P2-11-4-6

01466

CODE 6

PAPER 2

Time : 3 Hours **Maximum Marks : 240**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. The **question paper CODE** is printed on the right hand top corner of this sheet and on the back page (page No. 32) of this booklet.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed.
4. Write your name and registration number in the space provided on the back page of this booklet.
5. The answer sheet, a machine-gradable Optical Response Sheet (ORS), is provided separately.
6. DO NOT TAMPER WITH/MUTILATE THE ORS OR THE BOOKLET.
7. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.
8. This Question Paper contains 32 pages having 60 questions.
9. On breaking the seals, please check that all the questions are legible.

B. Filling the Right Part of the ORS :

10. The ORS also has a CODE printed on its Left and Right parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the codes do not match, ask for a change of the booklet.**
12. Write your Name, Registration No. and the name of centre and sign **with pen** in the boxes provided. **Do not write them anywhere else.** Darken the appropriate bubble **UNDER** each digit of your Registration No. with a **good quality HB pencil**.

C. Question paper format and Marking Scheme:

13. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections.**
14. In **Section I** (Total Marks: 24), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (-1)** mark will be awarded.
15. In **Section II** (Total Marks: 16), for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. There are **no negative marks** in this section.
16. In **Section III** (Total Marks: 24), for each question you will be awarded **4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are **no negative marks** in this section.
17. In **Section IV** (Total Marks: 16), for each question you will be awarded **2 marks** for each row in which you have darkened **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. Thus, each question in this section carries **a maximum of 8 marks**. There are **no negative marks** in this section.

Write your Name, registration number and sign in the space provided on the back page of this booklet

DO NOT BREAK THE SEALS WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

Useful Data

$R = 8.314\ JK^{-1}\ mol^{-1}$ or 8.206×10^{-2} L atm $K^{-1}\ mol^{-1}$

$1F = 96500\ C\ mol^{-1}$

$h = 6.626 \times 10^{-34}$ Js

$1\ eV = 1.602 \times 10^{-19}$ J

$c = 3.0 \times 10^{8}\ ms^{-1}$

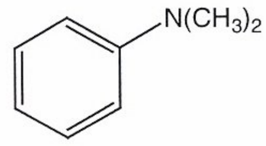
$N_A = 6.022 \times 10^{23}$

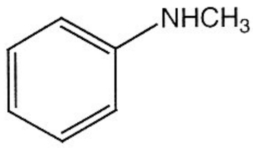
PART - I : CHEMISTRY

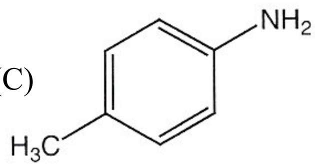
SECTION – I (Total Marks : 24)
Single Correct Answer Type

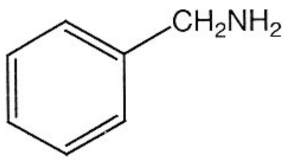
This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

1. Amongst the compounds given, the one that would form a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β–naphthol is

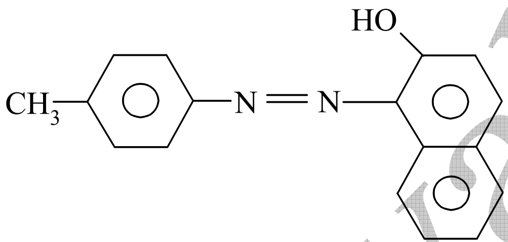
(A) 

(B) 

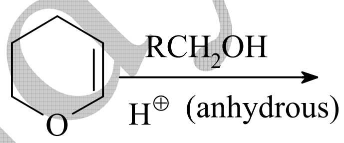
(C) 

(D) 

1. (C)

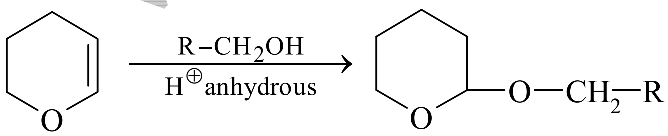


2. The major product of the following reaction is

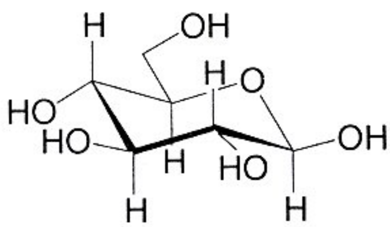


(A) a hemiacetal (B) an acetal (C) an ether (D) an ester

2. (B)



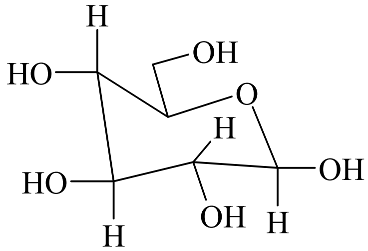
3. The following carbohydrate is



(A) a ketohexose (B) an aldohexose (C) an α – furanose (D) an α–pyranose

3. (B)

Carbohydrates is an aldohexase



β–pyranose

4. Oxidation states of the metal in the minerals haematite and magnetite, respectively, are

(A) II, III in haematite and III in magnetite
(B) II, III in haematite and II in magnetite
(C) II in haematite and II, III in magnetite
(D) III in haematite and II, III in magnetite

4. (D)

$Fe_2\,O_3 \longrightarrow$ 2 Fe + 3(0) = 0
(Haematite) 2 Fe + 3(−2) = 0
2 Fe = 6
Fe = +3

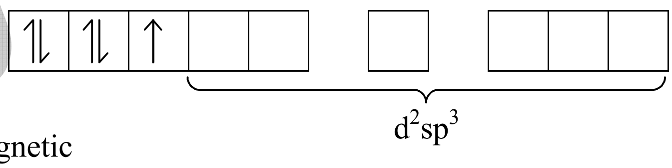
$Fe_3\,O_4 \longrightarrow FeO + FeO_3$
(Magnetite) +2 +3

5. Among the following complexes (K – P),
$K_3[Fe(CN)_6]$ (K) , $[Co(NH_3)_6]Cl_3$ (L) , $Na_3[Co(oxalate)_3]$ (M) , $[Ni(H_2O)_6]Cl_2$ (N), $K_2[Pt(CN)_4]$ (O) and $[Zn(H_2O)_6]\,(NO_3)_2$ (P)
The diamagnetic complexes are

(A) K, L, M, N (B) K, M, O, P (C) L, M, O, P (D) L, M, N, O

5. (C)

$K_3\,[Fe\,(CN)_6] \rightarrow Fe^{+3}$
Paramagnetic



$[Co(NH_3)_6]Cl_3$ → diamagnetic
$Na_3[Co(oxalate)_3]$ → diamagnetic
$[Zn(H_2O)_6]\,(NO_3)_2$ → diamagnetic
$K_2[Pt(CN)_4]$ → diamagnetic

6. Passing H_2S gas into a mixture of Mn^{2+} , Ni^{2+} , Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates

(A) CuS and HgS (B) MnS and CuS (C) MnS and NiS (D) NiS and HgS

6. (A)

Group II → Hg^{2+} and Cu^{+2}

7. Consider the following cell reaction :

$$2Fe_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + 2H_2O_{(\ell)} \qquad E° = 1.67\ V$$

At $[Fe^{2+}] = 10^{-3}$ M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25°C is

(A) 1.47 V (B) 1.77 V (C) 1.87 V (D) 1.57 V

7. (D)

$$E = 1.67 - \frac{0.0591}{4}\log_{10}\frac{[Fe^{+2}]^2}{[H^+]^4(P_{O_2})}$$

$$= 1.67 - 0.103 = 1.567\ V$$

8. The freezing point (in °C) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (Mol. Wt. 329) in 100 g of water (K_f = 1.86 K kg mol^{-1}) is

(A) -2.3×10^{-2} (B) -5.7×10^{-2} (C) -5.7×10^{-3} (D) -1.2×10^{-2}

8. (A)

$K_3[Fe(CN)_6] \longrightarrow 3K^+ + [Fe(CN)_6]^{3-}$

i = 4

$$\Delta T_f = iK_f m = 4 \times 1.86 \times \left(\frac{0.1 \times 1000}{329 \times 100}\right)$$

$$= \frac{4 \times 1.86}{329} = 0.0226 = 2.23 \times 10^{-2}$$

$T_f = -2.23 \times 10^{-2}$

SECTION – II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions**. Each question has four choices (A) , (B) , (C) and (D) out our which **ONE or MORE** may be correct.

9. The equilibrium

$2Cu^I \rightleftharpoons Cu^O + Cu^{II}$

In aqueous medium at 25°C shifts towards the left in the presence of

(A) NO_3^- (B) Cl^- (C) SCN^- (D) CN^-

9. (B), (C), (D)

Cl^-, SCN^-, CN^- forms ppt. with Cu^+.

