2$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) + (z^2 - 10z + 25) + (x^2 + 2x + 1) + (y^2 - 6y + 9) + (z^2 + 14z + 49) = 2K^2$$
$$2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2K^2 - 109$$

6. Consider a point $A(4,8,10)$ in space

(a) Find the distance of the point A from XY-plane

(b) Find the distance of the point A from x axis

Ans :

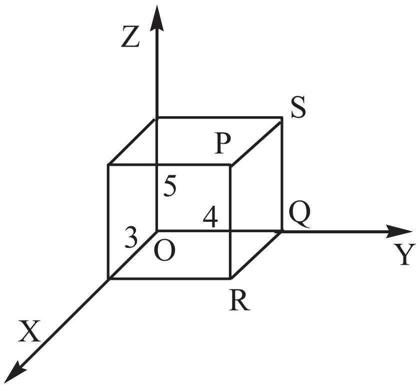
(a) Given $A(4,8,10)$

Distance from XY plane = modulus of Z coordinate = 10

(b) Distance from x axis $= \sqrt{y^2 + z^2}$

$$= \sqrt{8^2 + 10^2}$$
$$= \sqrt{164}$$

7. Consider the following figure. Find the distance PQ



Ans :

$P(3,4,5)$, $Q(0,4,0)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$PQ = \sqrt{(0-3)^2 + (4-4)^2 + (0-5)^2}$$

$$= \sqrt{9 + 0 + 25}$$

$$= \sqrt{34}$$

PRACTICE PROBLEMS

1. Name the octants in which the following points lie
   (i) $(-5,4,7)$ (ii) $(-5,-3,-2)$
   (iii) $(2,-5,-7)$ (iv) $(7,4,-3)$
2. Given three points $A(-4,6,10)$, $B(2,4,6)$ and $C(14,0,-2)$.
   (a) Find AB
   (b) Prove that the points A, B and C are collinear.
3. Verify the following
   (a) $(0,7,-10)$, $(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle.
   (b) (−1,2,1),(1,−2,5), (4,−7,8) and (2,−3,4) are the vertices of a parallelogram.
4. Find the equation of the set of points P such that its distance from the points $A(3,4,-5)$ and $B(-2,1,4)$ are equal.

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LIMITS AND DERIVATIVES

KEY NOTES

- $x \to a$ (x *tends to* a) means 'x' is very closer to a, may be just below or just above 'a' but never equal to a
- $x \to a \Longrightarrow x - a \neq 0$
- **Limit** : As $x \to a$, if $f(x) \to l \Longrightarrow \lim_{x \to a} f(x) = l$

  Eg : As $x \to 3$, $x^2 \to 9 \Longrightarrow \lim_{x \to 3} x^2 = 9$
- Evaluation of limit

  $\lim_{x \to a} f(x) = f(a)$, if $f(a) \in R$

  Case – I : If $f(a) = \frac{Non\ zero\ real\ number}{0}$, $\lim_{x \to a} f(x)$ does not exist.

  Case – II : If $f(a) = \frac{0}{0}$, $\lim_{x \to a} f(x)$ may or may not exist, we use different methods for evaluation

- Working rule to find $\lim_{x \to a} f(x)$

  Step – I : Find $f(a)$

  Step – II : Case –I : If $f(a) \in R \Longrightarrow \lim_{x \to a} f(x) = f(a)$

  Case – II : If $f(a) = \frac{Non\ zero\ real\ number}{0} \Longrightarrow \lim_{x \to a} f(x)$ does not exist.

  Case – III : If $f(a) = \frac{0}{0} \Longrightarrow$ we use different methods for evaluation.

