PLUS ONE PHYSICS

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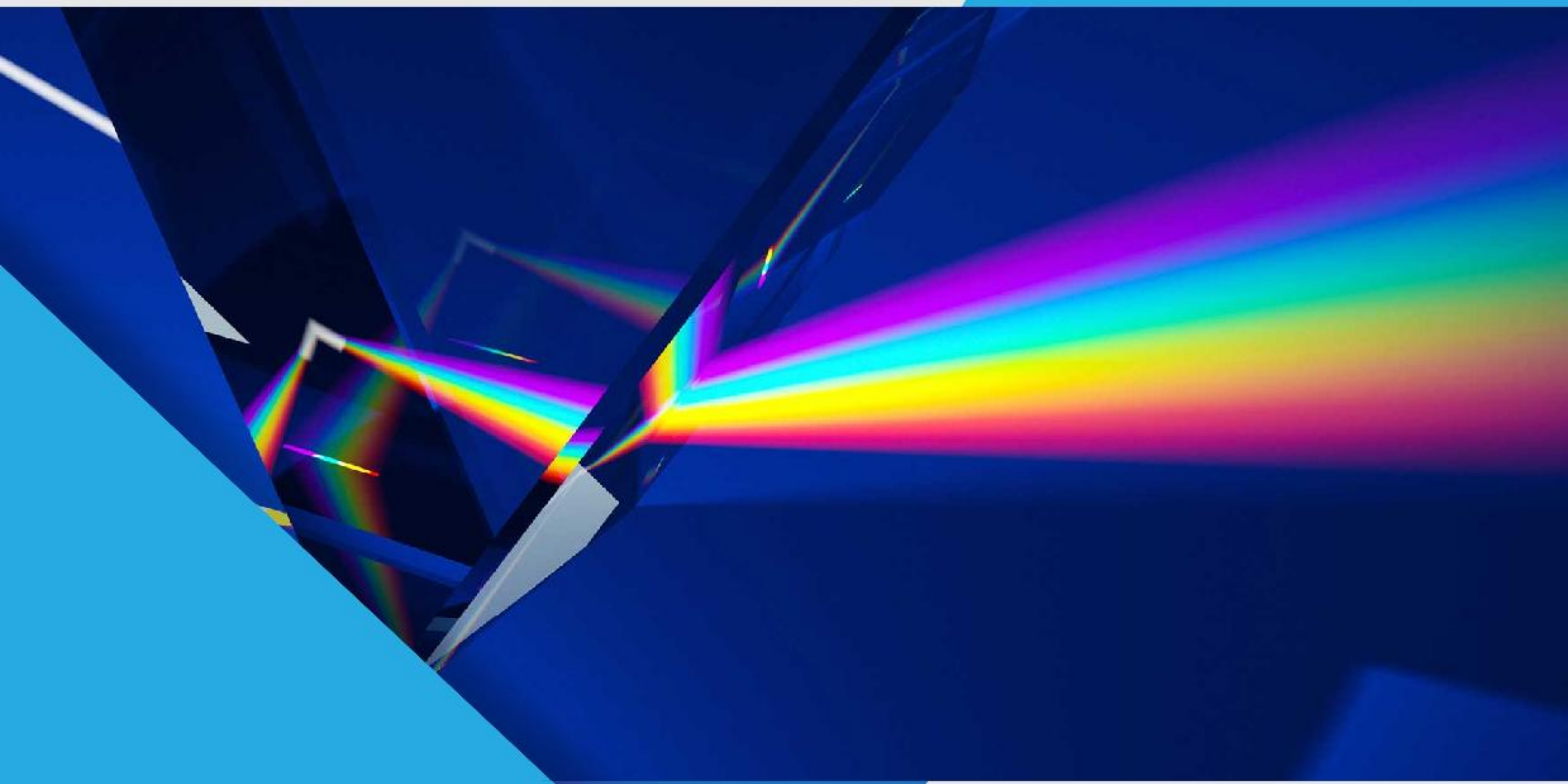
STUDY MATERIAL



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HIGHER SECONDARY NATIONAL SERVICE SCHEME

Chapter 1
Physical World

The word Science originates from the Latin verb **Scientia** meaning **'to know'**
The word Physics comes from a Greek word meaning nature.

Scientific Method

Scientific Method involves the following steps,
1. Systematic observations 2. Controlled experiments 3. Qualitative and quantitative reasoning
4. Mathematical modelling 5. Prediction and verification or falsification of theories

Scope and Excitement of Physics

Basically, there are two domains of interest : **macroscopic and microscopic.**

The macroscopic domain

The macroscopic domain includes phenomena at the laboratory, terrestrial and astronomical scales. Classical Physics deals mainly with macroscopic phenomena.
The branches of classical physics -Mechanics, Electrodynamics, Optics and Thermodynamics.

- Mechanics founded on Newton's laws of motion and the law of gravitation . It is concerned with the motion of particles, rigid and deformable bodies, and general systems of particles.
- Electrodynamics deals with electric and magnetic phenomena associated with charged and magnetic bodies.
- Optics deals with the phenomena involving light.
- Thermodynamics deals with systems in macroscopic equilibrium and is concerned with changes in internal energy, temperature, entropy, etc., of the system through external work and transfer of heat.

The microscopic domain

The microscopic domain includes atomic, molecular and nuclear phenomena. Classical physics is inadequate to handle this domain and **Quantum Theory** is currently accepted as the proper framework for explaining microscopic phenomena.

Fundamental Forces in Nature

There are four fundamental forces in nature- Gravitational Force, Weak Nuclear Force, Electromagnetic Force and Strong Nuclear Force

Forces	Gravitational Force	Weak Nuclear Force	Electromagnetic Force	Strong Nuclear Force
What ?	It is the force of mutual attraction between any two objects by virtue of their masses	The weak nuclear force appears only in certain nuclear processes such as the β-decay of a nucleus.	Electromagnetic force is the force between charged particles.	The strong nuclear force binds protons and neutrons in a nucleus.
Operate s among	All objects in nature	Some elementary particles, particularly electron and neutrino	Charged particles	Nucleons, heavier elementary particles
Range	Long range force; Infinite	Very short, Sub-nuclear size ($\sim 10^{-16}$m)	Long range force ; Infinite	Short, nuclear size ($\sim 10^{-15}$m)
Nature	Always attractive	Not attractive or repulsive	Similar charges repel and opposite charges attract.	Attractive for distances larger than 0.8 fm and repulsive if they are separated by distances les than 0.8 fm. Force between n-n, p-p, n-p are same. Nuclear force is charge independent.
Relative strength	**Weakest force in nature**	Stronger than gravitational force , but weaker than electromagnetic force	Stronger than gravitational and weak nuclear force, but weaker than strong nuclear force	**Strongest force in nature**
Relative strength	10^{-39}	10^{-13}	10^{-2}	1

Nature of Physical Laws

- The physical quantities that remain unchanged in a process are called conserved quantities.
- Some of the general conservation laws in nature include the laws of conservation of mass, energy, linear momentum, angular momentum, charge, parity, etc.
- **Some conservation laws are true for one fundamental force but not for the other.**
- Conservation laws have a deep connection with symmetries of nature. Symmetries of space and time, and other types of symmetries play a central role in modern theories of fundamental forces in nature.

Chapter 2
Units and Measurement

Fundamental and Derived Quantities

- **The physical quantities, which are independent of each other and cannot be expressed in terms of other physical qualities are called fundamental quantities.**
  Eg: length, mass, time.
- **The physical quantities , which can be expressed in terms of fundamental qualities are called derived quantities.** Eg: volume, velocity, force

Fundamental and Derived Units

- The units for the fundamental or base quantities are called fundamental or base units. The units of all other physical quantities can be expressed as combinations of the base units.
- The units of the derived quantities are called derived units.

Systems of Units

The base units for length, mass and time in these systems were as follows :

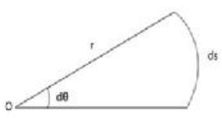
- CGS system - centimetre, gram and second.
- FPS system - foot, pound and second.
- MKS system - metre, kilogram and second.

The International System of Units

In SI system there are seven base units and two supplementary units.

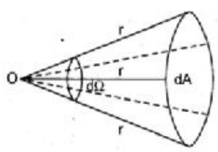
BASE QUANTITY	BASE UNIT	SYMBOL
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

SUPPLEMENTARY QUANTITY	SUPPLEMENTARY UNITS	SYMBOL
Plane Angle	radian	rad
Solid Angle	steradian	sr



$$\text{Angle} = \frac{arc}{radius}$$

$$d\theta = \frac{ds}{r}$$

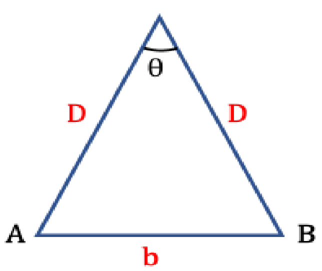


$$\text{Solid angle} = \frac{\text{Intercepted } Area}{Square\ of\ radius}$$

$$d\Omega = \frac{dA}{r^2}$$

Measurement of Large distances-Parallax Method

Large distances such as the distance of a planet or a star from the earth can be measured by parallax method.
To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance AB = b.



$$\text{Angle} = \frac{arc}{radius}$$

$$\theta = \frac{b}{D}$$

$$\boxed{D = \frac{b}{\theta}}$$

D=distance of a planet from the earth
b=distance between the observatories
θ= parallax angle

Ranges of Length

We also use certain special length units for short and large lengths.

1 fermi = 1 f = 10^{-15} m
1 angstrom = 1 Å = 10^{-10} m

1 astronomical unit = 1 AU (average distance of the Sun from the Earth)
= 1.496×10^{11} m

1 light year = 1 ly = 9.46×10^{15} m (distance that light travels with velocity of 3×10^{8} m s^{-1} in 1 year)

1 parsec = 3.08×10^{16} m (Parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)

Measurement of Mass

While dealing with atoms and molecules, unified atomic mass unit (u), is used.