10. Reduction of the metal centre in aqueous permanganate ion involves

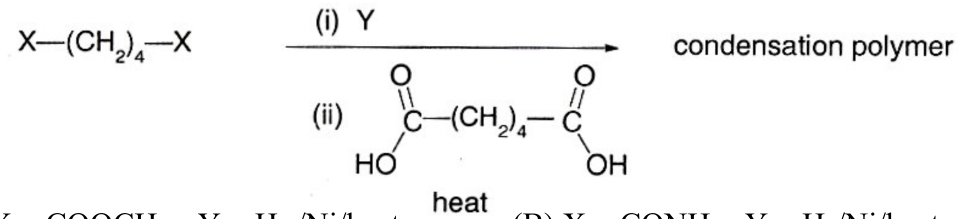
(A) 3 electrons in neutral medium (B) 5 electrons in neutral medium
(C) 3 electrons in alkaline medium (D) 5 electrons in acidic medium

10. (A), (C), (D)

$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{+2} + 4H_2O$ (Acidic)

$MnO_4^- + 2H_2O + 3e^- \longrightarrow MnO_2 + 4OH^-$ (Neutral and weak Alkaline)

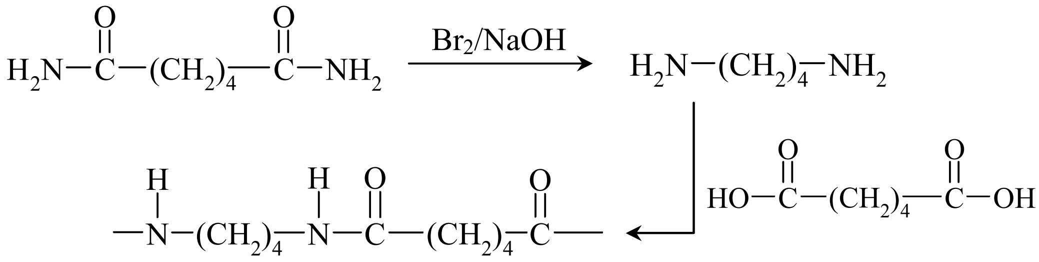
11. The correct functional group X and the reagent/reaction conditions Y in the following scheme are



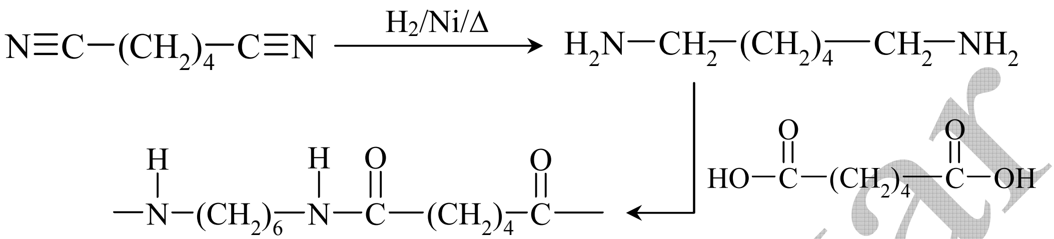
(A) X = $COOCH_3$, Y = H_2/Ni/heat (B) X = $CONH_2$, Y = H_2/Ni/heat
(C) X = $CONH_2$, Y = Br_2/NaOH (D) X = CN , Y = H_2/Ni/heat

11. (C), (D)

(C)



(D)



12. For the first order reaction

$$2N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$$

(A) The concentration of the reactant decreases exponentially with time.
(B) The half–life of the reaction decreases with increasing temperature.
(C) The half–life of the reaction depends on the initial concentration of the reactant.
(D) The reaction proceeds to 99.6% completion in eight half–life duration.

12. (A), (B), (D)

$C = C_0\, e^{-KT}$ for 1st order reaction

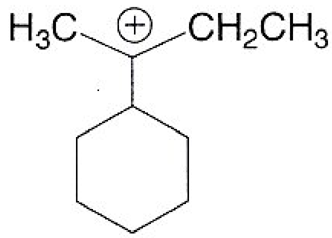
$$t = \frac{2.303}{K}\log_{10}\left(\frac{100}{0.4}\right)$$

$$t = 8\, t_{1/2}$$

SECTION – III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

13. The total number of contributing structures showing hyperconjugation (involving C–H bonds) for the following carbocation is



13. [6]
No. of hyperconjugative structure = 6

14. Among the following, the number of compounds than can react with PCl_5 to give $POCl_3$ is O_2, CO_2, SO_2, H_2O, H_2SO_4 , P_4O_{10}

14. [4]

$$PCl_5 + SO_2 \rightarrow POCl_3 + SOCl_2$$
$$PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$$
$$6PCl_5 + P_4O_{10} \rightarrow 10POCl_3$$

15. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to

15. [6]

No. of moles of $AgNO_3$ required $= 2\left(\frac{0.01\times 30}{1000}\right) = \frac{0.1\times V}{1000}$

$$\Rightarrow \quad \frac{0.01\times 30\times 2}{1000} = \frac{0.1\times V}{1000}$$
$$V = 6\ \text{mL}$$

16. In 1L saturated solution of AgCl $[K_{sp}(AgCl) = 1.6 \times 10^{-10}]$, 0.1 mol of CuCl $[K_{sp}(CuCl) = 1.0 \times 10^{-6}]$ is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of "x" is

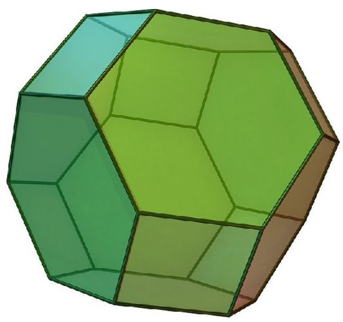
16. [7]

$$[Ag^+] = \frac{K_{sp}(AgCl)}{\sqrt{K_{sp}(CuCl) + K_{sp}(AgCl)}}$$
$$= \frac{1.6\times 10^{-10}}{\sqrt{1.6\times 10^{-10} + 10^{-6}}} = \frac{1.6\times 10^{-10}}{10^{-3}} = 1.6\times 10^{-7}$$

17. The number of hexagonal faces that are present in a truncated octahedron is

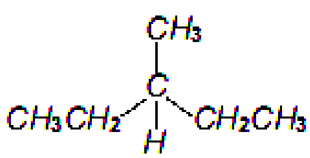
17. [8]

In geometry, the **truncated octahedron** is an Archimedean solid. It has 14 faces (8 regular hexagonal and 6 square), 36 edges, and 24 vertices. Since each of its faces has point symmetry the truncated octahedron is a zonohedron.

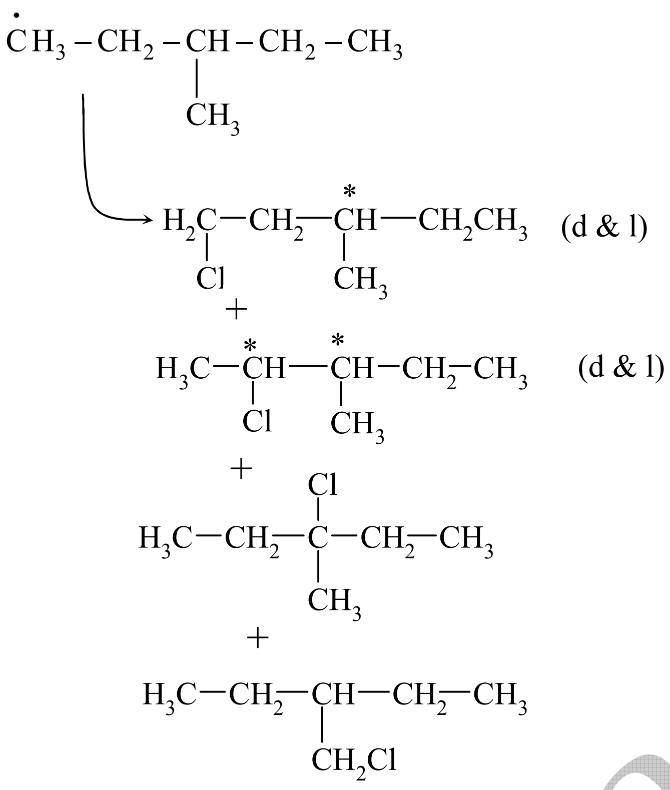


If the original truncated octahedron has unit edge length, its dual tetrakis cube has edge lengths $\frac{9}{8}\sqrt{2}$ and $\frac{3}{2}\sqrt{2}$.

18. The maximum number of isomers (including stereoisomers) that are possible on monochlorination of the following compound, is



18. [8]



SECTION – IV (Total Marks : 16)
(Matrix Match Type)

This section contains **2 questions.** Each question has four statements (A,B,C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with **ONE** or **MORE** statement(s) given in **Column II.** For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

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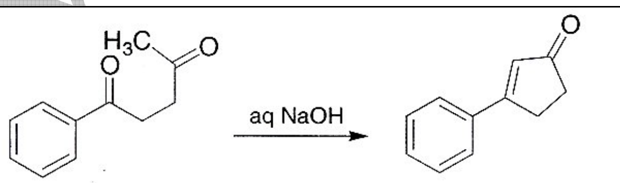
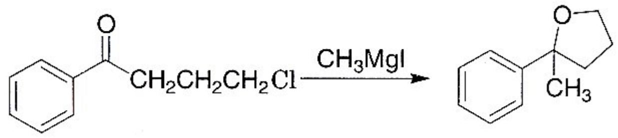
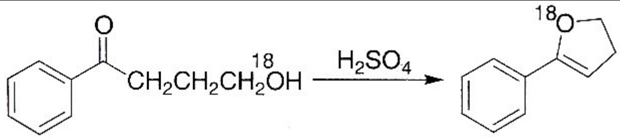
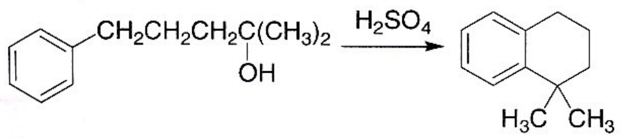
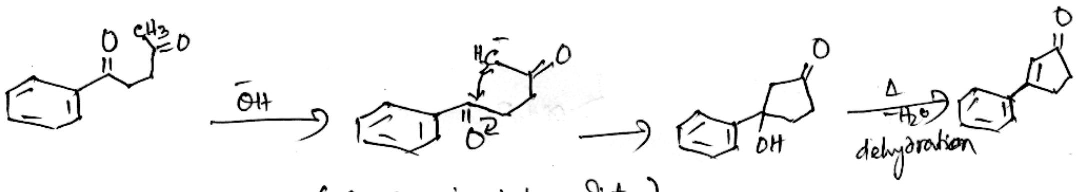
19. Match the reactions in **Column I** with appropriate types of steps/reactive intermediate involved in these reactions as given in **Column II**

