SETS

Representation of a set

- Roster or tabular form
- Set-builder form

⇒The Empty Set is denoted by the symbol $\emptyset$ or { }

⇒If a set has only one element - **Singleton Set.**

⇒The number of elements of a finite set is called **Cardinal Number** of the set.

Subsets: A set A is said to be a subset of a set B if every element of A is also an element of B. (written as $A \subset B$).

Number of subsets : 2^n

A = {1,2}

Subsets : {1}, {2}, {1,2}, { }

Proper Subset: Let A be a subset of B. We say that A is a proper subset of B if A ≠ B

A = {1,2}

Proper subsets : {1}, {2}, { }

Intervals as subsets of $\mathbb{R}$

Open interval (a, b) $\{x: x \in \mathbb{R}, a < x < b\}$

Closed interval $[a, b]$ $\{x: x \in \mathbb{R}, a \leq x \leq b\}$

$[a, b) = \{x: x \in \mathbb{R}, a \leq x < b\}$

$(a, b] = \{x: x \in \mathbb{R}, a < x \leq b\}$

Power set: The collection of all subsets of a set A is called the power set of A.

If a set has n elements, then its power set has 2^n elements.

A = {1,2}

Power set : { {1}, {2}, {1,2}, { } }

Union of sets:

The union of two sets A and B, denoted by $A \cup B$ is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.

If A = {2, 4, 6, 8} and B = {6, 8, 10, 12}, then A ∪ B = {2, 4, 6, 8, 10, 12}.

Properties of Union

(i) $A \cup B = B \cup A$ (Commutative law)

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \emptyset = A$ (Law of $\emptyset$)

(iv) $A \cup U = U$ (Law of U)

(v) $A \cup A = A$ (Idempotent law)

Intersection of sets:

The intersection of two sets A and B, denoted by $A \cap B$ is the set of all elements which are common to both A and B.

If A = {2,4,6,8} & B = {6, 8, 10, 12}, A ∩ B = {6, 8}.

Properties of Intersection

(i) $A \cap B = B \cap A$ (Commutative law)

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

(iii) $A \cap \emptyset = \emptyset$ (Law of $\emptyset$)

(iv) $A \cap U = A$ (Law of U)

(v) $A \cap A = A$ (Idempotent law)

(vi) Distributive laws:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

If $A \cap B = \emptyset$, then A and B are said to be **Disjoint Sets.**

Difference of sets

The difference of sets A and B, in this order, denoted by $A - B$ is the set of elements which belong to A but not to B.

Thus $A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly $B - A = \{x : x \in B \text{ and } x \notin A\}$.

If A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8}, then

A − B = {1, 3, 5} and

B − A = {8}.

Complement of a set

Let U be the universal set and A is a subset of U. Then the complement of A with respect to U, denoted by A' is the set of all elements of U which are not the elements of A.

Thus $A' = \{x: x \in U \text{ and } x \notin A\}$.

Properties of Complement Sets

(i) $A \cup A' = U$ **(ii)** $A \cap A' = \emptyset$ **(iii)** $(A')' = A$.

(iv) $\emptyset' = U$ **(v)** $U' = \emptyset$

De Morgan's Laws:

(i) $(A \cup B)' = A' \cap B'$ **(ii)** $(A \cap B)' = A' \cup B'$.

Practical Problems on Union & Intersection of Sets

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A - B) = n(A) - n(A \cap B)$

$n(B - A) = n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

RELATIONS AND FUNCTIONS

Ordered pair:

Two ordered pairs (a, b) and (c, d) are said to be equal $\Leftrightarrow a = c \;\&\; b = d$.

Cartesian products of sets:

The **Cartesian product** of two non-empty sets A and B is defined as the set of all ordered pairs (a, b), where $a \in A$, $b \in B$. Denoted as **A × B**

If A and B are two finite sets, then

$\mathbf{n(A \times B) = n(A).\, n(B)}$.

Relations: Subset of the Cartesian product $A \times B$.

If A contains m elements and B contains n elements, then the number of relations from A to B is 2^{mn}. It includes the **empty relation** $R = \phi$ and the **universal relation** R = $A \times B$.