  Different methods :
  - Direct method
  - Factorization method
  - Rationalisation method
  - Using algebraic limit
- Algebraic limit
  - $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
  - $\lim_{x \to 1} \frac{x^n - 1}{x - 1} = n$
  - $\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m} = \frac{n}{m} a^{n-m}$
  - $\lim_{x \to 1} \frac{x^n - 1}{x^m - 1} = \frac{n}{m}$
  - $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n$

- Algebra of limit
  - $\lim_{x \to a} k = k$ [k is any constant]
  - $\lim_{x \to a} k.f(x) = k \lim_{x \to a} f(x)$
  - $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
  - $\lim_{x \to a} [f(x).g(x)] = \lim_{x \to a} f(x). \lim_{x \to a} g(x)$
  - $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ provided $g(a) \neq 0$
- Derivatives
  - Consider $y = f(x)$, its derivative with respect to x is denoted by $\frac{dy}{dx}$ or $f'(x)$ or y'
  - $y = f(x) \Longrightarrow \frac{dy}{dx} = f'(x) = y'$
  - $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
  - As y changes to different functions, the process of finding $\frac{dy}{dx}$ is called differentiation.
- Results

$\frac{d}{dx}(k) = 0$, k is any constant	$\frac{d}{dx}(sin\ x) = cos\ x$	$\frac{d}{dx}(k.y) = k.\frac{dy}{dx}$, k is constant
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(cos\ x) = -sin\ x$	$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
$\frac{d}{dx}x = 1$	$\frac{d}{dx}(tan\ x) = sec^2\ x$	**PRODUCT RULE**
$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$	$\frac{d}{dx}(sec\ x) = sec\ x\ tan\ x$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$	$\frac{d}{dx}(cosec\ x) = -cosec\ x\ cot\ x$	**QUOTIENT RULE**
	$\frac{d}{dx}(cot\ x) = -cosec^2\ x$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

QUESTIONS AND ANSWERS

DIRECT METHOD

1. $\lim_{x\to3} x^2+1 = 3^2+1=10$

2. $\lim_{x\to-1}\left(\frac{x^2-1}{x-1}\right) = \frac{(-1)^2-1}{-1-1}$

$= \frac{0}{-2} = 0$

3. $\lim_{x\to0}(\sin x + \cos x) = \sin 0 + \cos 0$

$= 0+1 = 1$

4. Find $\lim_{x\to1}\left(\frac{ax^2+bx+c}{cx^2+bx+a}\right) = \frac{a\times1^2+b\times1+c}{c\times1^2+b\times1+a}$

$= \frac{a+b+c}{c+b+a}$

$= 1$

5. $\lim_{x\to2}\left(\frac{x+2}{x-2}\right) = \frac{2+2}{2-2} = \frac{4}{0}$, does not exist

FACTORIZATION METHOD

6. Find $\lim_{x\to3}\frac{x^2-9}{x-3}$

Ans :

$\lim_{x\to3}\frac{x^2-9}{x-3}$

$= \lim_{x\to3}\frac{(x+3)(x-3)}{x-3}$

$= \lim_{x\to3}(x+3)$

$= 3+3 = 6$

7. Find $\lim_{x\to3}\frac{x^2-9}{2x^2-5x-3}$

Ans :

$\lim_{x\to3}\frac{x^2-9}{2x^2-5x-3}$

$= \lim_{x\to3}\frac{(x+3)(x-3)}{(x-3)(2x+1)}$

$= \lim_{x\to3}\frac{(x+3)}{(2x+1)}$

$= \frac{3+3}{6+1}$

$= \frac{6}{7}$

RATIONALISATION METHOD

8. **Find** $\lim_{x\to0}\frac{\sqrt{x+1}-1}{x}$

Ans :

$\lim_{x\to0}\frac{\sqrt{x+1}-1}{x}$

$= \lim_{x\to0}\left(\frac{\sqrt{x+1}-1}{x}\right)\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)$

$= \lim_{x\to0}\frac{(x+1)-1}{x(\sqrt{x+1}+1)}$

$= \lim_{x\to0}\frac{x}{x(\sqrt{x+1}+1)}$

$= \lim_{x\to0}\frac{1}{(\sqrt{x+1}+1)}$

$= \frac{1}{1+1}$

$= \frac{1}{2}$

OR

$\lim_{x\to0}\frac{\sqrt{x+1}-1}{x}$

$= \lim_{x\to0}\frac{(1+x)^{\frac{1}{2}}-1}{x}$

$= \frac{1}{2}$

$[\because \lim_{x\to0}\frac{(1+x)^n-1}{x} = n\ ]$

USING ALGEBRAIC LIMIT

9. **Find** $\lim_{x\to3}\frac{x^4-81}{x-3}$

Ans :