Column I	Column II
(A) 	**(p)** Nucleophilic substitution
(B) 	**(q)** Electrophilic substitution
(C)	**(r)** Dehydration

	
(D) 	**(s)** Nucleophilic addition
	(t) Carbanion

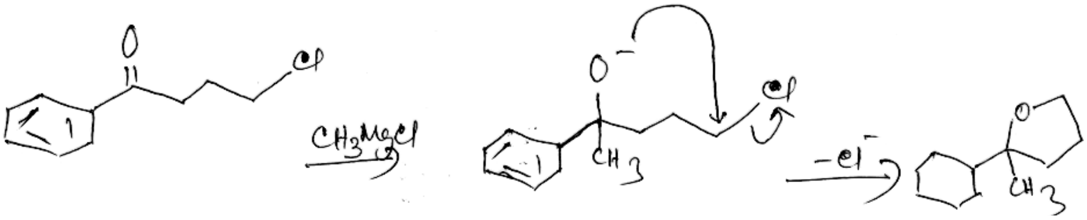
19. (A) → (r), (s), (t); (B) → (p), (s), (t); (C) → (r), (s); (D) → (q), (r)

(A)

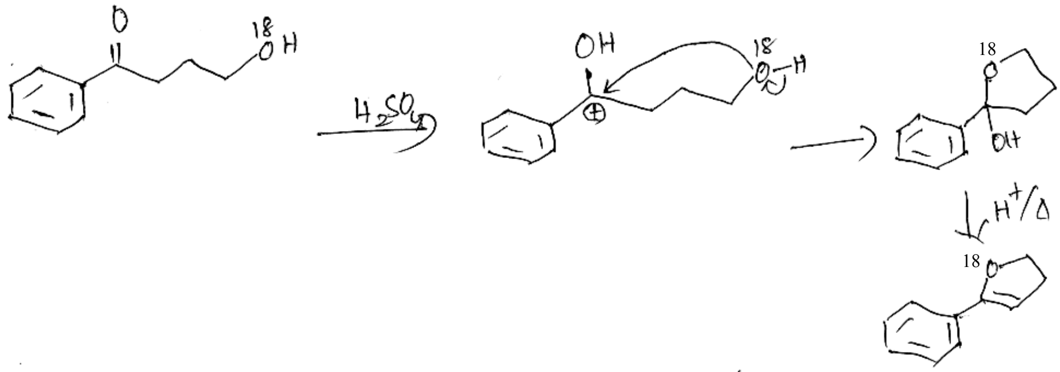


(Carbanion intermediate)

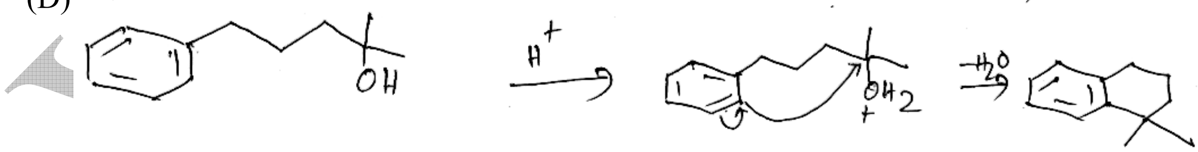
(B)



(C)



(D)



20. Match the transformations in Column I with appropriate options in Column II

Column I	**Column II**
(A) $CO_2(s) \rightarrow CO_2(g)$	**(p)** Phase transition
(B) $CaCO_{3(s)} \rightarrow CaO_{(s)} + CO_{2(g)}$	**(q)** Allotropic change
(C) $2H\bullet \rightarrow H_2(g)$	**(r)** ΔH is positive
(D) $P_{(white,\ solid)} \rightarrow P_{(red,\ solid)}$	**(s)** ΔS is positive
	(t) ΔS is negative

20. (A) → (p), (r), (s); (B) → (r), (s); (C) → (t); (D) → (p), (q), (t)

(A) $CO_{2(s)} \rightarrow CO_{2\,(g)}$

There is a phase change (transition) from solid to gas. ΔH is positive as heat is required to change from solid to gas.

ΔS : is also positive as randomness is increasing from solid to gas.

(B) ΔH is positive as that is required to decompose $CaCO_{3(s)}$ to $CaO_{(s)}$ and $CO_{2\,(g)}$.

ΔS is positive as randomness is increasing on the right hand side of the reactor.

(C) ΔS is negative as free radical is converted to $H_{2(s)}$ molecule.

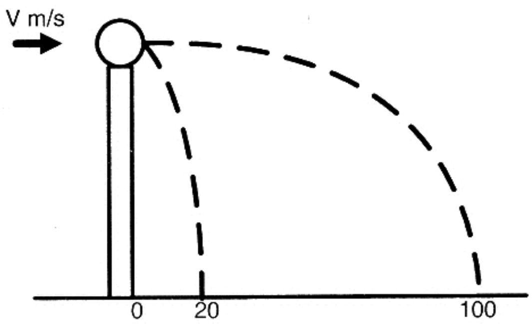
(D) Allotropic change is seen from P(white) → P(red) and since P(red) is a polymeric form the entropy of the product is decreasing. Therefore ΔS is negative.

PART II : PHYSICS

SECTION – I (Total Marks : 24)
(Single Correct Choice Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is



(A) 250 m/s (B) $250\sqrt{2}$ m / s (C) 400 m/s (D) 500 m/s

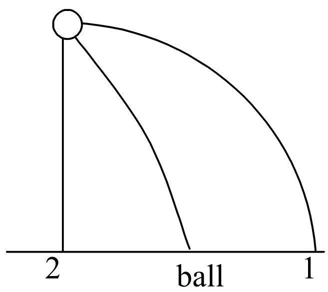
21. (D)

$$V_2\sqrt{\frac{2H}{g}} = 20$$

$$V_2\sqrt{\frac{10}{10}} = 20$$

$$V_2 = 20$$

$$V_1\sqrt{\frac{2H}{g}} = 100$$



$V_1 = 100$

(0.01) V = (0.2) (20) + (0.01) 100

0.01 V = 4 + 1

V = 500 m/s.

22. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(A) 0.9 % (B) 2.4 % (C) 3.1 % (D) 4.2 %

22. (C)

$$\rho = \frac{m}{V}$$

$\Delta\rho\ \% = \Delta m\ \% + \Delta V\ \%$

$\Delta V\ \% = 3\ (\Delta r)\ \%$

$\Delta\rho\ \% = \Delta m\ \% + 3\ (\Delta r)\ \%$

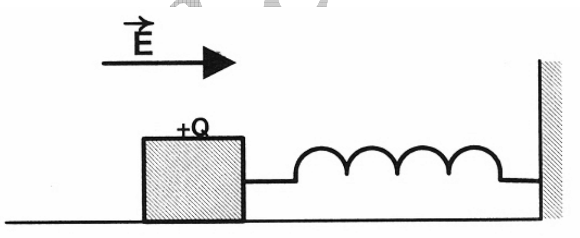
$$\text{L.C.} = \frac{0.5}{50} = (0.01)\ \text{min}$$

r = 2.5 + (20) (0.01) = 2.7

$$\Delta r\ \% = \frac{(0.01)}{2.7} \times 100 = \frac{1}{2.7} = \frac{10}{27} = 0.37$$

$\Delta\rho\ \% = (2 + 3\ (0.37)) = 3.1\ \%$.

23. A wooden block performs SHM on a frictionless surface with frequency, v_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched–on as shown, then the SHM of the block will be



(A) of the same frequency and with shifted mean position.

(B) of the same frequency and with the same mean position.

(C) of changed frequency and with shifted mean position.

(D) of changed frequency and with the same mean position.

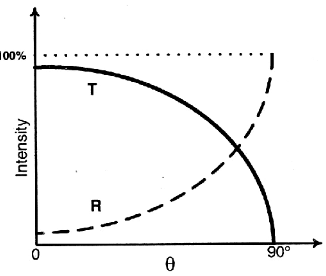
23. (A)

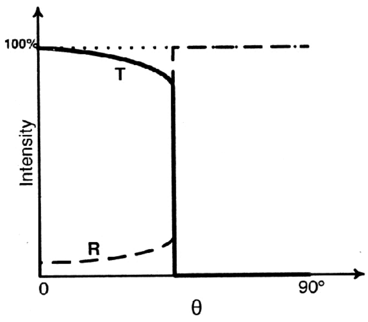
Force of E will be constant. So only mean position will shift to

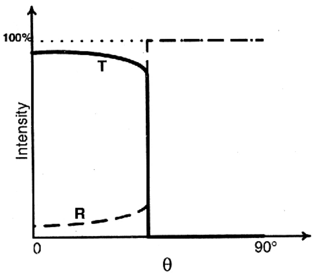
$$x_0 = \frac{QE}{K}$$

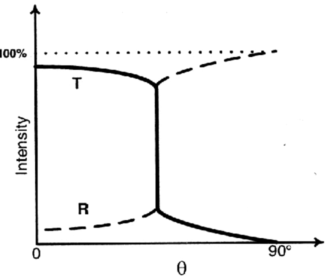
ω remain same.

24. A light traveling in glass medium is incident on glass–air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

(A) 

(B) 

(C) 

(D) 

24. (C)

At the θ_c

$i_T = 0$

$i_r = i_i$

at $\theta < \theta_c$ there will be partial reflection.

(B) cannot be correct as at $\theta = 0$

$i_T + i_R > i_i$ (according to graph) which is not possible.