Domain : The set of first elements of ordered pairs in R is called the ***domain*** of the relation R

Range : The set of second elements of ordered pairs in R is called the ***range*** of the relation R.

Codomain : If $f : A \to B$, then the whole set B is called the ***codomain*** of the relation R·

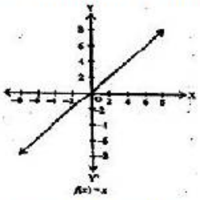
Functions: A relation f from a non-empty set A to a non-empty set B is said to be a ***function*** if every element of set A has only one image in set B.

If $f : A \to B$, then A is called the ***domain*** of f and B is called the ***codomain*** of f. The set of all image points of elements points of elements A is called the ***range*** of f.

Some functions and their graphs:

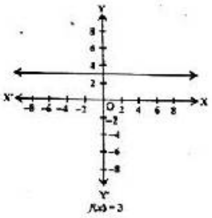
Identity function: $y = f(x) = x$

Domain and range of f are $\mathbb{R}$



Constant function: $y = f(x) = c$

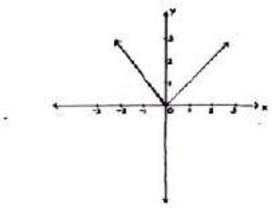
Domain of f is $\mathbb{R}$ and its range is $\{c\}$



Rational functions: Rational functions are functions of the type $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial functions of x defined in a domain, where $g(x) \neq 0$.

Modulus function: $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

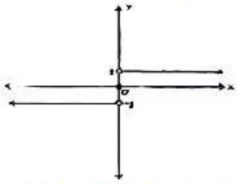
Its domain is $\mathbb{R}$ and its range is $[0, \infty)$



Signum function:

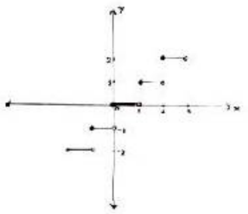
$$f(x) = \frac{|x|}{x} = \frac{x}{|x|} = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$$

Its domain is $\mathbb{R}$ and its range is {−1, 0, 1}



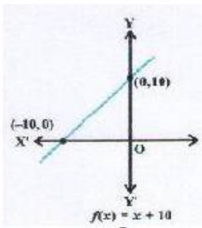
Greatest integer function: $f(x) = [x]$

Its domain is $\mathbb{R}$ and its range is the set of integers **Z**

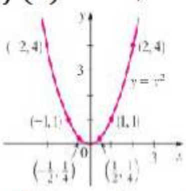


Other examples

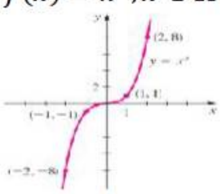
$f(x) = x + 10, x \in \mathbb{R}$ (**Linear function**)



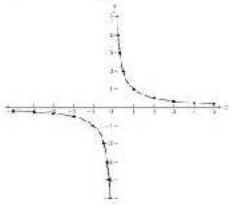
$f(x) = x^2 , x \in \mathbb{R}$ (**Quadratic function**)



$f(x) = x^3, x \in \mathbb{R}$ (**Cubic function**)



$f(x) = \frac{1}{x}, x \in \mathbb{R} - \{0\}$ (**Reciprocal function**)



Algebra of real functions

$(f + g)(x) = f(x) + g(x)$

$(f - g)(x) = f(x) - g(x)$

$(\alpha f)(x) = \alpha f(x)$

$(fg)(x) = f(x)g(x)$

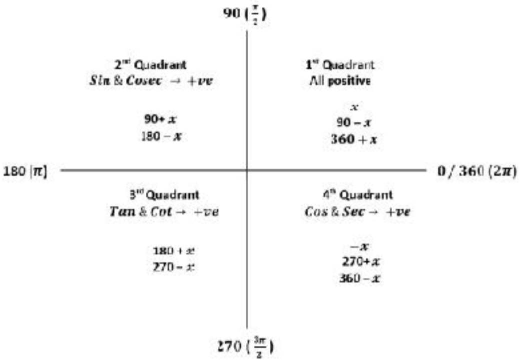
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