1 unified atomic mass unit = 1u
= (1/12) of the mass of an atom of carbon-12 isotope ($^{12}_{6}C$) including the mass of electrons

1u =1.66 x10^{-27} kg

The dimensional formulae of some derived quantities

Area - L^2
Volume - L^3
Density - ML^{-3}
Velocity- LT^{-1}
Acceleration - LT^{-2}
Momentum - MLT^{-1}
Force - MLT^{-2}
Work or energy - ML^2T^{-2}
Power- ML^2T^{-3}
Torque - ML^2T^{-2}
Pressure- $ML^{-1}T^{-2}$
Stress- $ML^{-1}T^{-2}$
Modulus of elasticity- $ML^{-1}T^{-2}$

Physical quantities having no dimension and no unit

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}} = \frac{[L]}{[L]} = [L^0]$$

$$\text{Relative Density} = \frac{\text{Density of substance}}{\text{Density of water}} = \frac{[ML^{-3}]}{[ML^{-3}]} = [L^0]$$

Physical quantities having units, but no dimension

Plane angle
Solid Angle
Angular Displacement

Dimensional analysis and its applications

1) Checking the Dimensional Consistency(correctness) of Equations
2) Deducing Relation among the Physical Quantities

1) Checking the Dimensional Consistency(correctness) of Equations

The principle called the principle of homogeneity of dimensions is used to check the dimensional correctness of an equation.

The principle of homogeneity states that, for an equation to be correct, the dimesions of each terms on both sides of the equation must be the same.

The magnitudes of physical quantities may be added or subtracted only if they have the same dimensions.

If X + Y = Z
[X] = [Y] = [Z]

We cannot add 5m and 10kg
Velocity cannot be added to force.

1.Check the dimensional correctness of the equation s=ut +1/2 at²

$$[s] = L$$
$$[ut] = LT^{-1} \times T$$
$$= L$$
$$[\tfrac{1}{2}at^2] = LT^{-2} \times T^2$$
$$= L$$

Since each term has the same dimension ,this equation is dimensionally correct.

2.Check the dimensional correctness of the equation ½ mv² = mgh

$$[\tfrac{1}{2}mv^2] = M\ [LT^{-1}]^2$$
$$= ML^2T^{-2}$$

$$[mgh] = M\ LT^{-2}\ L$$
$$= ML^2T^{-2}$$

Since each term has the same dimension ,this equation is dimensionally correct.

3.Check the dimensional correctness of the equation E= mc²

$$[E] = ML^2T^{-2}$$

$$[m\,c^2] = M\ [LT^{-1}]^2$$
$$= ML^2T^{-2}$$

Since each term has the same dimension ,this equation is dimensionally correct.

2) Deducing Relation among the Physical Quantities

We can deduce relation of a physical quantity which depends upto three physical quantities.

1.Derive the equation for kinetic energy (E) of a body of mass m moving with velocity v

$$E \alpha\ m^x\ v^y$$
$$E = k\,m^x v^y \longrightarrow (1)$$

Writing the dimensions on both sides,

$$M\,L^2\,T^{-2} = M^x\ (LT^{-1})^y$$
$$M^1L^2\ T^{-2} = M^x\ L^y\ T^{-y}$$

equating the dimensions on both sides,

$$x = 1$$
$$y = 2$$

Substituting in eq (1)

$$E = k\ m^1\ v^2$$
$$\boxed{E = k\,mv^2}$$

2) Suppose that the period of oscillation of the simple pendulum depends on its mass of the bob (m), length (l) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

$$T \propto m^x\ l^y\ g^z$$
$$T = k\,m^x\,l^y\,g^z \longrightarrow (1)$$

Writing the dimensions on both sides,

$$M^0L^0T^1 = M^x\ L^y\ (LT^{-2})^z$$
$$M^0L^0T^1 = M^x\ L^y\ L^z\ T^{-2z}$$
$$M^0L^0T^1 = M^x\ L^{y+z}\ T^{-2z}$$

equating the dimensions on both sides,

$$x = 0$$
$$y + z = 0$$
$$-2z = 1 \qquad z = \frac{-1}{2}$$
$$y + \frac{-1}{2} = 0 \qquad y = \frac{1}{2}$$
$$\boxed{T = km^0\, l^{1/2} g^{-1/2}}$$
$$T = k\frac{l^{1/2}}{g^{1/2}}$$

$$T = k\frac{\sqrt{l}}{\sqrt{g}}$$

$$\boxed{T = k\sqrt{\frac{l}{g}}}$$

Limitations of Dimensional Analysis

1) Dimensional analysis check only the dimensional correctness of an equation, but not the exact correctness.
2) The dimensionless constants cannot be obtained by this method.
3) We cannot deduce a relation, if a physical quantity depends on more than three physical quantities.
4) The method cannot be considered to derive equations involving more than one term
5) A formula containing trigonometric , exponential and logarithmic function can not be derived from it.
6) It does not distinguish between the physical quantities having same dimensions.

Errors In Measurement

Every measurement is approximate due to errors in measurement.

Errors

Systematic errors   Random errors.

Systematic errors

The systematic errors are those errors that tend to be in one direction, either positive or negative.

Random errors

The random errors are those errors, which occur irregularly and hence are random with respect to sign and size.

Some of the sources of systematic errors are :

a) Instrumental errors that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(b) Imperfection in experimental technique or procedure

(c) Personal errors that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness

True value or Mean value or Average value

Suppose the values obtained in several measurements are $a_1, a_2, a_3, \ldots \ldots, a_n$ **The arithmetic mean of these values is taken as the True value.**

$$a_{mean} = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

Absolute Error

The magnitude of the difference between the true value of the quantity and the individual measurement value is called the absolute error of the measurement.

$$\Delta a_1 = |a_1 - a_{mean}|$$

Mean Absolute Error

The arithmetic mean of all the absolute errors is taken as the final or mean absolute error.

$$\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 + \cdots + \Delta a_n}{n}$$

Relative Error

The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

$$\delta a = \frac{\Delta a_{mean}}{a_{mean}}$$

Percentage Error

Relative error expressed in percent is called Percentage Error.

$$\text{Percentage Error}=\frac{\Delta a_{mean}}{a_{mean}}\ x\ 100\%$$

Example

The centripetal force of a body is given by F=$\frac{mv^2}{r}$. Write an expression for percentage error in centripetal force.

$$\frac{\Delta F}{F} \text{ x } 100\% = \frac{\Delta m}{m} \text{ x } 100\% + 2 \text{ x } \frac{\Delta v}{v} \text{ x } 100\% + \frac{\Delta r}{r} \text{ x } 100\%$$

Significant Figures

The result of measurement is a number that includes all digits in the number that are known reliable plus the first digit that is uncertain.

The reliable digits plus the first uncertain digit in a measurement are known as significant digits or significant figures.

If the period of oscillation of a simple pendulum is 1.62 s, the digits 1 and 6 are reliable and certain, while the digit 2 is uncertain

Write the number of significant figures in following numbers

0.02380 - 4
23.08 -4
23.80-4
2380-3
43.00-4
4300-2
4.700×10^{2} - 4
4.700×10^{-3} -4

Chapter 3
Motion in a Straight Line

Path Length (Distance Travelled)

The total length of the path travelled by an object is called Path Length.

Displacement

Dispalcement is the change in position of the object.

Let x_1 and x_2 be the positions of an object at time t_1 and t_2 .

Displacement, Δx, $= x_2 - x_1$,

Differences between distance (path length) and displacement

1. Distance is a scalar, while displacement is a vector.
2. For a moving particle distance can never be zero or negative while displacement can be zero, positive or negative.
3. For a moving particle, distance can never decrease with time while displacement can. Decrease in displacement with time means that the body is moving towards the initial position.
4. Distance is always greater than or equal to displacement

Average Velocity

Average velocity is defined as the ratio of total displacement to the total time interval.

$$\text{Average velocity} = \frac{\textbf{Total displacement}}{\textbf{Total time interval}}$$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Average speed

Average speed is defined as the ratio of total path lenth (distance travelled) to the total time interval.

$$\text{Average speed} = \frac{\textbf{Total path length}}{\textbf{Total time interval}}$$

Differences between average speed and average velocity

1. Average speed is a scalar, while average velocity is a vector quantity.
2. For a moving body, speed can never be zero or negative while velocity can be zero, positive or negative.
3. Speed is always greater than or equal to velocity.

Instantaneous velocity

The velocity at an instant is called instantaneous velocity and is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$v = \frac{dx}{dt}$$

Instantaneous speed

Instantaneous speed or simply speed is the magnitude of velocity.

Uniform motion

If an object moving along the straight line covers equal distances in equal intervals of time, it is said to be in uniform motion along a straight line .

In uniform motion velocity of the object remains constant.

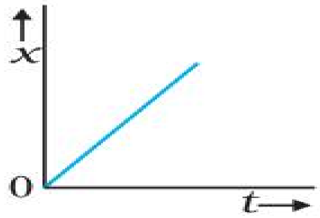
Note that for uniform motion, velocity is the same as the average velocity at all instants.

Position-time graph

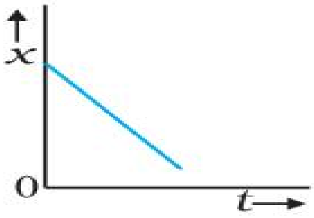
Motion of an object can be represented by a position-time graph .For motion along a straight line, say X-axis, only x-coordinate varies with time and we have an x-t graph.

Position-time graph for an object moving with:-

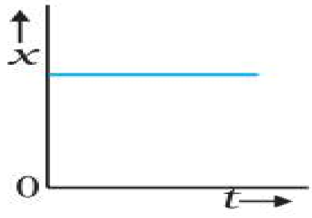
(a) uniform positive



(b) uniform negative velocity



(c) at rest.

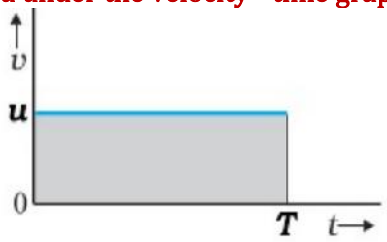


The slope of position-time graph gives the velocity

Velocity - time graph for uniform motion

In uniform motion, velocity is the same at any instant of motion.Therefore, the velocity - time graph is a straight line parallel to the time axis.

The area under the velocity - time graph is equal to the displacement of the particle.



Area = uT=displacement

Acceleration

Suppose the velocity itself is changing with time. In order to describe its effect on the motion of the particle, we require another physical quantity called acceleration. The rate of change of velocity of an object is called acceleration.

Average Acceleration

The average acceleration a over a time interval is defined as the change of velocity divided by the time interval .

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Uniform acceleration

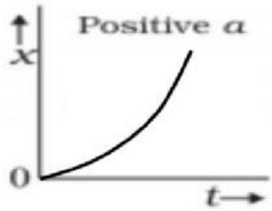
If the velocity of an object changes by equal amounts in equal intervals of time, it has uniform acceleration.