$\lim_{x\to3}\frac{x^4-81}{x-3} = \lim_{x\to3}\frac{x^4-3^4}{x-3}$

$= 4\times3^{4-1}$

$= 4\times3^3$

$= 108$

10. Find $\lim_{x\to2}\frac{x^5-32}{x^3-8}$

Ans :

$\lim_{x\to2}\frac{x^5-32}{x^3-8}$

$= \lim_{x\to2}\frac{x^5-2^5}{x^3-2^3}$

$= \frac{5}{3}\times2^{5-3}$

$= \frac{5}{3}\times2^2$

$= \frac{20}{3}$

11. **Find** $\lim_{x\to1}\frac{x^{15}-1}{x^{10}-1}$

Ans :

$\lim_{x\to1}\frac{x^{15}-1}{x^{10}-1} = \frac{15}{10} = \frac{3}{2}$

$[\because \lim_{x\to1}\frac{x^n-1}{x^m-1} = \frac{n}{m}\ ]$

12. **If** $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80$, Find the value of n.

Ans :

Given $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80$

i.e., $n.2^{n-1}=5\times 16$

$=5\times 2^4$

$\Longrightarrow \quad n=5$

DERIVATIVES

13. Find the derivative of x^2-2 at $x=10$

Ans :

$f(x)=x^2-2$

$f'(x)=\frac{d}{dx}(x^2-2)$

$=\frac{d}{dx}x^2-\frac{d}{dx}(2)$

$=2x-0$

$=2x$

$f'(10)=2\times 10=20$

14. Find the derivative of $\frac{1}{x^3}+x^3$ with respect to x.

Ans :

$\frac{d}{dx}\left(\frac{1}{x^3}+x^3\right)$

$=\frac{d}{dx}(x^{-3}+x^3)$

$=-3.x^{-3-1}+3x^2$

$=-3x^{-4}+3x^2$

15. For the function, $f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+\cdots+\frac{x^2}{2}+x+1$, show that $f'(1)=100f'(0)$?

Ans :

$f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+\cdots+\frac{x^2}{2}+x+1,$

$f'(x)=\frac{100x^{99}}{100}+\frac{99x^{98}}{99}+\cdots+\frac{2x}{2}+1+0$

$f'(x)=x^{99}+x^{98}+\cdots+x+1+0$ ..(1)

(1) $\Longrightarrow f'(1)=1+1+\cdots ...+1+1=100$

(1) $\Longrightarrow f'(0)=0+0+\cdots+0+1=1$

$f'(1)=100=100\times 1=100f'(0)$

16. Differentiate $(x-1)(x-2)$ with respect to x.

Ans :

$\frac{d}{dx}(x-1)(x-2)\Longrightarrow$

$=(x-1)\frac{d}{dx}(x-2)+(x-2)\frac{d}{dx}(x-1)$

$=(x-1)(1-0)+(x-2)(1-0)$

$=x-1+x-2$

$=2x-3$

17. Find $f'(x)$, if $f(x)=\frac{x^n-a^n}{x-a}$

Ans :

$f'(x)=\frac{d}{dx}\left(\frac{x^n-a^n}{x-a}\right)$

$=\frac{(x-a)\frac{d}{dx}(x^n-a^n)-(x^n-a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$

$=\frac{(x-a)(nx^{n-1})-(x^n-a^n)}{(x-a)^2}$

18. Find $f'(x)$ if $f(x)=\sin x\cos x$

Ans :

$f'(x)=\frac{d}{dx}(\sin x\cos x)$

$=\sin x\frac{d}{dx}(\cos x)+\cos x\frac{d}{dx}(\sin x)$

$=\sin x\times-\sin x+\cos x\times\cos x$

$=-\sin^2 x+\cos^2 x$

19. Find $f'(x)$ if $f(x)=\frac{x}{1+\tan x}$

Ans :