$i_T + i_R = i_1$ at any moment i.e., shown in (C).

25. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

25. (B)

$$KE + \left(-\frac{GMm}{r}\right) = 0$$

$$KE = +\frac{GMm}{r}$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \Rightarrow \quad \frac{GM}{r} = v^2$$

$\therefore$ KE = mv^2.

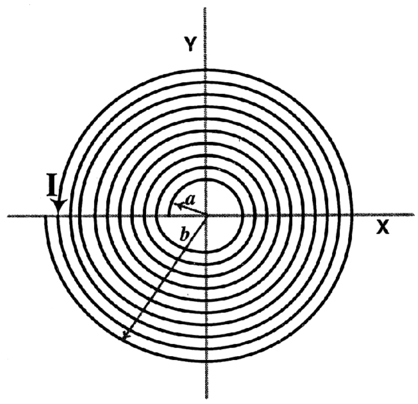
26. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the X–Y plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the center of the spiral is

(A) $\dfrac{\mu_0 N I}{2(b-a)} \ln\left(\dfrac{b}{a}\right)$

(B) $\dfrac{\mu_0 N I}{2(b-a)} \ln\left(\dfrac{b+a}{b-a}\right)$

(C) $\dfrac{\mu_0 N I}{2b} \ln\left(\dfrac{b}{a}\right)$

(D) $\dfrac{\mu_0 N I}{2b} \ln\left(\dfrac{b+a}{b-a}\right)$

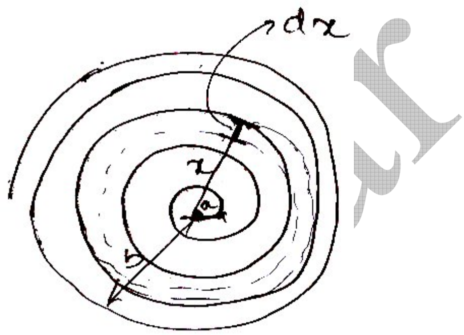


26. (A)

Current flowing per unit width of the spiral wire :

$$\frac{di}{dx} = \frac{N}{b-a} i$$

$$di = \frac{N}{b-a} i\, dx$$



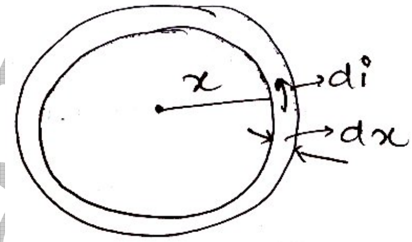
B at centre

$$\therefore \quad dB = \frac{\mu_0 di}{2x}$$

$$dB = \frac{\mu_0}{2x} \frac{Ni\,dx}{b-a}$$

$$B = \frac{\mu_0 Ni}{2(b-a)} \int_a^b \frac{dx}{x}$$

$$B = \frac{\mu_0 Ni}{2(b-a)} \ell n\left(\frac{b}{a}\right)$$



27. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \dfrac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

(A) $\sqrt{2}\,A, \dfrac{3\pi}{4}$ (B) $A, \dfrac{4\pi}{3}$ (C) $\sqrt{3}\,A, \dfrac{5\pi}{6}$ (D) $A, \dfrac{\pi}{3}$

27. (B)

Net displacement = 0.

$$\therefore \quad x_1(t) + x_2(t) + x_3(t) = 0$$

$$A \sin \omega t + A \sin\left(\omega t + \frac{2\pi}{3}\right) + B \sin(\omega t + \phi) = 0$$

$$A\left[\sin \omega t + \sin\left(\omega t + \frac{2\pi}{3}\right)\right] = -B \sin(\omega t + \phi)$$

L.H.S.

$$2A\sin\left(\frac{\omega t+\omega t+\frac{2\pi}{3}}{2}\right)\cos\left(\frac{\omega t-\omega t-\frac{2\pi}{3}}{2}\right)$$

$$2A\sin\left(\omega t+\frac{\pi}{3}\right)\cos\left(\frac{-\pi}{3}\right)$$

$$2A\left(\frac{1}{2}\right)\sin\left(\omega t+\frac{\pi}{3}\right) \Rightarrow A\sin\left(\omega t+\frac{\pi}{3}\right)$$

$$\Rightarrow -A\sin\left(\omega t+\frac{\pi}{3}+\pi\right) \Rightarrow -A\sin\left(\omega t+\frac{4\pi}{3}\right)$$

Comparing with R.H.S.

$B = A$

& $\phi = \frac{4\pi}{3}$

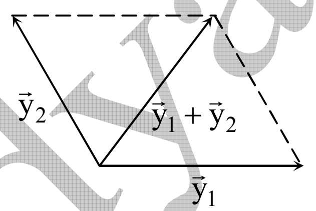
Alternate solution :

By phasor,

$$|\vec{y}_1+\vec{y}_2| = \sqrt{A^2+A^2+2A^2\cos\frac{2\pi}{3}}$$

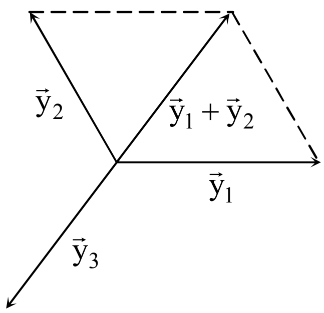
$$= A$$

& $\delta = \frac{\pi}{3}$

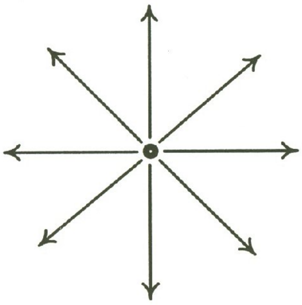


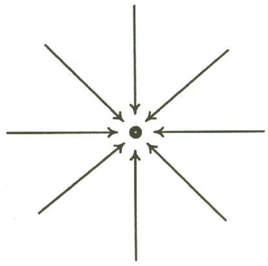
For $\vec{y}_1+\vec{y}_2+\vec{y}_3=0$

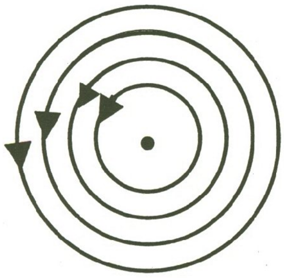
$|\vec{y}_3| = A$ & direction should be $\frac{4\pi}{3}$ (In 3rd quadrant).

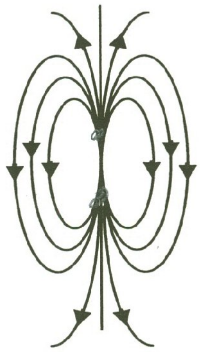


28. Which of the field patterns given below is valid for electric field as well as for magnetic field?

(A) 

(B) 

(C) 

(D) 

28. (C)

The given pattern is valid for magnetic field around a straight wire for which length is perpendicular to the plane of paper and current is flowing such that it is coming out of the paper. As well as it represents an induced electric field due to a varying magnetic field.

(A) & (B) are not valid as $\vec{B}$ should form close loop and in (D) one line terminates at second point as well as another one emerging from it. That is not possible.

SECTION – II (Total Marks : 16)

(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

---

29. A series R−C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$

(C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

29. (B), (C)

$$E = \sqrt{{V_R}^2 + {V_C}^2}$$

$$= I\sqrt{(R)^2 + {X_C}^2}$$

$$E = I\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$\therefore$ If C is greater, X_C will be less and therefore V_C will be less.

$$C = \frac{\varepsilon_0 KA}{d}$$

$\therefore \quad C_B = 4C_A$

$\therefore \quad C_B$ is greater than C_A

hence $V_C^A > V_C^B$ [C is correct]

Now,

$$E = I\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

E remains same for both cases.

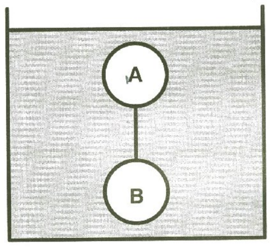
If C is greater X_C will be smaller. I will be greater.

$C_B > C_A$

$X_C^B < X_C^A$

$I_B > I_A$

30. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



(A) $d_A < d_F$ (B) $d_B > d_F$

(C) $d_A > d_F$ (D) $d_A + d_B = 2\, d_F$

30. (A), (B), (D)

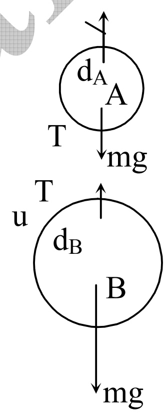
For B :

$$V d_B g - V d_F g - T = 0 \qquad \ldots\ldots(1)$$

$$T + V d_A g - V d_f g = 0 \qquad \ldots\ldots(2)$$

$$V d_B g - V d_F g = -V d_A g + V d_F g$$

$$d_B + d_A = 2 d_F \qquad \ldots\ldots(D)$$



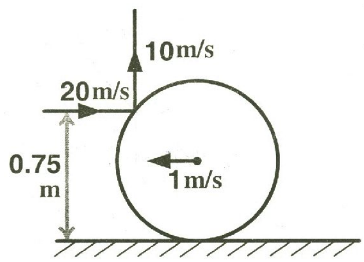
Now, A has a tendency to go up

$\therefore \quad d_A < d_F \qquad \ldots\ldots(A)$

B has a tendency to go down.

$d_B > d_F \qquad \ldots\ldots(B)$

31. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(A) the ring has pure rotation about its stationary CM.

(B) the ring comes to a complete stop.

(C) friction between the ring and the ground is to the left.