Instantaneous acceleration

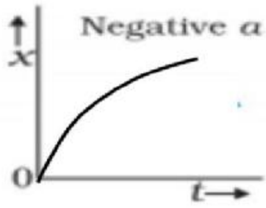
The acceleration of a particle at any instant of its motion is called instantaneous acceleration.

Position-time graph for motion with

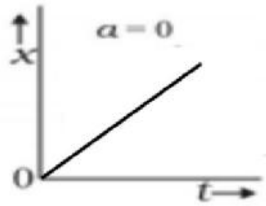
(a)positive acceleration



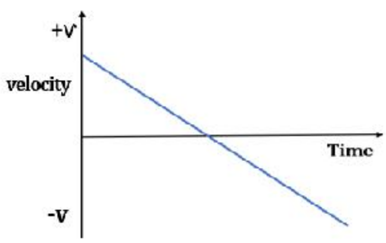
(b) negative acceleration



c)zero acceleration

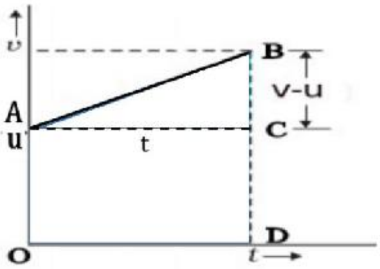


Velocity time graph of a body thrown vertically upwards and returns to ground



Kinematic Equations for Uniformly Accelerated Motion

Consider a body moving with uniform acceleration . The velocity – time graph is as shown in figure



(1) Velocity – time relation

From the graph , acceleration = slope

$$a = \frac{BC}{AC}$$

$$a = \frac{v-u}{t}$$

$$v-u = at$$

$$v = u + at \text{ -------------------- (1)}$$

(2) Position-time relation

Displacement = Area under the graph

s = Area of ▬ + Area of ▲

$$s = ut + \tfrac{1}{2}(v-u)\,t$$

But from equation (1)

$$v - u = at$$

$$s = ut + \tfrac{1}{2}\,at \times t$$

$$s = ut + \tfrac{1}{2}at^2 \text{ ------------------(2)}$$

(3)Position – velocity relation

Displacement = Average velocity x time

$$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \left(\frac{v^2-u^2}{2a}\right)$$

$$v^2 - u^2 = 2as$$

$$v^2 = u^2 + 2as \text{ ----------------(3)}$$

Stopping distance of vehicles

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.

$v^2 = u^2 + 2as$

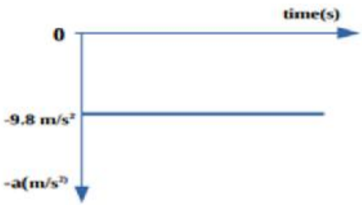
$$0 = u^2 + 2as$$

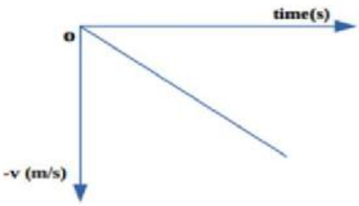
$$-u^2 = 2as$$

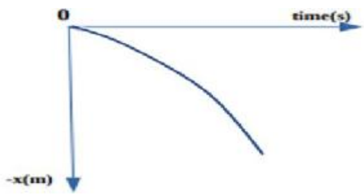
$$s = \frac{-u^2}{2a}$$

Motion of an object under Free Fall

(a)Variation of acceleration with time (b)Variation of velocity with time (c)Variation of distance with tim







Relative Velocity

Consider two objects A and B moving uniformly with average velocities v_A and v_B in one dimension, then

The velocity of object B relative to object A is $v_{BA} = v_B - v_A$

The velocity of object A relative to object B is $v_{AB} = v_A - v_B$

Chapter 4
Motion in a Plane

Scalars and Vectors

A scalar quantity has only magnitude and no direction. It is specified completely by a single number, along with the proper unit.

Eg. distance ,mass , temperature, time .

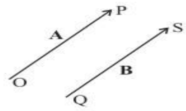
A vector quantity has both magnitude and direction and obeys the triangle law of addition or the parallelogram law of addition. A vector is specified by giving its magnitude by a number and its direction.

Eg.displacement, velocity, acceleration and force.

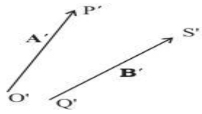
Equality of Vectors

Two vectors A and B are said to be equal if, and only if, they have the same magnitude and the same direction.

(a) Two equal vectors **A** and **B**.



(b) Two vectors **A**′ and **B**′ are unequal eventhough they are of same length



Null vector or a Zero vector

A Null vector or a Zero vector is a vector having zero magnitude and is represented by **O** or $\bar{O}$. The result of adding two equal and opposite vectors will be a Zero vector

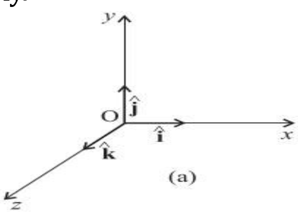
Eg: When a body returns to its initial position its displacement will be a zero vector.

The main properties of $\bar{O}$ are :

$$\bar{A} + \bar{O} = \bar{A}$$
$$\lambda \bar{O} = \bar{O}$$
$$\bar{O}\,\bar{A} = \bar{O}$$

Unit vectors

A unit vector is a vector of unit magnitude and points in a particular direction.
It has no dimension and unit. It is used to specify a direction only.
Unit vectors along the x-, y- and z-axes of a rectangular coordinate system are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$, respectively.

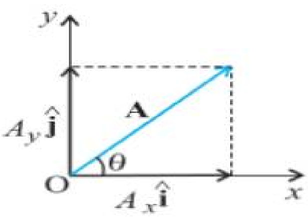


Since these are unit vectors, we have

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

These unit vectors are perpendicular to each other and are called orthogonal unit vectors

Resolution of a vector



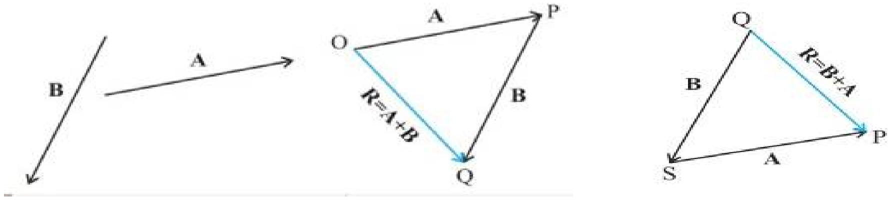
$$\bar{A} = A_x\ \hat{i}\ + A_y\ \hat{j}$$

where $A_x = A\cos\theta$
$A_y = A\sin\theta$

Addition and Subtraction of Vectors — Graphical Method

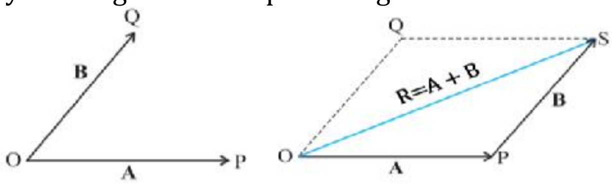
Triangle law of vector addition

If two vectors are represented in magnitude and direction by the two sides of a triangle , their resultant is given by the third side of the triangle.

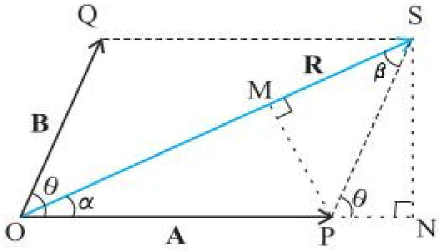


Parallelogram law of vector addition

If two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram ,then their resultant is given by the diagonal of the parallelogram.



Vector Addition – Analytical Method



SN is normal to OP and PM is normal to OS.

ΔSNP, $\cos\theta = PN / PS$ $\sin\theta = SN / PS$

$\cos\theta = PN / B$ $\sin\theta = SN / B$

$PN = B\cos\theta$ $SN = B\sin\theta$

From the geometry of the figure,

$$OS^2 = ON^2 + SN^2$$

$$\text{but } ON = OP + PN$$

$$= A + B\cos\theta$$

$$SN = B\sin\theta$$

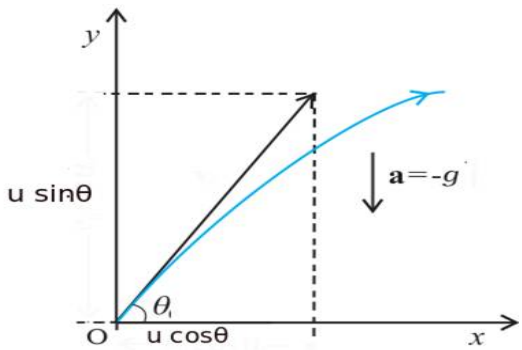
$$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R^2 = A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

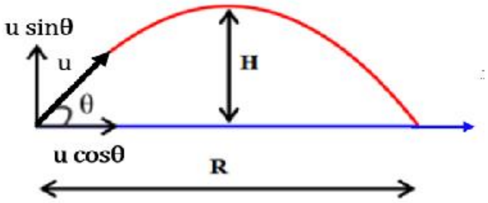
$$\mathbf{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

Motion in a Plane-Projectile Motion



- An object that is in flight after being thrown or projected is called a projectile.
- The path (trajectory)of a projectile is a parabole
- The components of initial velocity u are **u cos θ** along horizontal direction and **u sin θ** along vertical direction.
- The x-component of velocity(**u cos θ**) remains constant throughout the motion and hence there is no acceleration in horizontal direction,i.e., $a_x = 0$
- The y- component of velocity (**u sin θ**) changes throughout the motion. At the point of maximum height, **u sin θ** = 0. There is acceleration in horizontal direction, $a_y = -g$

Time of Flight of a projectile (T)



The total time T during which the projectile is in flight is called Time of Flight, T.
Consider the motion in vertical direction,

$$s = ut + \frac{1}{2} at^2$$

$$s=0,\ u = u\sin\theta,\ a = -g,\ t = T$$

$$0 = u\sin\theta\, T - \frac{1}{2} gT^2$$

$$\frac{1}{2} gT^2 = u\sin\theta\, T$$

$$T = \frac{2u\sin\theta}{g}$$

Horizontal range of a projectile (R)

The horizontal distance travelled by a projectile during its time of flight is called the horizontal range.