$f'(x)=\frac{d}{dx}\left(\frac{x}{1+\tan x}\right)$

$=\frac{(1+\tan x)\frac{d}{dx}(x)-x.\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$

$=\frac{(1+\tan x)\times 1-x.(0+\sec^2 x)}{(1+\tan x)^2}$

$=\frac{1+\tan x-x\sec^2 x)}{(1+\tan x)^2}$

PRACTICE PROBLEMS

LIMITS

1. Evaluate the following :

   (a) $\lim_{x\to 2}(ax^2+bx+c)$

   (b) $\lim_{x\to 3}\frac{x+3}{x-3}$

   (c) $\lim_{x\to 3}\frac{x-3}{x+3}$

   (d) $\lim_{x\to 2}\left(\frac{x^3-4x^2+4x}{x^2-4}\right)$

   (e) $\lim_{x\to a}\frac{x^2-a^2}{x-a}$

   (f) $\lim_{x\to a}\frac{x^7-a^7}{x-a}$

   (g) $\lim_{x\to 2}\frac{x^4-16}{x^3-8}$

   (h) $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

   (i) $\lim_{x\to 1}\frac{x^7-1}{x^6-1}$

   (j) $\lim_{x\to 0}\frac{(1+x)^7-1}{x}$

DERIVATIVES

2. Find $\frac{d}{dx}\left(\frac{x^n}{n}\right)$

3. If $f(x)=1+x+x^2+\cdots+x^{50}$, find $f'(1)$

4. Find $\frac{dy}{dx}$, $y=\frac{a}{x^4}-\frac{b}{x^2}+\cos x$

5. Find $\frac{dy}{dx}$, if

   (a) $y=(x-a)(x-b)$

   (b) $y=\frac{x-a}{x-b}$

   (c) $y=\frac{\sin x}{x+1}$

   (d) $y=\frac{x^5-\cos x}{\sin x}$

   (e) $y=x.\sin x$

6. If $xy=c^2$, prove that $x^2\frac{dy}{dx}+c^2=0$

7. Find the derivative of the following with respect to x.

   (a) $y=2x-\frac{3}{4}$

   (b) $y=(5x^3+3x-1)(x-1)$

   (c) $y=x^{-3}(5+3x)$

   (d) $y=x^{-4}(3-4x^{-5})$

   (e) $y=x^5(3-6x^{-9})$

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MATHEMATICAL REASONING

KEY NOTES

- **Statement**
  A statement is a sentence which is either always true or always false, but not both simultaneously.
- **Simple Statements**
  A statement is called simple if it cannot be broken down into two or more statements.
- **Compound Statements**
  A compound statement is the one which is made up of two or more simple statements.
- **Negation of a statement**
  The denial of a statement is called the negation of the statement. The negation of a statement p in symbolic form is written as "~p".
- **The Conditional Statement**
  If p and q are any two statements, then the compound statement "if p then q" formed by joining p and q by a connective 'if-then' is called a conditional statement or an implication and is written in symbolically $p \rightarrow q$ or $p \Rightarrow q$,
- **Converse of a Conditional Statement**
  The conditional statement "$q \rightarrow p$" is called the converse of the conditional statement "$p \rightarrow q$".
- **Contrapositive of Conditional Statement**
  The statement "$(\sim q) \rightarrow (\sim p)$ " is called the contrapositive of the statement $p \rightarrow q$.

QUESTIONS AND ANSWERS

1. Write the negation of the following statements

(a) $\sqrt{2}$ is irrational

(b) $\sqrt{2}$ is not a complex number

(c) Every natural number is greater than zero

Ans :

(a) It is false that $\sqrt{2}$ is irrational

(b) It is false that $\sqrt{2}$ is not a complex number

(c) It is false that every natural number is greater than zero

2. Write the converse and contrapositive of the following statements

(a) If a number is divisible by 9, then it is divisible by 3

(b) If the integer n is odd, then n^2 is odd

(c) If a triangle is equilateral ,then it is isosceles

Ans :