TRIGONOMETRIC FUNCTIONS

- To convert degree measure to radian measure, multiply with $\frac{\pi}{180}$
- To convert radian measure to degree measure, multiply with $\frac{180}{\pi}$
- Length of an arc of a circle, $l = r\theta$

Pythagoras' relation:

- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \operatorname{cosec}^2 x$



Any trigonometric function of $\left(\frac{n\pi}{2} \pm x\right)$ =

$$\begin{cases} \text{Trigonometric function of } x, \text{if } n \text{ is even} \\ \text{co} - \text{trigonometric function of } x, \text{if } n \text{ is odd} \end{cases}$$

and the sign depend upon the quadrant.

- The periods of $\sin, \cos, \sec, \operatorname{cosec}$ function are 2π
- The periods of $\tan, \cot$ function are π.

$\sin x \Rightarrow$ Domain $= R$ & Range $= [-1, 1]$

$\cos x \Rightarrow$ Domain $= R$ & Range $= [-1, 1]$

$\tan x \Rightarrow$ Domain $= R - \left\{(2n+1)\frac{\pi}{2}, n \epsilon Z\right\}$ & Range $= R$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$
$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$
$$\cot(x+y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$$
$$\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan x}{1 + \tan x}$$

$\sin 2x = 2\sin x \cos x$	$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$
$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$	$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$\cos 2x = \cos^2 x - \sin^2 x$	$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$
$\cos 2x = 1 - 2\sin^2 x$	$\cos x = 1 - 2\sin^2\frac{x}{2}$
$\cos 2x = 2\cos^2 x - 1$	$\cos x = 2\cos^2\frac{x}{2} - 1$
$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$	$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$
$1 + \cos 2x = 2\cos^2 x$	$1 + \cos x = 2\cos^2\frac{x}{2}$
$1 - \cos 2x = 2\sin^2 x$	$1 - \cos x = 2\sin^2\frac{x}{2}$
$\sin 3x = 3\sin x - 4\sin^3 x$ $\cos 3x = 4\cos^3 x - 3\cos x$	$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Principal solutions:

The solution of a trigonometric equation which lies between $0 \le x < 2\pi$ are called principal solutions.

Eg: PS of the equation $\sin x = \frac{1}{2}$ are $\frac{\pi}{6}, \frac{5\pi}{6}$

General solution:

The expression involving an **integer** (Z) n which gives all solutions of a trigonometric equation is called general solution.

General solution of some standard trigonometric equations:

$\sin x = 0 \Rightarrow x = n\pi$

$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$

$\tan x = 0 \Rightarrow x = n\pi$

$\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y$

$\cos x = \cos y \Rightarrow x = 2n\pi \pm y$

$\tan x = \tan y \Rightarrow x = n\pi + y$

Representation of sides of a triangle in terms of the sin of angles and vice-versa.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$\Rightarrow a = k\sin A, b = k\sin B, c = k\sin C$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$\Rightarrow \sin A = ka,\ \sin B = kb,\ \sin C = kc$

Law of cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos A = \frac{b^2+c^2-a^2}{2bc}$ $\qquad \cos B = \frac{a^2+c^2-b^2}{2ac}$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

In any $\Delta ABC, A + B + C = \pi$ then

$A + B = \pi - C$ & $C = \pi - (A+B)$ and

$\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$ & $\frac{C}{2} = \frac{\pi}{2} - \left(\frac{A+B}{2}\right)$

PRINCIPLE OF MATHEMATICAL INDUCTION

- Denote the given statement by P(n) and show that P(1) is true.
- Assume that P(k) is true for some natural number k.
- Show that P(k+1) is true, whenever P(k) is true
- Hence, by the principle of mathematical induction (PMI), P(n) is true for all natural number n.