Horizontal range = Horizontal component of velocity x Time of flight

$$R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$R = \frac{u^2 \times 2\sin\theta\cos\theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

R is maximum when sin2θ is maximum, i.e., when θ = 45 ^{0}.

$$R_{max} = \frac{u^2}{g}$$

For a given velocity of projection range will be same for angles *θ* and (90-*θ*)

Maximum height of a projectile (H)

It is the maximum height reached by the projectile.
Consider the motion in vertical direction to the highest point

$$v^2 - u^2 = 2as$$

$$u = u\sin\theta,\ v = 0,\ a = -g,\ s = H$$

$$0 - u^2\sin^2\theta = -2gH$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

Uniform Circular Motion

When an object follows a circular path at a constant speed, the motion of the object is called **uniform circular motion**. The word "uniform" refers to the speed, which is uniform (constant) throughout the motion.

Period

The time taken by an object to make one revolution is known as its time period T

Frequency

The number of revolutions made in one second is called its frequency.

$$\upsilon = \frac{1}{T}$$ unit - hertz (Hz)

Angular velocity (ω)

angular velocity is the time rate of change of angular displacement

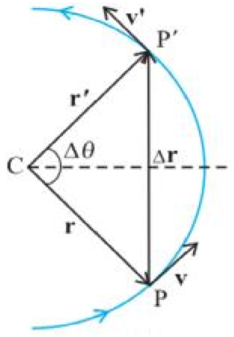
$$\omega = \frac{d\,\theta}{dt}$$

Unit is rad/s

During the time period T ,the angular displacement is 2π radian

$$\omega = \frac{2\pi}{T} \quad \text{or} \quad \omega = 2\pi\,\upsilon$$

Relation connecting angular velocity and linear velocity



$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

$$\Delta\,\theta = \frac{\Delta\,r}{r}$$

$$\Delta\,r = r\ \Delta\,\theta$$

Linear velocity $v = \frac{\Delta\,r}{\Delta t}$

$$v = \frac{r\,\Delta\,\theta}{\Delta t}$$

But $\omega = \frac{\Delta\,\theta}{\Delta t}$

$$v = r\,\omega$$

Angular Acceleration

The rate of change of angular velocity is called angular acceleration.

$$\alpha = \frac{d\,\omega}{dt}$$

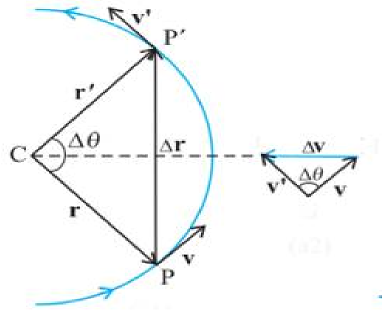
But $\omega = \frac{d\,\theta}{dt}$

$$\alpha = \frac{d}{dt}\left(\frac{d\,\theta}{dt}\right)$$

$$\alpha = \frac{d^2\,\theta}{dt}$$

Centripetal acceleration

A body in uniform circular motion experiences an acceleration , which is directed towards the centre along its radius .This is s called centripetal acceleration .



$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

$$\Delta v = \frac{v \Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v \Delta r}{r \Delta t}$$

$$a = \frac{v}{r} \text{ x } r$$

$$a = \frac{v^2}{r}$$

If R is the radius of circular path, then centripetal acceleration .

$$\mathbf{a_c = \frac{v^2}{R}}$$

Example

An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s.

(a) What is the angular speed, and the linear speed of the motion?
(b) Is the acceleration vector a constant vector ? What is its magnitude ?

Period, $T = \frac{100}{7}$ s

(a) The angular speed ω is given by

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{100}{7}} = \frac{2\pi \text{ x} 7}{100} = 0.44 \text{ rad/s}$$

The linear speed v is :

$$v = \omega R = 0.44 \times 0.12 = 5.3 \text{ x } 10^{-2} \text{ m s}^{-1}$$

(b) The direction of velocity v is along the tangent to the circle at every point. The acceleration is directed towards the centre of the circle. Since this direction changes continuously, acceleration here is not a constant vector.

$$a = \omega^2 R = (0.44)^2 \text{x} 0.12 = 2.3 \text{x} 10^{-2} \text{ m s}^{-2}$$

Chapter 5
Laws of Motion

Newton's First Law of Motion (Law of inertia)

Every body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to change that state.

Momentum

Momentum, P of a body is defined to be the product of its mass m and velocity v, and is denoted by p.

$$p = m\,v$$

Momentum is a vector quantity.

Unit = kgm/s

$[p] = ML\,T^{-1}$

Newton's Second Law f Motion

The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

$$F \propto \frac{\Delta p}{\Delta t}$$

$$F = \frac{dp}{dt}$$

Why a cricketer draws his hands backwards during a catch?

By Newton's second law of motion ,

$$F = \frac{\Delta p}{\Delta t}$$

When he draws his hands backwards, the time interval (Δt) to stop the ball increases . Then force decreases and it does not hurt his hands.

Force not only depends on the change in momentum but also on how fast the change is brought about.

Derivation of Equation of force from Newton's second law of motion

By Newton's second law of motion ,

$$F = \frac{dp}{dt}$$

For a body of fixed mass m, p=mv

$$F = \frac{d}{dt}mv$$

$$F = m\frac{dv}{dt}$$

$$F = ma$$

Force is a vector quantity

Unit of force is $kgms^{-2}$ or newton (N)

Impulse (I)

Impulse is the the product of force and time duration, which is the change in momentum of the body.

Impulse = Force × time duration

$$I = F\,x\,t$$

Unit = kg m s^{-1} $\quad$ $[I] = M\,L\,T^{-1}$

Impulsive force

A large force acting for a short time to produce a finite change in momentum is called an impulsive force.

Example: a ball hits a wall and bounces back.

Impulse momentum Principle

Impulse is equal to the change in momentum of the body.

By Newton's second law of motion,

$$F = \frac{dp}{dt}$$

$$F\,x\,dt = dp$$

$$I = dp$$

Impulse = change in momentum

Example

A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 m s^{-1} . If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball.

Impulse = change of momentum

Change in momentum = final momentum – initial momentum

Change in momentum = $0.15 \times 12 - (0.15 \times -12)$

Impulse = 3.6 N s

Newton's Third Law of Motion

To every action, there is always an equal and opposite reaction.

- Action and reaction forces act on different bodies, not on the same body. So they do not cancel each other , eventhough they are equal and opposite.

Law of Conservation of Momentum

The total momentum of an isolated system of interacting particles is conserved.

Or

When there is no external force acting on a system of particles ,their total momentum remains constant.

Proof of law of conservation of momentum

By Newton's second law of motion , $F=\frac{dp}{dt}$

When $F = 0$

$$\frac{dp}{dt} = 0$$

$$dp = 0 ,$$

$$p=\text{constant}$$

Thus when there is no external force acting on a system of particles, their total momentum remains constant.

Applications of law of conservation of linear momentum

1.Recoil of gun

When a bullet is fired from a gun , the backward movement of gun is called recoil of the gun.

If p_b and p_g are the momenta of the bullet and gun after firing

$$\mathbf{p_b + p_g = 0}$$

$$\mathbf{p_b = - p_g}$$

The negative sign shows that the gun recoils to conserve momentum.

Expression for Recoil velocity and muzzle velocity

Momentum of bullet after firing , $p_b = mv$

Recoil momentum of the gun after firing , $p_g = MV$

$$\mathbf{p_b = - p_g}$$

$$\mathbf{mv = -MV}$$

Recoil velocity of gun , $V=\frac{-mv}{M}$

Muzzle velocity of bullet , $v=\frac{-MV}{m}$

M= mass of gun, V= recoil velocity of bullet

m= mass of bullet, v=muzzle velocity of bullet

2. The collision of two bodies

By Newton's second law , $F = \frac{\Delta P}{\Delta t}$

$$F\Delta t = \Delta P$$

F_{AB} changes the momentum of body A

$$F_{AB}\,\Delta t = p'_A - p_A \text{-----------------}(1)$$

F_{BA} changes the momentum of body B

$$F_{BA}\,\Delta t = p'_B - p_B \text{-----------}(2)$$

By Newton's third law

$$F_{AB} = -F_{BA} \qquad \text{-----------}(3)$$

$$p'_A - p_A = -(p'_B - p_B)$$

$$p'_A + p'_B = p_A + p_B$$

Total Final momentum = Total initial momentum

i.e. , the total final momentum of the isolated system equals its total initial momentum.

Common Forces in Mechanics

There are two types of forces in mechanics- Contact forces and Non contact forces.

Contact forces

A contact force on an object arises due to contact with some other object: solid or fluid.

Eg: Frictional force, viscous force, air resistance

Non contact forces

A non contact force can act at a distance without the need of any intervening medium.

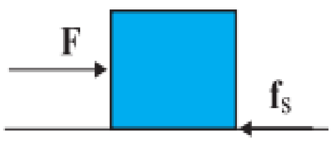
Eg: Gravitational force.

Friction

The force that opposes (impending or actual) relative motion between two surfaces in contact is called frictional force.

There are two types of friction-Static and Kinetic friction

Static friction f_s



Static friction is the frictional force that acts between two surfaces in contact before the actual relative motion starts. Or Static friction f_s opposes impending relative motion.

- The maximum value of static friction is $(f_s)_{max}$
- The limiting value of static friction $(f_s)_{max}$, is independent of the area of contact.
- The limiting value of static friction $(f_s)_{max}$, varies with the normal force(N)

$$(f_s)_{max}\ \alpha N$$

$$(f_s)_{max} = \mu_s N$$

Where the **constant μ_s is called the coefficient of static friction** and depends only on the nature of the surfaces in contact.

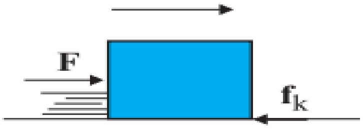
The Law of Static Friction

The law of static friction may thus be written as , $fs \leq \mu_s N$

Or

$$(f_s)_{max} = \mu_s N$$

Kinetic friction f_k



Frictional force that opposes (actual) relative motion between surfaces in contact is called kinetic or sliding friction and is denoted by f_k. .