(a) Converse : If a number is divisible by 3, then it is divisible by 9

Contrapositive: If a number is not divisible by 3, then it is not divisible by 9

(b) Converse: If the integer n^2 is odd, then n is odd

Contrapositive: If the integer n^2 is not odd, then n is not odd

(c) Converse: If a triangle is isosceles ,then it is equilateral

Contrapositive: If a triangle is not isosceles, then it is not equilateral

3. Verify by the method of contradiction that $\sqrt{2}$ is irrational

Ans :

To prove $\sqrt{2}$ is irrational

Assume that $\sqrt{2}$ is rational

$\therefore \sqrt{2} = \frac{a}{b}$, where a and b are integers having no common Factor

Squaring , we get $2 = \frac{a^2}{b^2}$

$\Rightarrow a^2 = 2b^2$

$\Rightarrow$ 2 divides a^2

$\Rightarrow$ 2 divides a

Let $a = 2k$

$\Rightarrow (2k)^2 = 2b^2$

$\Rightarrow 4k^2 = 2b^2$

$\Rightarrow 2k^2 = b^2$

$\Rightarrow b^2 = 2k^2$

$\Rightarrow$ 2 divides b^2

$\Rightarrow$ 2 divides b

Thus we get 2 divides a and 2 divides b
i.e., 2 is a common factor of a and b
Which is a contradiction to our assumption.
$\therefore \sqrt{2}$ is irrational

PRACTICE PROBLEMS

1. Write the negation of the following statements
   (a) Both diagonals of a rectangle have same length
   (b) Chennai is the capital of Tamil Nadu
2. Write the converse and contrapositive of the following statements
   (a) If the integer n is even, then n^2 is even
   (b) If x is prime number , then x is odd
3. Verify by the method of contradiction that $\sqrt{5}$ is irrational

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STATISTICS

KEY NOTES

Mean, variance and standard deviations

For ungrouped data

- n = number of observations
- Mean $\bar{x} = \frac{\Sigma x_i}{n}$
- Variance $= \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$
- SD = $\sigma = \sqrt{\text{variance}}$

For grouped data

- $N = \Sigma f_i$
- Mean $\bar{x} = \frac{\Sigma f_i x_i}{N}$
- Variance $= \sigma^2$

$$= \frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$$
$$= \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$$

- SD = $\sigma = \sqrt{\text{variance}}$

For commerce maths only :

Mean deviation about Mean

(1) Ungrouped Data

$$MD(\bar{x}) = \frac{1}{n}\sum |x_i - \bar{x}|$$

(2) Grouped data

$$MD(\bar{x}) = \frac{1}{N}\sum f_i|x_i - \bar{x}|$$

QUESTIONS AND ANSWERS

1. Consider the observations 2, 4, 6, 8 and 10.

(a) Find the Mean

(b) Find the Variance and standard deviations

Ans :

x_i	x_i^2
2	4
4	16
6	36
8	64
10	100
$\Sigma x = 30$	$\Sigma x^2 = 220$

Here $n = 5$

(a) Mean $\bar{x} = \frac{\Sigma x_i}{n} = \frac{30}{5} = 6$

(b) Variance $= \sigma^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$$= \frac{220}{5} - 36 = 8$$

$$\text{SD} = \sqrt{\text{variance}}$$
$$= \sqrt{8}$$

2. Consider the following frequency distribution.

x_i	3	8	13	18	23
f_i	7	10	15	10	6

(a) Find the mean

(b) Find the variance and standard deviation

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
3	7	21	9	63
8	10	80	64	640
13	15	195	169	2535
18	10	180	324	3240
23	6	138	529	3174
	48	**614**	**1095**	**9652**

Ans :

$N = 48$

(a) Mean $\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{614}{48} = 12.79$

(b) Variance $= \sigma^2$

$= \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$

$= \frac{9652}{48} - 12.79^2$

$= 37.45$

SD $= \sqrt{\text{variance}}$

$= \sqrt{37.45}$

$= 6.12$

3. Consider the Following distribution.

Classes	30-40	40-50	50-60	60-70	70-80
Freque-ncies	3	7	12	15	8

(a) Find the mean.