COMPLEX NUMBERS

$i^1 = i$	$i^2 = -1$	$i^3 = -i$	$i^4 = 1$	$i^{-1} = -i$
$i^{4n} = 1$	$i^{4n+1} = i$	$i^{4n+2} = -1$	$i^{4n+3} = -i$	
$(i^2)^{even} = 1$		$(i^2)^{odd} = -1$		

Modulus: If $z = a + ib$, then $|z| = \sqrt{a^2 + b^2}$

Conjugate: If $z = a + ib$, then $\bar{z} = a - ib$

Multiplicative inverse: $z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{a-ib}{a^2+b^2}$

$(a + ib)\,(a - ib) = a^2 + b^2$

Polar form: $z = r(cos\theta + isin\theta)$ where $r = |z|$

- If $z = x + iy$, then $r = \sqrt{x^2 + y^2}$ and θ is obtained from $\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$

$tan\alpha = \left|\frac{y}{x}\right|$

Quadrant	I	II	III	IV
θ	α	$\pi - \alpha$	$\alpha - \pi$	$-\alpha$

Quadratic equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Square root of a complex number:

$$\sqrt{a \pm ib} = \pm\left(\sqrt{\frac{|z|+a}{2}}, \pm i\sqrt{\frac{|z|-a}{2}}\right)$$

LINEAR INEQUALITIES

⇒ A statement involving variables and the sign of inequality ($<, \leq, >, \geq, \neq$) is called an inequality. If each term of an inequality is of first degree, then it is a linear inequality.

⇒ The solution of inequality in one variable is a value of the variable which makes it a true statement.

⇒ An inequality may or may not have a solution.

⇒ If the inequality has a solution, then it may have infinitely many solutions.

Algebraic solutions of linear inequalities in one variable

☑ Equal number may be added to or subtracted from both sides of an inequality without affecting the sign of the inequality.

☑ Both sides of an inequality can be multiplied with or divided by the same positive number. But when both sides are multiplied with or divided by a negative number, the sign of inequality is reversed.

Graphical solution of linear inequalities

PERMUTATIONS AND COMBINATIONS

Fundamental principle of counting:

If an event A can occur in $'m'$ different ways and another event B can occur in $'n'$ different ways, then the total number of different ways of

a) simultaneous occurrence of both events in a definite order is mn. (known as multiplication principle).

b) happening exactly one of the events is $m + n$ (known as addition principle).

Factorial :

For any natural number n, we define factorial as

$n! = n(n-1)(n-2)\ \dots\ 3 \times 2 \times 1$ and

$0! = 1! = 1$

Factorials of negative integers are not defined

Permutation

Each of the different arrangement which can be made by taking some or all of a number of things is called a permutation.

Mathematically The number of ways of arranging n distinct objects in a row taking r $(0 \leq r \leq n)$ at a time is denoted by $P(n, r)$ or ${}^nP_r$

$${}^nP_r = \frac{n!}{(n-r)!} \quad \text{or}$$

$${}^nP_r = n(n-1)(n-2)\ \dots (n-r+1)$$

- ${}^nP_n = n!$ ${}^nP_0 = 1$ ${}^nP_1 = n$
- Number of permutations of n different things taken r at a time, allowing repetitions is n^r.
- Number of permutations of n different things taken r at a time, not allowing repetitions is ${}^nP_r$
- Number of permutations (arrangements) of n distinct things taken all at a time is ${}^nP_n = n!$
- The number of permutations of n things taken all at a time, in which p are alike of one kind, q are alike of second kind and r are alike of third kind and rest are different is $\frac{n!}{p!\,q!\,r!}$

Combinations

A combination is a group or selection of a number of objects irrespective of the order in which they occur.

The number of combination of 'n' objects taken 'r' at a time is denoted by ${}^nC_r$.

$${}^nC_r = \frac{n!}{r!\,(n-r)!} = \frac{{}^nP_r}{r!}$$

- ${}^nC_n = {}^nC_0 = 1$ ${}^nC_1 = n$
- ${}^nC_r = {}^nC_{n-r}$ or
- ${}^nC_x = {}^nC_y$ then $x = y$ or $x + y = n$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

BINOMIAL THEOREM

$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^n b + {}^nC_2\, a^{n-2} b^2 + \ldots + {}^nC_n\, b^n$

$(a-b)^n = {}^nC_0\, a^n - {}^nC_1\, a^n b + {}^nC_2\, a^{n-2} b^2 - \ldots + (-1)^n\, {}^nC_n\, b^n$