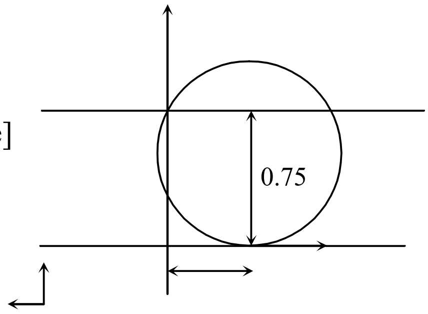
(D) there is no friction between the ring and the ground.

31. (A), (C)

Horizontal momentum is conserved.

$P_i = 0.1\ \text{Kg} \times 20 - 2 \times 1 = 0$

Final horizontal momentum of 0.1 Kg = 0



$\therefore$ Final horizontal momentum of ring = 0

$L_i = 0.75 \times 0.1 \times 20 = 1.5$ (of 0.1 Kg) [Into the plane of figure]

$L_i = I\omega = (MR^2 + MR^2) \times \dfrac{V}{R} = 2MVR$

$= 2 \times 2 \times 1 \times 0.5 = 2$ [out of the plane]

$L_i = 0.5$ (out of plane)

After collision :

$L_1 = 10 \times 0.1 \times \dfrac{\sqrt{3}}{2} \times \left(\dfrac{1}{2}\right)$ (of 0.1 kg) [into the plane]

$= \dfrac{\sqrt{3}}{4}$

$L_2 = (mR^2)\,\omega = 2 \times 0.25 \times \omega = 0.5\,\omega$ (as it is pure rotation about centre)

By conservation of angular momentum

$$0.5 = -\frac{\sqrt{3}}{4} + \frac{\omega}{2}$$

$$\frac{\omega}{2} = 0.5 + \frac{\sqrt{3}}{4}$$

$\Rightarrow$ $\omega > 0 \Rightarrow \omega$ is out of the plane.

Thus rotation is anti–clockwise.

Thus friction is towards left.

32. Which of the following statement(s) is/are correct?

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(D) The work done by the external force is moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

32. (C), (D)

Gauss law tells $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{in}}{\varepsilon_0}$ [i.e., constant w.r.t. distance from a point charge]

$\Rightarrow$ Electric flux from a point charge $\propto \dfrac{1}{\sqrt{r}}$ [in new condition & that is not constant]

Hence (A) is incorrect.

Gauss' law is used to calculate amount of flux but not the field distribution.

Hence (B) is incorrect.

If there are only two charges then (C) is correct.

By definition, (D) is correct.

SECTION – III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

33. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is A × 10^Z (where 1<A< 10). The value of 'Z' is

33. [7]
The loss of electrons, charges the sphere positive.
The final potential acquired by the sphere is indeed the slopping potential.

$$eV_s = K_{max} = \frac{hc}{\lambda} - \phi = \frac{1242\text{nm} - \text{eV}}{200\text{nm}} - 4.7 = 6.21 - 4.7$$

$$eV_s = 1.51 \text{ eV}$$

$$V_s = 1.51 \text{ volts.}$$

$V_s = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ where q is the charge acquired by the sphere.

$$1.51 = 9 \times 10^9 \times \frac{q}{1\times10^{-2}} \text{m}$$

$$q = \frac{1.51}{9}\times10^{-11}$$

The no. of electrons = $\frac{q}{e} = \frac{1.51\times10^{-11}}{9\times1.6\times10^{-19}} = \frac{1.51}{14.4}\times10^{8} \approx 1 \times 10^7$. $\therefore$ Z = 7.

34. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

34. [5]
The ball is thrown w.r.t. to the boy at 10 m/s at angle of 60°.

Time of flight = $\frac{2u\sin\theta}{g} = \frac{2\times10\sin60°}{g} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

The projectile has an deceleration of a in the frame of the boy.

$$R = u\cos\theta \times t - \frac{1}{2}a\,t^2$$

$$1.15 = 10 \times \cos 60 \times \sqrt{3} - \frac{1}{2}\times a\times3$$

$$1.15 = 5\sqrt{3} - 1.5a$$

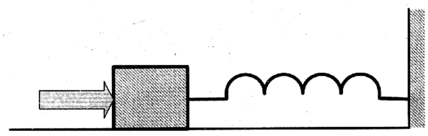
$$1.5a = 5 \times 1.73 - 1.15$$

$$1.5\,a = 8.65 - 1.15$$

$$1.5\,a = 7.5$$

$$a = \frac{7.5}{1.5} = 5 \text{ m/s}^2.$$

35. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is



35. [4]

$$W_{ext} = \Delta K + \Delta U$$

$$-\mu N\, x = 0 - \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

$$-0.1 \times m\, g\, x = -\frac{1}{2}\, mv^2 + \frac{1}{2} kx^2$$

$x = 0.06$

$\mu = 0.1$

$m = 0.18$

$k = 2$

$$-0.1 \times 0.18 \times 10 \times 0.06 = -\frac{1}{2} \times 0.18\, v^2 + \frac{1}{2} \times 2 \times (0.06)^2$$

$$-1.8 \times 6 \times 10^{-3} = -0.09\, v^2 + 36 \times 10^{-4}$$

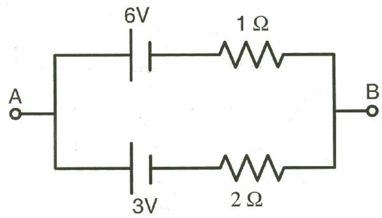
$$0.09\, v^2 = 10^{-4}\,(144)$$

$$v^2 = 10^{-2} \times \frac{144}{9}$$

$$v = 10^{-1} \times \frac{12}{3} = 4 \times 10^{-1}$$

$\therefore$ N = 4.

36. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



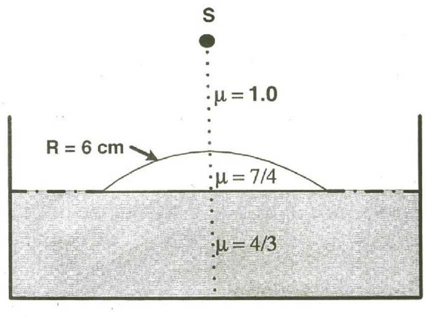
36. [5]

Total EMF = (6 – 3) V = 3 V

$$i = \frac{3V}{3\Omega} = 1A$$

$$V_A - V_B = 3V + i \times 2 = 3V + 2V = 5V.$$

37. Water $\left(\text{with refractive index } = \frac{4}{3}\right)$ in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



37. [2]

For surface (I)

$$\frac{\mu_2}{V_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad \ldots\ldots(1)$$

It will be object for second surface and by applying same formula on surface (II).

$$\frac{\mu_3}{V} - \frac{\mu_2}{V_1} = \left(\frac{\mu_3 - \mu_1}{\infty}\right) \qquad \ldots\ldots(2)$$

by adding (1) and (2)

$$\frac{\mu_3}{V} - \frac{\mu_1}{u} = \left(\frac{\mu_2 - \mu_1}{R_1}\right)$$

$$\frac{4}{3V} - \frac{1}{(-24)} = \frac{\left(\frac{7}{4} - 1\right)}{+6}$$

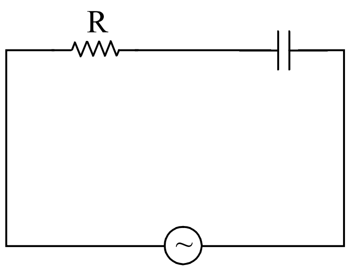
$$\frac{4}{3V} = \frac{1}{8} - \frac{1}{24} = \frac{1}{12}$$

$\Rightarrow$ V = 16 below surface

$\Rightarrow$ 2 cm above surface.

38. A series R–C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R–C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

38. [4]



$\omega = 500$ rad/s.

$$X_C = \frac{1}{\omega C}$$

$$Z^2 = R^2 + X_C^2 \Rightarrow 1.25R^2 = R^2 + X_C^2$$

$$\Rightarrow \; X_C = 0.5R = \frac{1}{\omega C}$$

$$\Rightarrow \; RC = \frac{1}{\omega \times 0.5} = \frac{2}{\omega} = \frac{2}{500} = \frac{2}{5} \times 10^{-2} = \frac{20}{5} \times 10^{-3}\,s$$

RC = 4 ms.

SECTION – VI (Total Marks : 16)

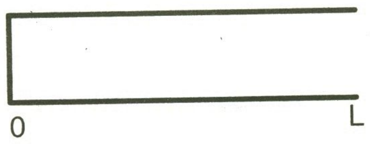
(Matrix-Match Type)

This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE or MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

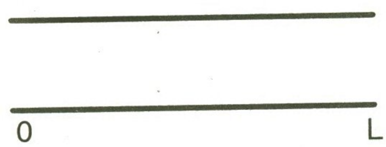
39. Column I shows for systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves.