- Kinetic friction is independent of the area of contact.
- Kinetic friction is nearly independent of the velocity.
- Kinetic friction , f_k varies with the normal force(N)

$$f_k \alpha N$$

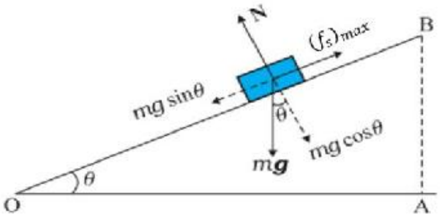
$$\mathbf{f_k = \mu_k N}$$

where μ_k the **coefficient of kinetic friction**, depends only on the surfaces in contact.
μ_k is less than μ_s

The Law of Kinetic Friction

The law of kinetic friction can be written as , $\mathbf{f_k = \mu_k N}$
where μ_k the **coefficient of kinetic friction**,

Body on an inclined surface



The forces acting on a block of mass m When it just begins to slide are
(i) the weight mg
(ii) the normal force N
(iii) the maximum static frictional force $(\mathbf{f_s})_{\mathbf{max}}$

In equilibrium, the resultant of these forces must be zero.

$$m\,g \sin\theta = (f_s)_{max}$$

$$\text{But } (f_s)_{max} = \mu_s N$$

$$mg \sin\theta = \mu_s N \text{------------}(1)$$

$$m\,g \cos\theta = N \text{-------------}(2)$$

$$\text{Eqn}\frac{(1)}{(2)} \text{ --------- } \frac{mg \sin\theta}{m\,g \cos\theta} = \frac{\mu_s N}{N}$$

$$\mathbf{\mu_s = \tan\theta}$$

This angle whose tangent gives the coefficient of friction is called angle of friction.

Rolling Friction

It is the frictional force that acts between the surfaces in contact when one body rolls over the other.
Rolling friction is much smaller than static or sliding friction

Disadvantages of friction

Friction is undesirable in many situations, like in a machine with different moving parts, friction opposes relative motion and thereby dissipates power in the form of heat, etc. Friction produces wear and tear.

Advantages of friction

In many practical situations friction is critically needed. Kinetic friction is made use of by brakes in machines and automobiles. We are able to walk because of static friction. It is impossible for a car to move on a very slippery road. On an ordinary road, the friction between the tyres and the road provides the necessary external force to accelerate the car.

Methods to reduce friction

(1)Lubricants are a way of reducing kinetic friction in a machine.
(2)Another way is to use ball bearings between two moving parts of a machine.
(3) A thin cushion of air maintained between solid surfaces in relative motion is an effective way of reducing friction.

Circular Motion

The acceleration of a body moving in a circular path is directed towards the centre and is called centripetal acceleration.

$$\mathbf{a = \frac{v^2}{R}}$$

The force f providing centripetal acceleration is called the centripetal force and is directed towards the centre of the circle.

$$\mathbf{f_s = \frac{mv^2}{R}}$$

where m is the mass of the body, R is the radius of circle.

Motion of a car on a curved level road

Three forces act on the car.

(i) The weight of the car, mg

(ii) Normal reaction, N

(iii) Frictional force, f_s

As there is no acceleration in the vertical direction

$$N = mg$$

The static friction provides the centripetal acceleration

$$f_s = \frac{mv^2}{R}$$

But , $f_s \leq \mu_s N$

$$\frac{mv^2}{R} \leq \mu_s mg \quad (N=mg)$$

$$\mathbf{v^2 \leq \mu_s Rg}$$

$$\mathbf{v_{max} = \sqrt{\mu_s Rg}}$$

This is the maximum safe speed of the car on a circular level road.

Motion of a car on a banked road

Raising the outer edge of a curved road above the inner edge is called banking of curved roads.

$$N \cos\theta = mg + f \sin\theta$$

$$N \cos\theta - f \sin\theta = mg \text{ --------------(1)}$$

The centripetal force is provided by the horizontal components of N and f_s.

$$N \sin\theta + f \cos\theta = \frac{mv^2}{R} \text{ -------------(2)}$$

$$\frac{Eqn(1)}{Eqn(2)} \text{ -----} \frac{N \cos\theta - f \sin\theta}{N \sin\theta + f \cos\theta} = \frac{mg}{\frac{mv^2}{R}}$$

Dividing throughout by N cos θ

$$\frac{1 - \frac{f}{N} \tan\theta}{\tan\theta + \frac{f}{N}} = \frac{Rg}{v^2}$$

But , $\frac{f}{N} = \mu_s$ for maximum speed

$$\frac{1 - \mu_s \tan\theta}{\tan\theta + \mu_s} = \frac{Rg}{v^2}$$

$$v^2 = \frac{Rg(\mu_s + \tan\theta)}{1 - \mu_s \tan\theta}$$

$$\mathbf{v_{max} = \sqrt{\frac{Rg(\mu_s + \tan\theta)}{1 - \mu_s \tan\theta}}}$$

This is the maximum safe speed of a vehicle on a banked curved road.

If friction is absent, $\mu_s = 0$

Then Optimum speed, $\mathbf{v_{op}}$

Units of Work and Energy

- Work and Energy are scalar quantities.
- Work and energy have the same dimensions, $[ML^2 T^{-2}]$.
- The SI unit is **kgm^2s^{-2}** or **joule (J)**, named after the famous British physicist James Prescott Joule.

Alternative Units of Work/Energy in J

erg	10^{-7} J
electron volt (eV)	1.6×10^{-19} J
calorie (cal)	4.186 J
kilowatt hour (kWh)	3.6×10^{6} J

Kinetic Energy

The kinetic energy is the energy possessed by a body by virtue of its motion.

If an object of mass m has velocity v, its kinetic energy K is

$$\mathbf{K = \frac{1}{2} m\bar{v} \cdot \bar{v} = \frac{1}{2} mv^2}$$

Kinetic energy is a scalar quantity.

The Work-Energy Theorem

The work-energy theorem can be stated as :**The change in kinetic energy of a particle is equal to the work done on it by the net force.**

Proof

For uniformly accelerated motion

$$v^2 - u^2 = 2as$$

Multiplying both sides by $\frac{1}{2}m$, we have

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas = Fs$$

$$K_f - K_i = W$$

Change in KE = Work

Potential Energy

Potential energy is the 'stored energy' by virtue of the position or configuration of a body.

- A body at a height h above the surface of earth possesses potential energy due to its position.
- A Stretched or compressed spring possesses potential energy due to its state of strain.

Gravitational potential energy of a body of mass m at a height h above the surface of earth is mgh.

Gravitational Potential Energy , V =mgh

Conservative Force

A force is said to be conservative, if it can be derived from a scalar quantity.

$$F = \frac{-dV}{dx} \text{ where V is a scalar}$$

Eg: Gravitational force, Spring force.

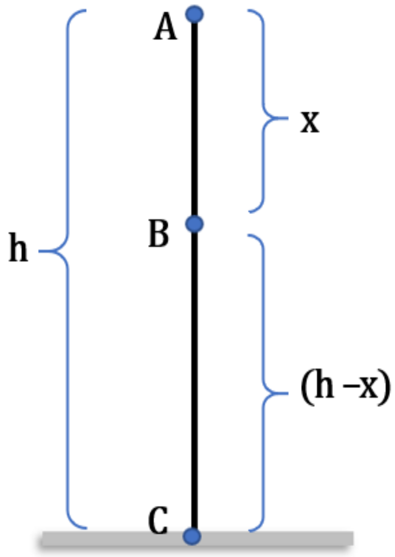
- The work done by a conservative force depends only upon initial and final positions of the body
- The work done by a conservative force in a cyclic process is zero

Note: Frictional force , air resistance are non conservative forces.

The Conservation of Mechanical Energy

The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.

Conservation of Mechanical Energy for a Freely Falling Body



At Point A

PE = mgh

KE = 0 (since v=0)

TE = PE + KE

= mgh + 0

TE = mgh----------(1)

At Point B

PE = mg (h-x)

KE = ½ mv^2

$v^2 = 2gx$

KE = ½ m x 2gx

KE= mgx

TE = PE + KE

TE = mg (h-x) + mgx

TE = mgh-------------(2)

At Po int C

PE = 0 (Since h=0)

KE = ½ mv^2

$v^2 = 2gh$

KE = ½ m x 2gh

KE= mgh

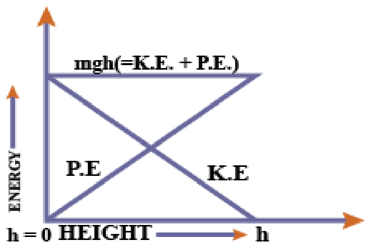
TE = PE + KE

TE = 0 + mgh

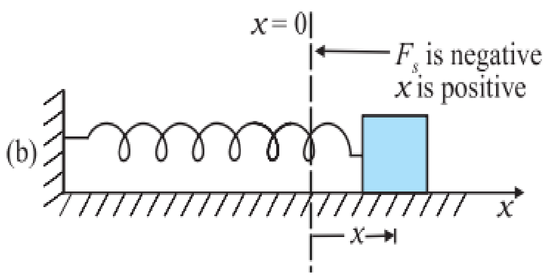
TE = mgh-------------(3)

From eqns (1), (2) and (3), it is clear that the total mechanical energy is conserved during the free fall.

Graphical variation of KE and PE with height from ground



The Potential Energy of a Spring



The the spring force **F = – kx**

The work done by the spring force is

$$W = \int_0^x F\,dx$$

$$W = -\int_0^x kx\,dx$$

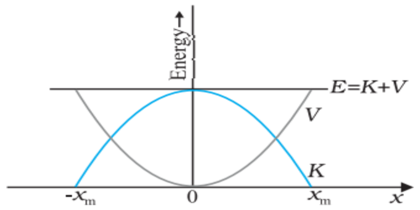
$$W = -\frac{1}{2}kx^2$$

his work is stored as potential energy of spring

$$\mathbf{PE = \frac{1}{2}kx^2}$$

- **At equilibrium position PE is zero and KE is max.**
- **At extreme ends, the PE is maximum and KE is zero.**
- The kinetic energy gets converted to potential energy and vice versa, however, the total mechanical energy remains constant.

Graphical variation of kinetic Energy and potential of a spring



The Equivalence of Mass and Energy

Mass and energy are equivalent and are related by the relation

$$E = m\,c^2$$

This is called Einstein's mass energy relation.

where c, the speed of light in vacuum is approximately $3 \times 10^8 \mathrm{m}\, s^{-1}$.

The Principle of Conservation of Energy

Energy can neither be created, nor destroyed. Energy may be transformed from one form to another but the total energy of an isolated system remains constant.