(b) Find the variance and standard deviation

Ans :

f_i	x_i	$f_i x_i$	x_i^2	$f_i x_i^2$
3	35	105	1225	3675
7	45	315	2025	14175
12	55	660	3025	36300
15	65	975	4225	63375
8	75	600	5625	45000
45		**2655**		**162525**

(a) Mean $= \frac{\Sigma f_i x_i}{N} = \frac{2655}{45} = 59$

(b) Variance $= \sigma^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$

$= \frac{162525}{45} - 59^2$

$= 130.66$

SD $= \sqrt{\text{variance}}$

$= \sqrt{130.66}$

$= 11.43$

4. Find the mean deviation about mean for the following data :

x	f
2	2
5	8
6	10
8	7
10	8
12	5

Ans :

x	f	fx	$\lvert x - \bar{x} \rvert$	$f\lvert x - \bar{x} \rvert$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
	N=40	**300**		**92**

$\bar{x} = \frac{\Sigma f_i x_i}{N}$

$= \frac{300}{40} = 7.5$

$MD = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{N} = \frac{92}{40} = 2.3$

PRACTICE PROBLEMS

1. Find the mean , mean deviation about mean, variance and standard deviation for the following data

(a) 6,7,10,12,13,4,8,12

(b) 6, 8,10,12,14,16,18,20,22 ,24

2. Find the mean , variance and standard deviation for the following data

(i)

x_i	5	10	15	20	25
f_i	7	4	6	3	5

(ii)

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

3. Find the mean ,mean deviation about mean, variance and standard deviation for the following data

(i)

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

(ii)

Classes	10-20	20-30	30-40	40-50	50-60
Frequencies	6	15	13	7	9

For commerce only:

1. Find the mean deviation about mean for the data, 6,7,10,12,13,4,8,12
2. Find the mean deviation about mean for the data

x_i	5	10	15	20	25
f_i	7	4	6	3	5

3. Find the mean deviation about mean for the data

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

PROBABILITY

KEY NOTES

- **Random experiment:** An experiment is called random experiment if it satisfies the following conditions
  (i) It has more than one possible outcome
  (ii) It is not possible to predict the outcomes in advance
- **Sample space (S)** : The set of all possible outcomes of a random experiment
- **Sample points** : Elements of sample space
- **Event :** A subset of sample space
- **Impossible event** : Null set
- **Sure event** : The whole sample space
- **Complimentary event or not event** : The set $A' = S - A$
- **Event A or B:** The set $A \cup B$
- **Event A and B :** The set $A \cap B$
- **Two events A and B are**
  Mutually exclusive Events if $A \cap B = \Phi$
  Exhaustive events if $A \cup B = S$
- **Equally likely outcomes** : All outcomes with equal probability
- **Probability of an event**
  For a finite sample space with equally likely out comes, the probability of an event A is given by

  $\boldsymbol{P(A) = \frac{n(A)}{n(S)}}$, $n(A)$ is number of elements of A &
  $n(S)$ is the number of elements of S

- $P(S) = 1 \text{ and } P(\Phi) = 0$
- $0 \leq P(A) \leq 1$
- $P(not\ A) = P(A') = 1 - P(A)$
- $P(A$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

(ii) Let A= At least two heads
$= \{HHH, HHT, HTH, THH\}, n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iii) Let $A =$ At most two heads $=$ $\{THH, HTH, HHT, HTT, THT, TTH, TTT\}$

$$n(A) = 7,$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

(iv) Let $A =$ At least one head = $\{HHH, THH, HTH, HHT, HTT, THT, TTH\}$

$$n(A) = 7, \; P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

(v) Let $A =$ At most one head

$= \{HTT, TTH, THT, TTT\}$
$n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

5. A die is thrown .Find the probability of getting
(a) a prime number
(b) a number greater than 4

Ans : Here $S = \{1,2,3,4,5,6\}$, $n(S) = 6$

(a) Let A be the event of getting a Prime number, then
$A = \{2,3,5\}, n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(b) Let $A =$ A number greater than 4
$= \{5,6\}, n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

6. Two dice are thrown.
Find the probability of
(a) Getting a doublet
(b) Getting sum of the numbers on the dice is 8
(c) Getting an odd number on the 1st die