Column I

(A) Pipe closed at one end

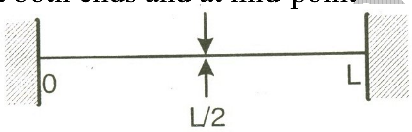


(B) Pipe open at both ends



(C) Stretched wire clamped at both ends

(D) Stretched wire clamped at both ends and at mid-point



Column II

(p) Longitudinal waves

(q) Transverse waves

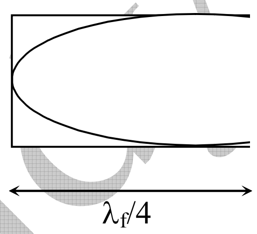
(r) $\lambda_f = L$

(s) $\lambda_f = 2L$

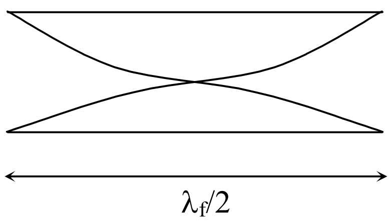
(t) $\lambda_f = 4L$

39. (A) → (p), (t); (B) → (p), (s); (C) → (q), (s); (D) → (q), (r)

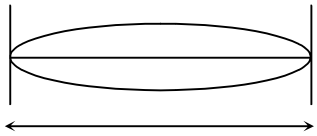
(A) → (p), (t)



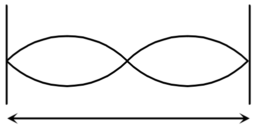
(B) → (p), (s)



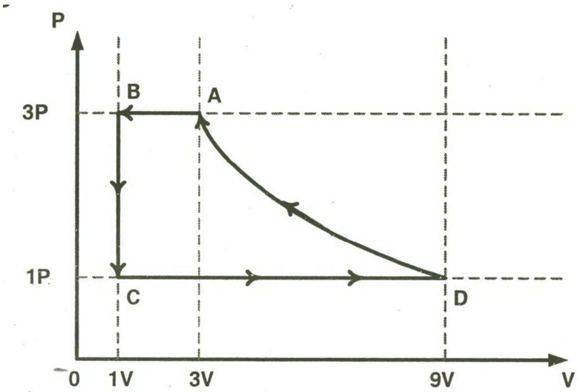
(C) $\rightarrow$ (q), (s)



(D) $\rightarrow$ (q), (r)



40. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



Column I	Column II
(A) Process A $\rightarrow$ B	(p) Internal energy decreases.
(B) Process B $\rightarrow$ C	(q) Internal energy increases.
(C) Process C $\rightarrow$ D	(r) Heat is lost.
(D) Process D $\rightarrow$ A	(s) Heat is gained.
	(f) Work is done on the gas.

40. (A) $\rightarrow$ (p), (r), (t); (B) $\rightarrow$ (p), (r); (C) $\rightarrow$ (q), (s); (D) $\rightarrow$ (r), (t)

(A) $\rightarrow$ (p), (r), (t)
Process [A $\rightarrow$ B] is a isobaric process in which volume decreases. Hence temperature also decreases. Hence Internal Energy decreases. Also $\Delta T < 0 \Rightarrow \Delta U < 0$ i.e., Heat is lost. Also work to be done on gas.
(B) $\rightarrow$ (p), (r)
Process [B $\rightarrow$ C] is isochoric and pressure falls.
Hence temperature falls. $\therefore \Delta u < 0$ and $\Delta Q < 0$.

(C) $\rightarrow$ (q), (s)
Process [C $\rightarrow$ D] is isobaric and volume increases. Hence temperature also increases.
$\therefore\ \Delta Q > 0$ & $\Delta u > 0$.

(D) $\rightarrow$ (r), (t)
Process [D $\rightarrow$ A] volume decreases. Thus work is done on gas.

$$T_A = \frac{3P \times 3V}{R} = \frac{9PV}{R}$$

$$T_D = \frac{9V \times P}{R} = \frac{9PV}{R}$$

$\therefore\ \Delta T = 0 \quad \Rightarrow \Delta u = 0$
$\therefore\ \Delta Q = W \quad \Rightarrow \Delta Q < 0$

PART III - MATHEMATICS

SECTION – I (Total Marks : 24)
(Single Correct Answer Type)

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

41. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x\, f(x)\, dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x–axis. Then

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

41. (C)

$f(x) = f(1-x)$

$$R_1 = \int_{-1}^{2} x\, f(x)\, dx \qquad \ldots\ldots\text{(i)}$$

$$R_1 = \int_{-1}^{2} (1-x) f(1-x)\, dx$$

$$R_1 = \int_{-1}^{2} (1-x) f(x)\, dx \qquad \ldots\text{(ii)}$$

$f(1-x) = f(x)$

adding (i) and (ii)

$$2R_1 = \int_{-1}^{2} f(x)$$

$2R_1 = R_2$

42. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is

(A) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, \ldots\}$ (B) $\pm\sqrt{n\pi}$, , $n \in \{1, 2, \ldots\}$

(C) $\frac{\pi}{2} + 2n\pi$, $n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (D) $2n\pi$, $n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

42. (A)

$f(x) = x^2$

$g(x) = \sin x$

$$f\left(g\left(g\left(f(x)\right)\right)\right) = g\left(g\left(f(x)\right)\right) = \sin\left(\sin\left(x^2\right)\right)$$

$$\left(\sin\left(\sin x^2\right)\right)^2 = \sin\left(\sin x^2\right)$$

$$\sin\left(\sin x^2\right) = 0, 1$$

$\sin\left(x^2\right)$ can not be $\frac{\pi}{2}$

So, $\sin x^2 = 0$

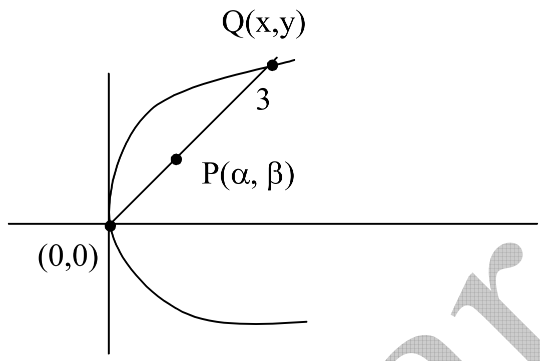
$x^2 = n\pi$

$x = \pm\sqrt{n\pi}$

$x = 0, 1, 2, \ldots\ldots.$

43. Let (x, y) be any point on the parabola $y^2 = 4x$. Let p be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is
(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

43. (C)
By section formula
$P(\alpha, \beta) = \left(\frac{x}{4}, \frac{y}{4}\right)$



So, $\alpha = \frac{x}{4}$ & $\beta = \frac{y}{4}$
$\Rightarrow$ $x = 4\alpha$ & $y = 4\beta$
As (x, y) lies on $y^2 = 4x$
$\Rightarrow (4\beta)^2 = 4(4\alpha)$
$\Rightarrow \beta^2 = \alpha$
Hence locus $y^2 = x$

44. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x−axis at (9, 0), then the eccentricity of the hyperbola is
(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

44. (B)
Slope of tangent at $P(x_1, y_1)$ is = $\frac{x_1 b^2}{y_1 a^2}$

Slope of normal at $P(x_1, y_1)$ is = $-\frac{y_1 a^2}{x_1 b^2}$

So, slope of normal at P(6, 3) $= -\frac{a^2}{2b^2}$

So, equation of normal $y - 3 = -\frac{a^2}{2b^2}(x - 6)$

Given it passes through (9, 0)

$\Rightarrow 0 - 3 = -\frac{a^2}{2b^2}(9 - 6)$

$\Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$

$\therefore e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$.

45. A value of b for which the equations $x^2 + bx - 1 = 0$
$x^2 + x + b = 0$
have one root in common is
(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

45. Let α be the common root.
$\alpha^2 + b\alpha - 1 = 0$ …..(i)
$\alpha^2 + \alpha + b = 0$ …..(ii)

(i) – (ii) $\Rightarrow$ $\alpha = \frac{b+1}{b-1}$

Substituting, α in (i) is

$b^2 + 3 = 0$

$b = \pm i\sqrt{3}$

46. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non–singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$ where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2 (B) 6 (C) 4 (D) 8

46. (A)

$\Delta = 1\,(1 - c\omega - a\,(\omega - \omega^2 c) + b\,(0))$

$\Delta = 1 - c\omega - a\omega + \omega^2 ac$

$\Delta = 1 - \omega\,(c + a) + \omega^2 ac$

$c = \omega$	$a = \omega^2$	singular
$c = \omega^2$	$a = \omega$	singular
$c = \omega$	$a = \omega$	non singular
$c = \omega^2$	$a = \omega^2$	singular

for every pair (a, c) there are two possible values of b hence 2 matrices.

47. The circle passing through the point (–1, 0) and touching the y–axis at (0, 2) also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

47. (D)

$(x - \alpha)^2 + (y - 2)^2 = \alpha^2$

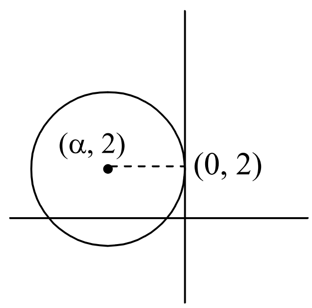
$\Rightarrow x^2 - 2x\alpha + \alpha^2 + y^2 - 4y + 4 = \alpha^2$

$\Rightarrow x^2 + y^2 - 2x\alpha - 4y + 4 = 0$

The circle passes through (–1, 0)

$\therefore 1 + 0 + 2\alpha + 4 = 0$

$\Rightarrow \alpha = -\frac{5}{2}$



The equation of circle

$x^2 + y^2 + 5x - 4y + 4 = 0$

Now, the point (–4, 0) satisfy the above equation.

Hence the point (–4, 0) lies on the circle.

48. If $\lim_{x\to 0}\left[1 + x\ln\left(1 + b^2\right)\right]^{\frac{1}{x}} = 2b\sin^2\theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$ (C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

48. (D)

$$\lim_{x\to 0}\left[1+x\,ln\left(1+b^2\right)\right]^{\frac{1}{x}}$$

$= e^{ln\left(1+b^2\right)}$

$= (1 + b^2)$

Hence, $1 + b^2 = 2\,b\,\sin^2\theta$

$\Rightarrow \sin^2\theta = \dfrac{1+b^2}{2b}$ … (1)

Now,

$\dfrac{1+b^2}{2b} \ge 1$

Hence (1) is true only when

$\sin^2\theta = 1$

$\Rightarrow \theta = \pm\dfrac{\pi}{2}$

SECTION – II (Total Marks : 16)

(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE** or **MORE** may be correct.