Power

Power is defined as the time rate at which work is done or energy is transferred.
The average power of a force is defined as the ratio of the work, W, to the total time t taken.

$$P_{av} = \frac{W}{t}$$

The instantaneous power

The instantaneous power is defined as the limiting value of the average power as time interval approaches zero.

$$P = \frac{dW}{dt}$$

The work done, $dW = F.\,dr$.

$$P = F\,.\frac{dr}{dt}$$

$$P = F\,.\,v$$

- SI unit of power is called a watt (W). 1W = 1 J/s
- Another unit of power is the horse-power (hp) . **1 hp = 746 W**
  This unit is still used to describe the output of automobiles, motorbikes, etc

kilowatt hour

Electrical energy is measured in kilowatt hour (kWh).

$$1\mathrm{kWh} = 3.6 \times 10^6\ \mathrm{J}$$

Collisions

In all collisions the total linear momentum is conserved; the initial momentum of the system is equal to the final momentum of the system. There are two types of collisions Elastic and Inelastic.

Elastic Collisions

The collisions in which both linear momentum and kinetic energy are conserved are called elastic collisions.
Eg: Collision between sub atomic particles

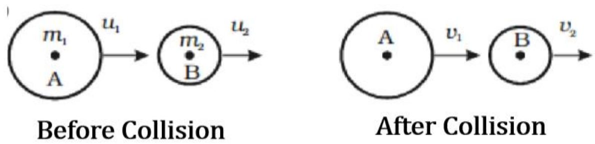
Inelastic Collisions

The collisions in which linear momentum is conserved, but kinetic energy is not conserved are called inelastic collisions. . Part of the initial kinetic energy is transformed into other forms of energy such as heat,sound etc..
Eg: Collision between macroscopic objects

A collision in which the two particles move together after the collision is a perfectly inelastic collision.

Elastic Collisions in One Dimension

If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a **one-dimensional collision, or head-on collision.**



Consider two masses m_1 and m_2 making elastic collision in one dimension.
By the conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{--------------}(1)$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)\text{----------------}(2)$$

By the conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\text{-----------}(3)$$

$$\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$$

$$\frac{1}{2}m_1\,(u_1^2 - v_1^2) = \frac{1}{2}m_2(v_2^2 - u_2^2)$$

$$m_1\,(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{-------------}(4)$$

$$\text{Eqn}\,\frac{(4)}{(2)} \quad \text{------------} \quad \frac{m_1\,(u_1^2 - v_1^2)}{m_1(u_1 - v_1)} = \frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)}$$

$$\frac{(u_1+v_1)\,(u_1-v_1)}{(u_1-v_1)} = \frac{(v_2+u_2)(v_2-u_2)}{(v_2-u_2)}$$

$$u_1 + v_1 = v_2 + u_2 \quad \text{-------------}(5)$$

$$\mathbf{u_1 - u_2 = -(v_1 - v_2)\text{--------}(6)}$$

i.e., relative velocity before collision is numerically equal to relative velocity after collision.

From eqn(5), $\quad v_2 = u_1 + v_1 - u_2$

Substituting in eqn (1)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$$

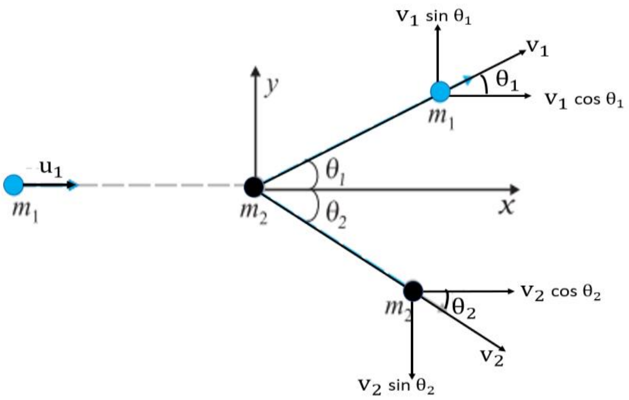
$$m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2 = m_1v_1 + m_2v_1$$

$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

$$\mathbf{v_1 = \frac{(m_1-m_2)u_1}{m_1+m_2} + \frac{2m_2u_2}{m_1+m_2}} \text{ ------- } \mathbf{(7)}$$

Similarly, $$\mathbf{v_2 = \frac{(m_2-m_1)u_2}{m_1+m_2} + \frac{2m_1u_1}{m_1+m_2}} \text{ ------- } \mathbf{(8)}$$

Elastic Collisions in Two Dimensions



Consider the elastic collision of a moving mass m_1 with the stationary mass m_2.
Since momentum is a vector ,it has 2 equations in x and y directions.
Equation for conservation of momentum in x direction

$$\mathbf{m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2}$$

Equation for conservation of momentum in y direction

$$0 = \mathbf{m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2}$$

Equation for conservation of kinetic energy,(KE is a scalar quantity)

$$\frac{1}{2}\mathbf{m_1u_1^2} = \frac{1}{2}\mathbf{m_1v_1^2} + \frac{1}{2}\mathbf{m_2v_2^2}$$

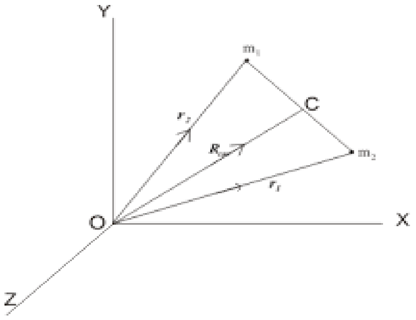
Chapter 7
Systems of Particles and Rotational Motion

Rigid Body

Ideally a rigid body is a body with a perfectly definite and unchanging shape. The distances between different pairs of such a body do not change.

Centre Of Mass

The centre of is a hypothetical point where the entire mass of an object may be assumed to be concentrated to visualise its motion.



Consider a two particle system. Let C be the centre of mass which is at a distancev X from origin.

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M} \quad \text{where } M = m_1 + m_2$$

Motion of Centre of Mass

- **Position vector of centre of mass**

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots \ldots\ldots + m_n\vec{r}_n}{M} \quad \text{--------(1)}$$

where $M = m_1 + m_2 + \ldots\ldots + m_n$

- **Velocity of centre of mass**

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots \ldots\ldots + m_n\vec{v}_n}{M} \quad \text{--------------(2)}$$

- **Acceleration of centre of mass**

$$\vec{A} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots \ldots\ldots + m_n\vec{a}_n}{M} \quad \ldots\ldots\ldots\ldots(3)$$

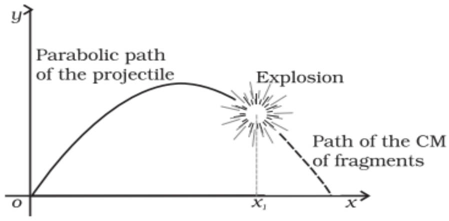
- **Force on centre of mass**

$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots \ldots \ldots . . + m_n\vec{a}_n$$

$$\vec{F}_{ext} = \vec{F}_1 + \vec{F}_2 + \cdots \ldots \ldots . . + \vec{F}_n$$

$$\vec{F}_{ext} = M\vec{A}$$

The centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.



The centre of mass of the fragments of the projectile continues along the same parabolic path which it would have followed if there were no explosion.

- **Linear Momentum of centre of mass**

  Velocity of centre of mass

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots \ldots\ldots + m_n\vec{v}_n}{M}$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots \ldots \ldots .. + m_n\vec{v}_n$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots \ldots \ldots .. + \vec{p}_n$$

Law of Conservation of Momentum for a System of Particles

If Newton's second law is extended to a system of particles,

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

When the sum of external forces acting on a system of particles is zero

$$\vec{F}_{ext} = 0$$

$$\frac{d\vec{P}}{dt} = 0$$

$$\vec{P} = \text{constant}$$

Thus, when the total external force acting on a system of particles is zero, the total linear momentum of the system is constant. This is the law of conservation of the total linear momentum of a system of particles.

$$\text{But } \vec{P} = M\vec{V}$$

$$M\vec{V} = \text{constant}$$

$$\vec{V} = constant$$

When the total external force on the system is zero the velocity of the centre of mass remains constant or the CM of the system is in uniform motion.

Vector Product or Cross product of Two Vectors

Vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as $\boldsymbol{\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}}$

where A and B are magnitudes of $\vec{A}$ and $\vec{B}$

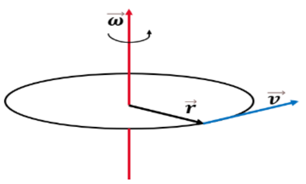
$\boldsymbol{\theta}$ is the angle between $\vec{A}$ and $\vec{B}$

$\hat{n}$ is the unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$

The direction of $\vec{A} \times \vec{B}$ is given by right hand screw rule or right hand rule.

- $\hat{\imath} \times \hat{\imath} = 0$, $\hat{\jmath} \times \hat{\jmath} = 0$, $\hat{k} \times \hat{k} = 0$
- $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$
- $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, $\hat{k} \times \hat{\jmath} = -\hat{\imath}$, $\hat{\imath} \times \hat{k} = -\hat{\jmath}$

Angular Velocity and its Relation with Linear Velocity



The angular velocity is a vector quantity. $\vec{\omega}$ is directed along the fixed axis as shown.

The linear velocity of the particle is

$$\vec{v} = \vec{\omega} \times \vec{r}$$

It is perpendicular to both $\vec{\omega}$ and $\vec{r}$ and is directed along the tangent to the circle described by the particle.

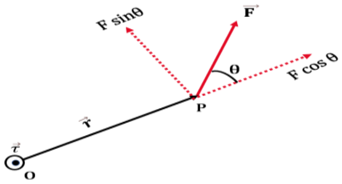
Angular acceleration

Angular acceleration $\vec{\alpha}$ is defined as the time rate of change of angular velocity.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

Torque or Moment of Force

The rotational analogue of force is torque or moment of force .



If a force $\vec{F}$ acts on a single particle at a point P whose position with respect to the origin O is $\vec{r}$,then torque about origin o is

$$\vec{\tau} = r\,F\,\sin\theta$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- Torque has dimensions $M\,L^2T^{-2}$
- Torque is a vector quantity
- The SI unit of moment of force is Newton-metre (Nm)

Angular momentum of a particle

Angular momentum is the rotational analogue of linear momentum.