- The expansion of $(a+b)^n$ contains $n+1$ terms
- The **General Term** in the expansion of $(a+b)^n$ is $T_{r+1} = {}^nC_r\, a^{n-r} b^r$
- The **General Term** in the expansion of $(a-b)^n$ is $T_{r+1} = (-1)^r\, {}^nC_r\, a^{n-r} b^r$
- If n is even, then number of terms in the expansion is odd. $\therefore \left(\frac{n}{2}+1\right)^{th}$ term is middle term
- If n is odd, then number of terms in the expansion is even. $\therefore \left(\frac{n+1}{2}\right)^{th}$ term and its next terms are middle terms.
- If the general term in the binomial expansion is a constant, then that term is called the term independent of the variable.
- ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$
- ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$
- ${}^nC_0 - {}^nC_1 + {}^nC_2 - \ldots + (-1)^n\, {}^nC_n = 0$

SEQUENCES AND SERIES

ARITHMETIC PROGRESSION (A.P): A sequence is said to be an **Arithmetic progression** if the difference between every two consecutive terms is always same. This fixed number is called **common difference** of the A.P and is denoted by **d.** first term is denoted by **a.**

General form of an A.P

$a, a+d, a+2d, \ldots\ldots a+(n-1)d$

$d = \frac{Term\ difference}{Position\ difference}$

n^th term of an A.P

$a_n = a+(n-1)d$ or $a_n = dn + a - d$

No of terms: $n = \left(\frac{a_n - a_1}{d}\right) + 1$

Sum to n terms

$S_n = \frac{n}{2}(a_1 + a_n)$ or

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n$

✓ Arithmetic mean between a and b is $\frac{a+b}{2}$.

GEOMETRIC PROGRESSION (G.P): A sequence of non zero numbers is said to be a **Geometric progression**, if the ratio of each term, except the first one, by its preceeding term is always the same. The ratio is called the **common ratio** of the G.P and is generally denoted by **r.**

General form of a G.P

$a, ar, ar^2, ar^3, \ldots\ldots ar^{n-1}$

n^th term of an G.P

$a_n = ar^{n-1}$

Sum to n terms of a G.P

$S_n = \frac{a(1-r^n)}{1-r}$, if r < 1

$S_n = \frac{a(r^n-1)}{r-1}$, if r > 1

$S_n = na$, if r = 1

- If a, b, c are in G.P, then $b^2 = ac$
- Three consecutive terms of G.P are $\frac{a}{r}, a, ar$

Sum to n terms of special series

1. **Sum of first n natural numbers**

   $\sum n = 1+2+3+\ldots\ldots\ldots = \frac{n(n+1)}{2}$

2. **Sum of squares of first n natural numbers**

   $\sum n^2 = 1^2+2^2+3^2+\ldots\ldots\ldots n^2 = \frac{n(n+1)(n+2)}{6}$

3. **Sum of cubes of first n natural numbers**

   $\sum n^3 = 1^3+2^3+3^3+\ldots\ldots\ldots n^3 = \left(\frac{n(n+1)}{2}\right)^2$

STRAIGHT LINES

Distance formula,

$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

Section formula

- Internal division : $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$
- External division : $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$
- Midpoint formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Area of triangle

$= \frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$

Centroid of a triangle: $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Slope of a line (m):

$m = tan\theta$ or $m = \frac{y_2-y_1}{x_2-x_1}$ or $m = -\frac{a}{b}$

- Slope of a line parallel to x-axis is zero
- Slope of a line parallel to y-axis is not defined
- Slopes of parallel lines are equal.
- If two lines are perpendicular, then the slope of one line is negative reciprocal of the other. $(m_1 \times m_2 = -1)$
- 3 points A,B,C are collinear, then slope of AB = slope of BC.