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49. Let E and F be two independent events. The probability that exactly one of them occurs is $\dfrac{11}{25}$ and the probability of none of them occurring is $\dfrac{2}{25}$. If P(T) denotes the probability of occurrence of the even T, then

(A) $P(E) = \dfrac{4}{5}$, $P(F) = \dfrac{3}{5}$ (B) $P(E) = \dfrac{1}{5}$, $P(F) = \dfrac{2}{5}$

(C) $P(E) = \dfrac{2}{5}$, $P(F) = \dfrac{1}{5}$ (D) $P(E) = \dfrac{3}{5}$, $P(F) = \dfrac{4}{5}$

49. (A), (D)

$P\left(E\overline{F}\right)+P\left(F\overline{E}\right) = \dfrac{11}{25}$ …………. (i)

$P\left(\overline{E}\,\overline{F}\right) = \dfrac{2}{25}$ …………….. (ii)

Using (i)

$P(E)\ P(\overline{F}) + P(F)\ P(\overline{E}) = \dfrac{11}{25}$

$P(E)\ (1 - P(F)) + P(F)\ (1 - P(E)) = \dfrac{11}{25}$

$P(E) + P(F) - 2\,P(E)\,P(F) = \dfrac{11}{25}$ …………… (iii)

$(1 - P(E))\ (1 - P(F)) = \dfrac{2}{25}$ …………….. (iv)

(A) and (D) are satisfying (iii) and (iv)

50. If $f(x)=\begin{cases}-x-\dfrac{\pi}{2}, & x\le -\dfrac{\pi}{2}\\ -\cos x, & -\dfrac{\pi}{2}<x\le 0\\ x-1, & 0<x\le 1\\ \ln x, & x>1,\end{cases}$ then

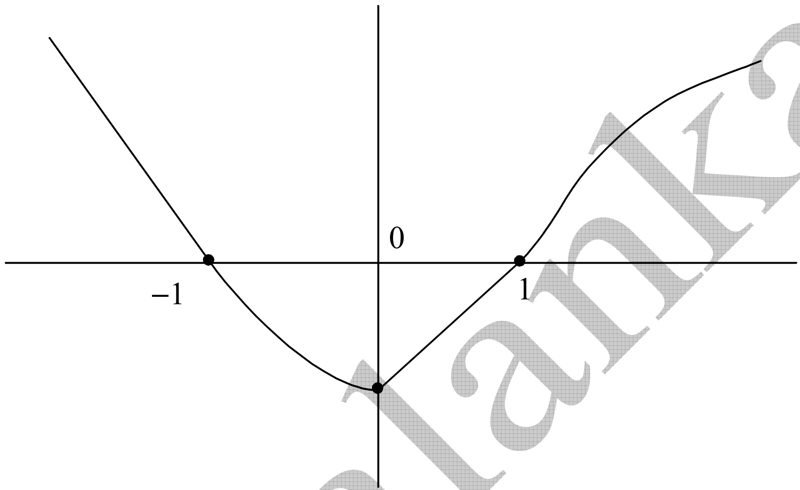
(A) f(x) is continuous at $x = -\frac{\pi}{2}$

(B) f(x) is not differentiable at x = 0

(C) f(x) is differentiable at x = 1

(D) f(x) is differentiable at $x = -\frac{3}{2}$

50. (A), (B), (C), (D)



f(x) is continuous everywhere, not differentiable at x = 0, hence (A), (B), (C), (D) are correct.

51. Let f : (0, 1) → $\mathbb{R}$ be defined by f(x) = $\frac{b-x}{1-bx}$, where b is a constant such that 0 < b < 1. Then

(A) f is not invertible on (0, 1)

(B) $f \ne f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$

(C) $f = f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$

(D) f^{-1} is differentiable on (0, 1)

51. (C), (D)

$f(x) = \frac{b-x}{1-bx}$

$$f'(x) = \frac{(1-bx)(-1)-(b-x)(-b)}{(1-bx)^2}$$

$$= \frac{-1+bx+b^2-bx}{(1-bx)^2} = \frac{b^2-1}{(1-bx)^2} \qquad \ldots (1)$$

$\therefore f'(x) < 0$ $\qquad\qquad \therefore b < 1$

$\Rightarrow$ f is decreasing strictly $\qquad \Rightarrow b^2 < 1$

$\therefore f^{-1}$ exists

$y = \dfrac{b-x}{1-bx}$

$\Rightarrow \quad y - ybx = b - x$

$y - b = ybx - x$

$y - b = x\,(yb - 1)$

$\dfrac{y-b}{yb-1} = x$

$x = \dfrac{y-b}{yb-1}$

$\Rightarrow \quad f^{-1}(x) = \dfrac{x-b}{xb-1}$

$\Rightarrow \quad f^{-1}(x) = \dfrac{b-x}{1-bx}$

$\therefore\ f = f^{-1}$ on (0, 1)

$f'(b) = \dfrac{1}{b^2-1}$ $\qquad\qquad f'(0) = b^2 - 1$

$\therefore\ f'(b) = \dfrac{1}{f'(0)}$

52. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$
(C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

52. (A), (B), (D)

$t\,x + y = 2t + t^3$

Since normal passes through (9, 6)

$9t + 6 = 2t + t^3$

$\Rightarrow\ t^3 - 7t - 6 = 0$

$\Rightarrow\ (t - 3)(t + 1)(t + 2) = 0$

For t = 3, –1, –2 normals are

$y + 3x - 33 = 0$, $\quad y - x + 3 = 0$ and $\quad y - 2x + 12 = 0$ respectively.

SECTION – III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions.** The answer to each of the questions is a **single–digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

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53. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If $S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$, then the number of point(s) in S lying inside the smaller part is

53. (2)

If (x, y) belongs to smaller part then it satisfies

$x^2 + y^2 \le 6$ $\qquad$ and $\qquad 2x - 3y - 1 > 0$

a) $\left(2, \frac{3}{4}\right)$ satisfies $x^2 + y^2 \le 6$

and also $2x - 3y - 1$

$= 2(2) - 3\left(\frac{3}{4}\right) - 1$

$= 4 - \frac{9}{4} - 1 > 0$

$\therefore (2, 3) \in$ smaller required region.

b) $\left(\frac{5}{2}, \frac{3}{4}\right)$ does not satisfy $x^2 + y^2 \le 6$

c) $\left(\frac{1}{4}, \frac{-1}{4}\right)$ satisfies both $x^2 + y^2 \le 6$ and $2x - 3y - 1 > 0$

d) $\left(\frac{1}{8}, \frac{1}{4}\right)$ does not satisfy $2x - 3y - 1 > 0$

54. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non–zero complex numbers such that

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z$$

Then the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

54. [solution will depends on the value of a, b & c]

$\omega = e^{i\pi/3}$

Let a = b = c = 1. $\because$ a, b, c $\in$ I and non–zero.

We have, $x = 3$

$y = 1 + \omega + \omega^2 = z$

Let us see the value of

$$\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = \frac{3^2 + \left|1 + \sqrt{3}\,i\right|^2 + \left|1 + \sqrt{3}\,i\right|^2}{|a|^2 + |b|^2 + |c|^2}$$

$$= \frac{9 + 4 + 4}{3} = \frac{17}{3} \notin \mathbb{Z}$$

So for a = b = c = 1, the final solution is non–integral.

Value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ depends on values taken by a, b, c.

And hence the problem is ambiguous.

55. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

55. [2]

$f(x) = x^4 - 4x^3 + 12x^2 + x - 1$

$f'(x) = 4x^3 - 12x^2 + 24x + 1$

$f''(x) \;=\; 12x^2 - 24x + 24 \;=\; 12(x^2 - 2x + 2) > 0 \;\forall\, x \in R$

$\therefore$ $f'(x)$ is always increasing and hence f(x) = 0 will have only one root.

$\because$ $f'(x) = 0$ has only one real root and f(x) is taking both positive and negative values, f(x) = 0 has exactly two real roots.

56. Let $y'(x) + y(x)\, g'(x) = g(x)\, g'(x)$, $y(0) = 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\frac{d f(x)}{dx}$ and g(x) is a given non–constant differentiable function on $\mathbb{R}$ with g(0) = g(2) = 0. Then the value of y(2) is

56. [0]

$\frac{dy}{dx} + g'(x) y = g(x). g'(x)$

Integrating factor = $e^{\int g'(x)dx} = e^{g(x)}$

$\therefore$ Solution is $y.e^{g(x)} = \int e^{g(x)}.g(x).\, g'(x)\, dx$

$\Rightarrow\; y\, e^{g(x)} = e^{g(x)}\, g(x) - e^{g(x)} + C$

$\Rightarrow\; y\, e^{g(0)} = e^{g(0)}\, g(0) - e^{g(0)} + C$

$\Rightarrow\; 0 = 0 - e^{\circ} + C$

$\Rightarrow C = 1$

$\therefore$ for x = 2,

$y\, e^{g(2)} = e^{\,g(2)}.g(2) - e^{\,g(2)} + 1$

$\Rightarrow\; y \times 1 = 0 - 1 + 1$

$\Rightarrow y = 0$

57. Let M be a 3 × 3 matrix satisfying $M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$, $M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$ and $M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}$.