Angular momentum is a vector quantity. It could also be referred to as moment of (linear) momentum.

$$\vec{l} = \vec{r} \times \vec{p}$$

$$\vec{l} = rp\sin\theta$$

Relation connecting Torque and Angular momentum

$$\vec{l} = \vec{r} \times \vec{p}$$

Differentiating

$$\frac{d\vec{l}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$\frac{d\vec{l}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\vec{p} = m\vec{v} \quad , \frac{d\vec{r}}{dt} = \vec{v}\,, \frac{d\vec{p}}{dt} = \vec{F}$$

$$\frac{d\vec{l}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times \vec{F}$$

$$\vec{v} \times \vec{v} = 0\,, \;\; (\vec{r} \times \vec{F} = \vec{\tau}\,)$$

$$\frac{d\vec{l}}{dt} = 0 + \vec{\tau}$$

$$\vec{\tau} = \frac{d\vec{l}}{dt}$$

Thus, the time rate of change of the angular momentum of a particle is equal to the torque acting on it.

This is the rotational analogue of the equation $\vec{F} = \frac{d\vec{p}}{dt}$, which expresses Newton's second law for the translational motion of a single particle.

Relation connecting Torque and Angular momentum for a system of particles

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

where $\vec{L} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_n$

Law of Conservation of Angular momentum

For a system of particles

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

If external torque $\vec{\tau}_{ext} = 0$,

$$\frac{d\vec{L}}{dt} = 0$$

$$\vec{L} = \text{constant}$$

If the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved i.e, remains constant.

Equilibrium of a Rigid Body

A rigid body is said to be in mechanical equilibrium, if it is in both translational equilibrium and rotational equilibrium.

i.e, for a body in mechanical equilibrium its linear momentum and angular momentum are not changing with time.

Translational Equilibrium

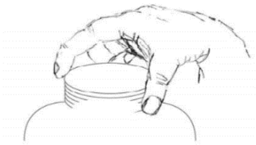
When the total external force on the rigid body is zero, then the total linear momentum of the body does not change with time and the body will be in translational equilibrium .

Rotational Equilibrium

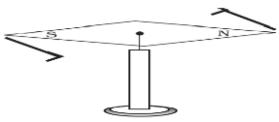
When the total external torque on the rigid body is zero, the total angular momentum of the body does not change with time and the body will be in rotational equilibrium .

Couple

A pair of equal and opposite forces with different lines of action is known as a couple. A couple produces rotation without translation.

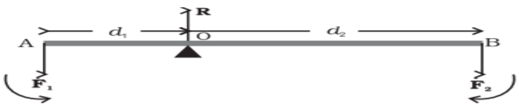


Our fingers apply a couple to turn the lid



The Earth's magnetic field exerts equal and opposite forces on the poles of a compass needle. These two forces form a couple.

Principles of Moments



The lever is a system in mechanical equilibrium.

For rotational equilibrium the sum of moments must be zero,

$$\mathbf{d_1F_1 - d_2F_2 = 0}$$

The equation for the principle of moments for a lever is

$$\mathbf{d_1F_1 = d_2F_2}$$

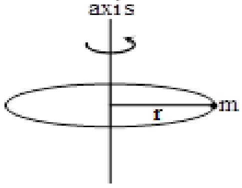
$$\textbf{load arm} \times \textbf{load} = \textbf{effort arm} \times \textbf{effort}$$

$$\text{Mechanical Advantage MA} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

Moment of Inertia

Moment of Inertia is the rotational analogue of mass.

Moment of inertia is a measure of rotational inertia



The moment of inertia of a particle of mass m rotating about an axis is

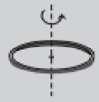
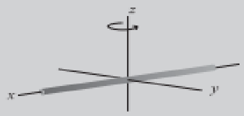
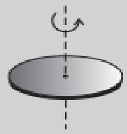
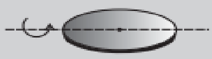
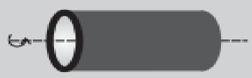
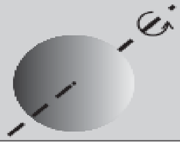
$$\mathbf{I = mr^2}$$

The moment of inertia of a rigid body is

$$I = \sum_{i=1}^{n} m_i r_i^2$$

The moment of inertia of a rigid body depends on the mass of the body, its shape and size; distribution of mass about the axis of rotation, and the position and orientation of the axis of rotation.

Moments of Inertia of some regular shaped bodies about specific axes

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius R	Perpendicular to plane, at centre		MR^2
(2)	Thin circular ring, radius R	Diameter		$MR^2/2$
(3)	Thin rod, length L	Perpendicular to rod, at mid point		$ML^2/12$
(4)	Circular disc, radius R	Perpendicular to disc at centre		$MR^2/2$
(5)	Circular disc, radius R	Diameter		$MR^2/4$
(6)	Hollow cylinder, radius R	Axis of cylinder		MR^2
(7)	Solid cylinder, radius R	Axis of cylinder		$MR^2/2$
(8)	Solid sphere, radius R	Diameter		$2MR^2/5$

Rotational Kinetic energy

Cosider a particle of mass m rotating about an axis of radius r with angular velocity ω
The kinetic energy of motion of this particle is

$$kE = \frac{1}{2}mv^2$$

But $v = r\,\omega$

$$kE = \frac{1}{2}mr^2\omega^2$$

$I = mr^2$

$$\textbf{Rotational kE} = \frac{1}{2}I\omega^2$$

Radius of Gyration (k)

The radius of gyration can be defined as the distance of a mass point from the axis of roatation whose mass is equal to the whole mass of the body and whose moment of inertia is equal to moment of inertia of the whole body about the axis.

If K is the radius of gyration, we can write

$$I = Mk^2$$

$$k = \sqrt{\frac{I}{M}}$$

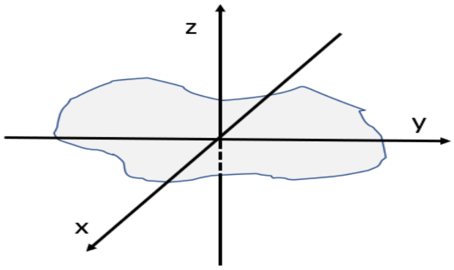
Theorems of Perpendicular and Parallel Axes

Perpendicular Axes Theorem

The moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

Or

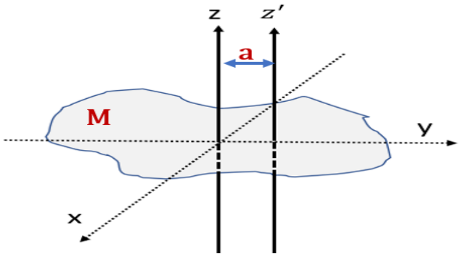
The moment of inertia of a plane lamina about z axis is equal to the sum of its moments of inertia about x-axis and y-axis, if the lamina lies in xy plane.



$$\mathbf{I_z = I_x + I_y}$$

(2) Parallel Axes Theorem

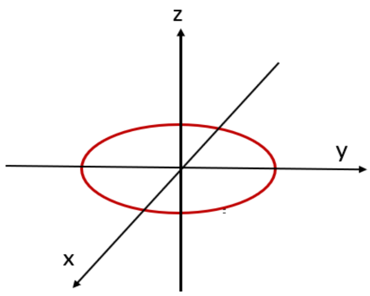
The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.



$$\mathbf{I_{z'} = I_z + Ma^2}$$

Applications Theorems of Momemt of Inertia

(1) Moment of Inertia of a Ring about One of its Diameter



By perpendicular axis theorem

$$I_z = I_x + I_y$$

But $I_x = I_y$

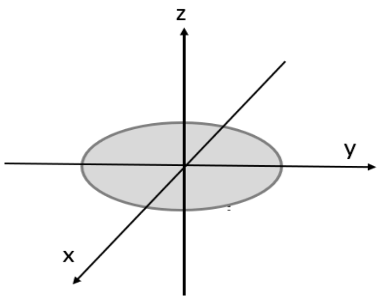
$$I_z = 2I_x$$

$$I_x = \frac{I_z}{2}$$

But $I_z = MR^2$

$$\mathbf{I_x = \frac{MR^2}{2}}$$

(2) Moment of Inertia of a Disc about One of its Diameter



By perpendicular axis theorem

$$I_z = I_x + I_y$$

But $I_x = I_y$

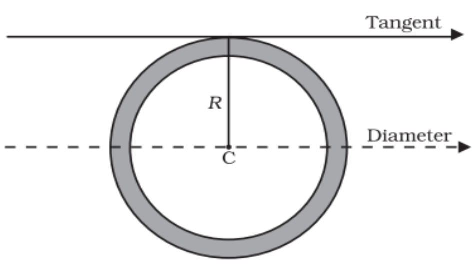
$$I_z = 2I_x$$

$$I_x = \frac{I_z}{2}$$

But $I_z = \frac{MR^2}{2}$

$$\mathbf{I_x = \frac{MR^2}{4}}$$

(3)Moment of inertia of a ring about a tangent to the circle of the ring



By parallel axis theorem

$$I_{z'} = I_z + Ma^2$$

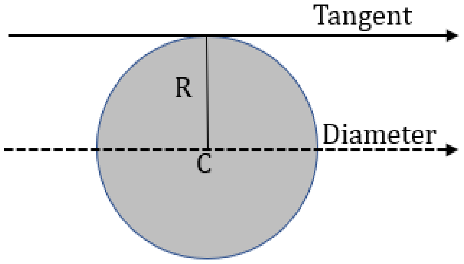
$$I_{tangent} = I_{\text{diameter}} + MR^2$$

But, $I_{\text{diameter}} = \frac{MR^2}{2}$

$$I_{tangent} = \frac{MR^2}{2} + MR^2$$

$$\mathbf{I_{tangent} = \frac{3}{2} MR^2}$$

(4)Moment of inertia of a disc about a tangent to the circle of the disc



By parallel axes theorem

$$I_{z'} = I_z + Ma^2$$

$$I_{tangent} = I_{diameter} + MR^2$$

$$\text{But, } I_{diameter} = \frac{MR^2}{4}$$

$$I_{tangent} = \frac{MR^2}{4} + MR^2$$

$$\mathbf{I_{tangent} = \frac{5}{4}MR^2}$$

Comparison of Translational and Rotational Motion

	Linear Motion	Rotational Motion about a Fixed Axis
1	Displacement x	Angular displacement θ
2	Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
3	Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
4	Mass M	Moment of inertia I
5	Force $F = Ma$	Torque $\tau = I\,\alpha$
6	Work $dW = F\,ds$	Work $W = \tau\, d\theta$
7	Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
8	Power $P = F\,v$	Power $P = \tau\omega$
9	Linear momentum $p = Mv$	Angular momentum $L = I\omega$

Conservation of angular momentum

If the external torque is zero, angular momentum is constant.

$$\vec{L} = \text{constant}$$

$$\text{But } \vec{L} = I\vec{\omega}$$

i.e., $\mathbf{I\vec{\omega} = \text{constant}}$

When I increases ,ω decreases and vice versa, so that Iω is constant.