Angle between two lines: $tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$

Equation of line:

$y - y_1 = m(x - x_1)$

General form : $ax + by + c = 0$

Slope intercept form : $y = mx + c$

Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$

[a = x-intercept, b = y-intercept]

Normal form : $xcos\omega + ysin\omega = p$

Equation of x-axis: $y = 0$ & y-axis : $x = 0$

Equation of line parallel to x-axis : $y = k$

Equation of line parallel to y-axis : $x = k$

The **foot of the perpendicular** (m, n) from (x_1, y_1) to the line ax+by+c=0 is given by

$\frac{m-x_1}{a} = \frac{n-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$

The **image or reflection** (m, n) of the point (x_1, y_1) in the line ax+by+c=0 is given by

$\frac{m-x_1}{a} = \frac{n-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$

⇒Distance of a point from a line: $d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

⇒Distance between two parallel lines : $d = \frac{|c_2-c_1|}{\sqrt{a^2+b^2}}$

⇒The equation of line passing through the point of intersection of two given lines $L_1=0$ and $L_2=0$ may be written as $L_1+KL_2=0$

Shifting of origin : New origin = old coordinate – coordinate of new origin

INTRODUCTION TO 3D GEOMETRY

Name	XOYZ	X¹OYZ	X¹OY¹Z	XOY¹Z
Octant	1st	2nd	3rd	4th
Sign	+ + +	− + +	− − +	+ − +
Eg	$(2, 3, 5)$	$(-2, 3, 5)$	$(-2, -3, 5)$	$(2, -3, 5)$

Name	XOYZ¹	X¹OYZ¹	X¹OY¹Z¹	XOY¹Z¹
Octant	5th	6th	7th	8th
Sign	+ + −	− + −	− − −	+ − −
Eg	$(2, 3, -5)$	$(-2, 3, -5)$	$(-2, -3, -5)$	$(2, -3, -5)$

The coordinates of any point on

x − axis : (x, 0,0)

y − axis : (0, y, 0)

z − axis : (0,0, z)

XY − plane : (x, y, 0)

XZ − plane : (x, 0, z)

YZ − plane : (0, y, z)

Distance formula,

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Section formula

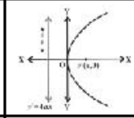
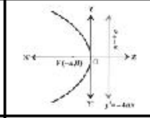
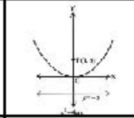
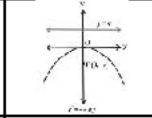
- Internal division : $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$
- External division : $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$
- Midpoint formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Centroid of a triangle: $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

CONIC SECTIONS

- **Equation of a circle** with centre (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$
- Equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, then centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$

PARABOLA

Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
				
Axis of symmetry	X-axis	X-axis	Y-axis	Y-axis
Vertex	(0,0)	(0,0)	(0,0)	(0,0)
Focus	(a, 0)	(−a, 0)	(0, a)	(0, −a)
Equation of Directrix	$x+a=0$	$x-a=0$	$y+a=0$	$y-a=0$
Length of Latus Rectum	4a	4a	4a	4a
Equation of Latus Rectum	x = a	x = −a	y = a	y = −a
Quadrants	1st & 4th	2nd & 3rd	1st & 2nd	3rd & 4th

ELLIPSE

Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a^2 > b^2$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $a^2 > b^2$
Centre	(0,0)	(0,0)
Foci	$(\pm c, 0)$ or $(\pm ae, 0)$	$(0, \pm c)$ or $(0, \pm ae)$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Equation of latus rectum	$x = \pm c$ or $x = \pm ae$	$y = \pm c$ or $y = \pm ae$
Equation of major axis	$y = 0$	$x = 0$
Equation of minor axis	$x = 0$	$y = 0$
Length of major axis	$2a$	$2a$
Length of minor axis	$2b$	$2b$
Equation of directrices	$x = \frac{a}{e}, x = -\frac{a}{e}$	$y = \frac{a}{e}, y = -\frac{a}{e}$
Relation b/w a, b & e	$b^2 = a^2(1-e^2)$	$b^2 = a^2(1-e^2)$
Relation b/w a, b & c	$c^2 = a^2 - b^2$	$c^2 = a^2 - b^2$
Eccentricity	$e = \frac{c}{a}$	$e = \frac{c}{a}$

HYPERBOLA

Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Centre	(0,0)	(0,0)
Foci	$(\pm c, 0)$	$(0, \pm c)$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Length of transverse axis	$2a$	$2a$
Length of conjugate axis	$2b$	$2b$
Equation of directrices	$x = \frac{a}{e}, x = -\frac{a}{e}$	$y = \frac{a}{e}, y = -\frac{a}{e}$
Relation b/w a, b & c	$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$
Eccentricity	$e = \frac{c}{a}$	$e = \frac{c}{a}$