Then the sum of the diagonal entries of M is

57. [9]

Let M = $\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$

$\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}.\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$

$\Rightarrow\; a_2 = -1, \quad b_2 = 2, \quad c_2 = 3$

Also,

$\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix} \quad \Rightarrow\; a_1 - a_2 = 1$

$\Rightarrow\; a_1 = 0,\; b_1 = 3,\; c_1 = 2$

$\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}.\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix} \quad \Rightarrow\; c_1 + c_2 + c_3 = 12$

$\Rightarrow\; c_3 = 12 - 5 = 7$

$\therefore$ Sum of diagonal elements = $a_1 + b_2 + c_3 = 0 + 2 + 7 = 9$

58. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then the value of $\vec{r} \cdot \vec{b}$ is

58. [9]

$$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$$

$$\left(\vec{r} - \vec{c}\right) \times \vec{b} = 0$$

$$\vec{r} - \vec{c} = \lambda \vec{b}$$

$$\vec{r} = \vec{c} + \lambda \vec{b}$$

$$\vec{r}.\vec{a} = \vec{c}.\vec{a} + \lambda\, \vec{b}.\vec{a}$$

$$0 = \vec{c}.\vec{a} + \lambda\, \vec{b}.\vec{a}$$

$$\lambda = -\frac{\vec{c} + \vec{a}}{\vec{b}.\vec{a}}$$

$$\lambda = +\frac{(4)}{1}$$

$$\vec{r} = \vec{c} + 4\vec{b}$$

$$\vec{r}.\vec{b} = \vec{c}.\vec{b} + 4\vec{b}.\vec{b} \quad = 1 + 4 \times 2$$

$$\vec{r}.\vec{b} = 9$$

SECTION – IV (Total Marks : 16)
(Matrix-Match Type)

This section contains **2** questions. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with **ONE** or **MORE** statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question against statement B, darken the bubbles corresponding to q and r in the ORS.

59. Match the statements given in **Column I** with the intervals/union of intervals given in **Column II**

	Column I		**Column II**
(A)	The set $\left\{ \operatorname{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \lvert z \rvert = 1, z \neq \pm 1 \right\}$ is	(p)	$(-\infty, -1) \cup (1, \infty)$
(B)	The domain of the function f(x) = $\sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q)	$(-\infty, 0) \cup (0, \infty)$
(C)	If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{ f(\theta) : 0 \le \theta < \frac{\pi}{2} \right\}$ is	(r)	$[2, \infty)$
(D)	If $f(x) = x^{3/2}(3x - 10)$, $x \ge 0$, then f(x) is increasing in	(s)	$(-\infty, -1] \cup [1, \infty)$
		(t)	$(-\infty, 0] \cup [2, \infty)$

59.

(A) $\rightarrow$ (S)

$|z| = 1 \quad \Rightarrow \quad z = e^{i\theta}$

$$\frac{2iz}{1-z^2} = \frac{2ie^{i\theta}}{1-e^{2i\theta}} = \frac{2i(\cos\theta + i\sin\theta)}{1-\cos 2\theta - i\sin 2\theta}$$

$$= \frac{2i(\cos\theta + i\sin\theta)}{2\sin^2\theta - i2\sin\theta\cos\theta} = -\frac{2i(\cos\theta + i\sin\theta)}{2i\sin\theta(\cos\theta + i\sin\theta)}$$

$$= -\frac{1}{\sin\theta} = -\operatorname{cosec}\theta$$

So, $\quad A \rightarrow \quad (-\infty, -1] \cup [1, \infty)$

(B) $\rightarrow$ (t)

$$f(x) = \sin^{-1}\left(\frac{8.3^{x-2}}{1-3^{2(x-1)}}\right)$$

domain, $\quad -1 \le \dfrac{8.3^{x-2}}{1-3^{2x-2}} \le 1$

$$-1 \le \frac{8.3^x}{9-3^{2x}} \le 1$$

taking, $\quad t = 3^x$

$$\Rightarrow -1 \le \frac{8t}{9-t^2} \le 1$$

Case – 1

$$0 \le \frac{8t}{9-t^2} \le 1$$

$$\Rightarrow \frac{8t}{9-t^2} - 1 \le 0$$

$$\Rightarrow \frac{t^2+8t-9}{t^2-9} \ge 0$$

$$\Rightarrow \frac{(t+9)(t-1)}{(t-3)(t+3)} \ge 0$$

$$\Rightarrow \frac{t-1}{t-3} \ge 0$$

$t \in (-\infty, 1] \cup (3, \infty)$

$\Rightarrow 3^x \in (0, 1] \cup (3, \infty)$

$\Rightarrow x \in (-\infty, 0] \cup (1, \infty)$ …..(i)

Case – 2

$$-1 \le \frac{8t}{9-t^2}$$

$$\Rightarrow \frac{8t}{9-t^2} + 1 \ge 0$$

$$\Rightarrow \frac{t^2-8t-9}{t^2-9} \geq 0$$

$$\Rightarrow \frac{(t-9)(t+1)}{(t+3)(t-3)} \geq 0$$

$$\Rightarrow \frac{t-9}{t-3} \geq 0$$

$t \in (-\infty, 3) \cup [9, \infty)$

$\Rightarrow 3^x \in (0, 3) \cup [9, \infty)$

$\Rightarrow x \in (-\infty, 1) \cup [2, \infty)$ ….(ii)

Intersection of (i) & (ii) is

$x \in (-\infty, 0)] \cup [2, \infty)$

(C) → (r)

$$f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$$

$= \sec^2\theta + \sec^2\theta$

$= 2\sec^2\theta$

(D) → (r)

$f(x) = x^{3/2}(3x-10)$

$$f^{1}(x) = \frac{3}{2}x^{1/2}(3x-10) + x^{3/2}.(3)$$

$$= \frac{3\sqrt{x}}{2}\left[(3x-10)+2x\right] = \frac{3\sqrt{x}}{2}[5x-10]$$

Given increasing function $\Rightarrow f'(x) \geq 0 \Rightarrow x \geq 2$

60. Match the statements given in **Column I** with the values given in **Column II.**

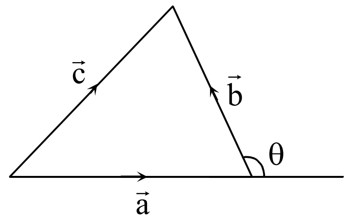
	Column I		Column II
(A)	If $\vec{a} = \hat{j}+\sqrt{3}\hat{k}$, $\vec{b} = -\hat{j}+\sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p)	$\frac{\pi}{6}$
(B)	If $\int_a^b (f(x)-3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q)	$\frac{2\pi}{3}$
(C)	The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)\,dx$ is	(r)	$\frac{\pi}{3}$
(D)	The maximum value of $\left\lvert \text{Arg}\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z\rvert = 1$, $z \neq 1$ is given by	(s)	π
		(t)	$\frac{\pi}{2}$

60. (A) $\rightarrow$ (q)

$\vec{a} = \hat{j} + \sqrt{3}\hat{k}$

$\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$

$\vec{c} = 2\sqrt{3}\hat{k}$



$\vec{a} \cdot \vec{b} = |a||b| \cos\theta$

$\Rightarrow$ $-1 + 3 = 4 \cos \theta$

$\Rightarrow$ $4 \cos \theta = 2$

$\Rightarrow$ $\cos \theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{3}$

Hence the angle is $\left(\pi - \dfrac{\pi}{3}\right) = \dfrac{2\pi}{3}$

(B) $\rightarrow$ (p)

$$\int_a^b (f(x) - 3x).dx = a^2 - b^2$$

let b = x, a = 0

$$\int_0^x (f(t) - 3t).dt = -x^2$$

$f(x) - 3x = -2x$

$f(x) = x$

$f\left(\dfrac{\pi}{6}\right) = \dfrac{\pi}{6}$

(C) $\rightarrow$ (s)

$$\frac{\pi^2}{\text{In}3}\int_{7/6}^{5/6} \sec(\pi x)dx = \frac{\pi^2}{\text{In}3} \cdot \frac{1}{\pi}\ \text{In}\left|\tan\left(\frac{\pi}{4} + \frac{\pi x}{2}\right)\right|_{7/6}^{5/6}$$

$$= \frac{\pi}{\text{In}3}\left[\text{In}\left|\tan\left(\frac{\pi}{4} + \frac{5\pi}{12}\right) - \text{In}\left(\frac{\pi}{4} + \frac{7\pi}{12}\right)\right|\right] = \frac{\pi}{\text{In}3}\ \text{In}\left|\frac{\tan\frac{8\pi}{12}}{\tan\frac{10\pi}{12}}\right|$$

$$= \frac{\pi}{\text{In}3}\ \text{In}\left|\frac{\tan\frac{2\pi}{3}}{\tan\frac{5\pi}{6}}\right| = \frac{\pi}{\text{In}3}\ \text{In}\left|\frac{-\sqrt{3}}{\frac{-1}{\sqrt{3}}}\right| = \frac{\pi}{\text{In}3} \cdot \ln 3 \quad \pi$$

(D) $\rightarrow$ (s)

Let $z = \cos\theta + i \sin\theta$

$$\left|\text{Arg}\left(\frac{1}{1 - (\cos\theta + i\sin\theta)}\right)\right|$$

$$\left|\text{Arg}\left(\frac{1}{2\sin\frac{\theta}{2}}\frac{1}{\left(\sin\frac{\theta}{2}-i\cos\frac{\theta}{2}\right)}\right)\right|$$

$$\left|\text{Arg}\left(\frac{1}{-i2\sin\frac{\theta}{2}}\frac{1}{\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}\right)\right|$$

$$\left|\text{Arg}\left(\frac{i}{2\sin\frac{\theta}{2}}\left(e^{-i\theta/2}\right)\right)\right|$$

$$\left|\text{Arg}\left(\frac{1}{2\sin\frac{\theta}{2}}e^{i(\pi/2-\theta/2)}\right)\right|$$

$$\left|\frac{\pi}{2}-\frac{\theta}{2}\right|$$

$$-\pi < \theta \le \pi$$

$$0 \le \frac{\pi}{2}-\frac{\theta}{2} < \pi$$

Maximum value = π

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