While the chair is rotating with considerable angular speed , if you stretch your arms horizontally, moment of inertia(I) increases and as a result, the angular speed(ω) is reduced.

If you bring back your arms closer to your body, moment of inertia(I) decreases and as a result, the angular speed(ω) increases again.

Kinetic Energy of Rolling Motion

Rolling motion is a combination of rotation and translation. The kinetic energy of a rolling body is the sum of kinetic energy of translation and kinetic energy of rotation.

$$\text{Total KE} = \text{Translational KE} + \text{Rotational KE}$$

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$I = mk^2, \quad \text{where k} = \text{the radius of gyration}$$

$$\text{and } v = R\,\omega,\ \omega = \frac{v}{R}$$

$$K = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mk^2v^2}{R^2}$$

$$\mathbf{K = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)}$$

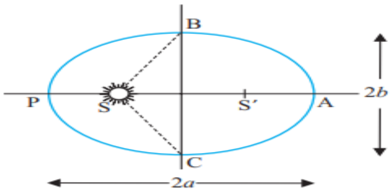
Here v is the velocity of centre of mass.

Chapter 8
Gravitation

Kepler's Laws

1.Law of orbits

All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse.



PA is the major axis
BC is the minor axis

2.Law of areas

The line that joins any planet to the sun sweeps equal areas in equal intervals of time. i.e, areal velocity $\frac{\Delta\vec{A}}{\Delta t}$ is constant

The planets move slower when they are farther from the sun than when they are nearer.

The law of areas is a consequence of conservation of angular momentum.

3.Law of periods

The square of the time period of revolution of a planet is proportional to the cube of the semi- major axis of the ellipse traced out by the planet.

$$T^2 \propto a^3$$

Universal Law of Gravitation

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them .

$$F = G\frac{m_1 m_2}{r^2}$$

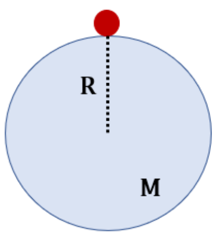
where G is the universal gravitational constant.

The Gravitational Constant

The value of the gravitational constant G was determined experimentally by English scientist Henry Cavendish in 1798. $G = 6.67\times10^{-11}\ N\ m^2/kg^2$

Acceleration due to gravity of the Earth

Consider a body of mass m on the surface of earth of mass M and radius R.



$$F = \frac{GMm}{R^2} \text{ -----------(1)}$$

By Newton's second law

$$F=mg \text{ -----------(2)}$$

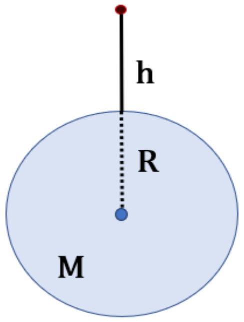
From Eq (1) and (2)

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2}$$

- **Acceleration due to gravity is independent of mass of the body.**
- The average value of g on the surface of earth is $9.8ms^{-2}$.

1.Acceleration due to gravity at a height h above the surface of the earth.



Acceleration due to gravity on the surface of earth

$$g = \frac{GM}{R^2} \text{ ------------(1)}$$

Acceleration due to gravity at a height above the surface of earth

$$g_h = \frac{GM}{(R+h)^2} \text{ ----------(2)}$$

for , h << R, $\quad g_h = \frac{GM}{R^2(1+\frac{h}{R})^2}$

$$g_h = \frac{GM}{R^2}(1+\frac{h}{R})^{-2}$$

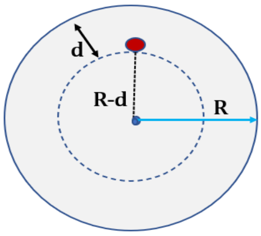
$$g_h = g(1+\frac{h}{R})^{-2}$$

Using binomial expression and neglecting higher order terms.

$$\mathbf{g_h \cong g\,(1-\frac{2h}{R})}$$

Thus, as we go above earth's surface, the acceleration due gravity decreases by a factor $(\mathbf{1-\frac{2h}{R}})$

2.Acceleration due to gravity at a depth d below the surface of the earth



Mass =volume x density

$$M = \frac{4}{3}\pi R^3 \rho \text{ ------------(1)}$$

Acceleration due to gravity on the surface of earth

$$g = \frac{GM}{R^2} \text{ -------------(2)}$$

$$g = \frac{G}{R^2}(\frac{4}{3}\pi R^3 \rho)$$

$$g = \frac{4}{3}\pi R \rho G \text{ ---------------(3)}$$

Acceleration due to gravity at a depth d below the surface of earth

$$g_d = \frac{4}{3}\pi(R-d)\rho G \text{ ---------(4)}$$

$$\frac{eq(4)}{eq(3)} \text{ ------ } \quad \frac{g_d}{g} = \frac{\frac{4}{3}\pi(R-d)\rho G}{\frac{4}{3}\pi R \rho G}$$

$$\frac{g_d}{g} = \frac{(R-d)}{R}$$

$$\mathbf{g_d = g(1-\frac{d}{R})}$$

Thus, as we go down below earth's surface, the acceleration due gravity decreases by a factor$(\mathbf{1-\frac{d}{R}})$

- The value of acceleration due to earth's gravity is maximum on its surface and decreases whether you go up or down.
- **At the centre of earth acceleration due to earth's gravity is zero.**

Example

At what height the value of acceleration due to gravity will be half of that on surface of earth. (Given the radius of earth R =6400km)

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$$g_h = g(1+\frac{h}{R})^{-2}$$

$$g_h = \frac{g}{2}$$

$$\frac{g}{2} = g(1+\frac{h}{R})^{-2}$$

$$\frac{1}{2} = (1+\frac{h}{R})^{-2}$$

$$2 = (1+\frac{h}{R})^{2}$$

$$\sqrt{2} = 1+\frac{h}{R}$$

$$\frac{h}{R} = \sqrt{2}-1$$

$$h = (\sqrt{2}-1)\,R = (1.414-1)\,6400 = 2650 \text{ km}$$

Gravitational Potential Energy

Gravitational potential energy at point is defined as the work done in displacing the particle from infinity to that point without acceleration.

The work done to give a displacement dx to the mass

$$dW=Fdx$$

$$dW=\frac{GMm}{x^2}dx$$

Total work done to move the mass from ∞ **to r**

$$W=\int_{\infty}^{r}\frac{GMm}{x^2}dx$$

$$W=\frac{-GMm}{r}$$

This work is is stored as gravitational PE in the body.

$$\mathbf{U=\frac{-GMm}{r}}$$

For unit mass m=1

So gravitational potential, $\mathbf{V=\frac{-GM}{r}}$

Escape speed

The minimum speed required for an object to reach infinity i.e. to escape from the earth's gravitational pull is called escape speed.

Let the body thrown from the surface of earth to infinity with velocity v_i.

$$\frac{1}{2}mv_i^2 - \frac{GMm}{r} = 0$$

$$\frac{1}{2}mv_i^2 = \frac{GMm}{R}$$

$$v_i^2 = \frac{2GM}{R}$$

$$\mathbf{v_e = \sqrt{\frac{2GM}{R}}}$$

- **Escape velocity is independent of mass of the body.**
- **Escape speed (o**

Relation Connecting Escape Velocity and Orbital Velocity

Orbital Velocity ,$v_0 = \sqrt{\frac{GM}{R}}$

Escape Velocity ,$v_e = \sqrt{\frac{2GM}{R}}$

$$\mathbf{v_e = \sqrt{2}\, v_o}$$

Escape Velocity = $\sqrt{2}$ x Orbital Velocity

Period of a Satellite

Period of a satellite is the time required for a satellite to complete one revolution around the earth in a fixed orbit.

$$\text{Period T} = \frac{\text{circumference of the orbit}}{\textit{orbital speed}}$$

$$T = \frac{2\pi\,(R+h)}{\sqrt{\frac{GM}{(R+h)}}}$$

$$\mathbf{T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}}$$

Energy of an orbiting satellite

$$KE = \frac{1}{2} m v_o^2$$

$$v_o = \sqrt{\frac{GM}{R+h}}\ , \quad v_o^2 = \frac{GM}{R+h}$$

$$KE = \frac{1}{2} m\ x \frac{GM}{R+h}$$

$$KE = \frac{GMm}{2(R+h)}$$

$$PE = \frac{-GMm}{R+h}$$

$$\text{Energy} = KE + PE$$

$$E = \frac{GMm}{2(R+h)} + \frac{-GMm}{R+h}$$

$$\mathbf{E = \frac{-GMm}{2(R+h)}}$$

The total energy of an circularly orbiting satellite is negative, which means that the satellite is bound to the planet .If the total energy is positive or zero, the object escapes to infinity. Satellites are always at finite distance from the earth and hence their energies cannot be positive or zero.

Geostationary Satellites

Satellites in a circular orbits around the earth in the equatorial plane with period, T = 24 hours are called Geostationery Satellites .

- As the earth rotates with the same period, the satellite would appear fixed from any point on earth.
- Geostationary satellites are at a height of e 35800 km from the surface of earth.
- Geostationary satellites are widely used for telecommunications.
- Eg. The INSAT group of satellites sent up by India are Geostationary satellites.

Polar Satellites

Satellites which revolve around earth along poles in a north-south direction are called polar satellites.

- These are low altitude (h= 500 to 800 km) satellites.
- Since its time period is around 100 minutes it crosses any altitude many times a day.
- These satellites can view polar and equatorial regions at close distances with good resolution.
- Polar satellites are useful for remote sensing, meterology as well as for environmental studies of the earth.

Weightlessness

When an object is in free fall, it is weightless and this is called the phenomenon of weightlessness.

In a satellite around the earth, every part of the satellite has an acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position. **Thus in the satellite everything inside it is in a state of free fall. Thus, in a manned satellite, people inside experience no gravity.**