Ans :

Here two dice are thrown ,

∴ sample space is

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$
$n(S) = 36$

(a) Let A be the event of getting a Doublet, then
$A = \{(1,1), (2,2), (3,3)(4,4)(5,5)(6,6)\},$
$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(b) Let $A =$ Sum of numbers on the dice is 8
$= \{(2,6), (3,5), (4,4)(5,3)(6,2)\}$
$n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

(c) Let $A =$ odd number on the 1st die
$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$,
$n(A) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

7. One card is drawn at random from a pack of 52 playing cards. Find the probability that,

(a) The card drawn is Diamond
(b) The card drawn is an ace
(c) The card drawn is black.
(d) The card drawn is a face card
(e) The card drawn is not diamond

Ans :

Here $n(S) = 52$
(One card can be selected from 52 cards in $52C_1 = 52$ ways)

(a) Let A be the event of getting a Diamond card ,then

$$n(A) = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(b) Let A =Ace card , $n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(c) Let A =Black card , $n(A) = 26$

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(d) Let A =Face card , $n(A) = 12$

$$P(A) = \frac{n(A)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(e) Let A = not Diamond card , $n(A) = 39$

$$P(A) = \frac{n(A)}{n(S)} = \frac{39}{52} = \frac{3}{4}$$

Or

From part (a)

$P(\text{Diamond card}) = P(A) = \frac{1}{4}$

$\therefore$ $P(\text{not Diamond}) = P(A')$
$= 1 - P(A)$
$= 1 - \frac{1}{4} = \frac{3}{4}$

8. A bag contains 9 balls of which 4 are red, 3 are blue and 2 are yellow. A ball is drawn at random from the bag: Calculate the probability that it will be

(a) Red
(b) Not blue
(c) Either red or blue.

Ans: Total number of balls in the bag is 9

$\therefore n(S) = 9$

(a) Let A be the event that the ball is red, then

$n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{9}$$

(b) Let A be the event that the ball is not blue , ie ball is red or yellow , then

$n(A) = 4\,red + 2\text{ yellow} = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{9} = \frac{2}{3}$$

(c) Let A be the event that the ball is either red or blue ,then

$n(A) = 4 + 3 = 7$

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{9}$$

9. A committee of two persons is selected from two men and two women. What is the probability that the committee will have

(a) no men?
(b) One man
(c) Two men

Ans :

Total number of persons = 2 + 2 = 4
Out of the 4 persons, two can be selected in ${}^4C_2$ ways.

$\therefore$ $n(S) = {}^4C_2 = 6$

(a) Let A be the event of selecting no men
i.e ., selecting two women.
Two women can be selected from 2 women in ${}^2C_2$ ways

$\therefore \quad n(A) = {}^2C_2 = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

(b) Let A =Selecting One man (ie we have to select one man and one women)

$$n(A) = {}^2C_1 \times {}^2C_1 = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

(c) Let A =Selecting two men

$$n(A) = {}^2C_2 = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

PRACTICE PROBLEMS

1. A coin is tossed twice. Find the probability of getting

(a) Exactly one head

(b) Three heads

(c) At most one head

(d) At least one head

2. A coin is tossed three times. Find the probability of getting

(a) No head

(b) At least two tails

(c) At most one tail

3. Two students, Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

(a) The problem is solved [Hint $P(A \cup B)$]

(b) Both will not qualify the examination [Hint $P(A' \cap B')$]

4. In a class of 60 students; 30 selected for NCC, 32 selected for NSS and 24 selected both NCC and NSS. If one of these students is selected at random, find the probability that :

(a) The student selected for NCC or NSS.

(b) The student has selected neither NCC nor NSS

5. Three cards are drawn from a well shuffled pack of 52 cards. find the probability that

(a) All the three cards are diamond

(b) At least one of the cards is non-diamond

(c) One card is king and two are jacks

6. If A and B are two events of a sample space with $P(A) = 0.54, P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find $P(A')$, $P(A \cup B), P(A' \cap B')$ and $P(A' \cup B')$