LIMITS AND DERIVATIVES

Limit of a function

$\lim_{x\to a} f(x) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

Properties

- ✓ $\lim_{x\to a} k = k$
- ✓ $\lim_{x\to a} x = a$
- ✓ $\lim_{x\to a} \left(f(x)+g(x)\right) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
- ✓ $\lim_{x\to a} \left(f(x)-g(x)\right) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
- ✓ $\lim_{x\to a} k\, f(x) = k \lim_{x\to a} f(x)$
- ✓ $\lim_{x\to a} \left(f(x).g(x)\right) = \lim_{x\to a} f(x) \,.\, \lim_{x\to a} g(x)$
- ✓ $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
- ✓ $\lim_{x\to c} f(x) = \lim_{h\to 0} f(c+h)$
- ✓ $\lim_{x\to a} \frac{x^n-a^n}{x-a} = n\, a^{n-1}$ $\qquad \lim_{x\to a} \frac{x^m-a^m}{x^n-a^n} = \frac{m}{n} a^{m-n}$
- ✓ $\lim_{x\to 0} \frac{\sin x}{x} = 1$ $\qquad \lim_{x\to 0} \frac{\sin mx}{x} = m$
- ✓ $\lim_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n}$ $\qquad \lim_{x\to 0} \frac{x}{\sin x} = 1$
- ✓ $\lim_{x\to 0} \frac{\tan x}{x} = 1$
- ✓ $\lim_{x\to 0} \frac{e^x-1}{x} = 1$

Derivatives

$f^1(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Product rule:

$\frac{d}{dx}\left(f(x)g(x)\right) = f(x)\frac{d}{dx}\, g(x) + g(x)\frac{d}{dx}\, f(x)$

Quotient rule:

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$

Derivatives of some functions

$\frac{d}{dx}k = 0$

$\frac{d}{dx}x = 1$

$\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = sec^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}cot\ x = -cosecx$

$\frac{d}{dx}cosec\ x = -cosecx\ cotx$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$

$\frac{d}{dx}\log x = \frac{1}{x}$

$\frac{d}{dx}e^x = e^x$

STATISTICS

Mean ($\bar{x}$):

Individual series (Ungrouped data) : $\bar{x} = \frac{\Sigma X}{N}$

Discrete & Continuous series (Grouped data) :

$\bar{x} = \frac{\Sigma fx}{N}$

Median(M):

Individual series (Ungrouped data) :

- Median is the number present in the middle when the numbers in a set of data are arranged in ascending or descending order.
- If the number of numbers in a data set is even, then the median is the mean of the two middle numbers.

Discrete series :

If N is odd, M = $\left(\frac{N+1}{2}\right)^{th}$ item

If N is even, M = $\frac{\left(\frac{N}{2}\right)^{th} \text{item} + \left(\frac{N}{2}+1\right)^{th} \text{item}}{2}$

Continuous series

Median = $l + \frac{\frac{N}{2} - Cf}{f} \times h$

Range = Maximum value – Minimum value

Mean deviation for ungrouped data [Individual series]:

a) Mean deviation about the mean: $\frac{\Sigma|x-\bar{x}|}{n}$

b) Mean deviation about the median: $\frac{\Sigma|x-M|}{n}$

Mean deviation for grouped data [Discrete series & Continuous series]:

a) Mean deviation about the mean: $\frac{\Sigma f\,|x-\bar{x}|}{N}$

b) Mean deviation about the median: $\frac{\Sigma f\,|x-M|}{N}$

Variance and Standard Deviation:

Variance and Standard Deviation for ungrouped data:

Variance, $\sigma^2 = \frac{\Sigma(x-\bar{x})^2}{n}$ or

$\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

Standard Deviation , $\sigma = \sqrt{variance}$

Variance and Standard Deviation for grouped data:

Variance, $\sigma^2 = \frac{\Sigma f(x-\bar{x})^2}{N}$ or

$\sigma^2 = \frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2$

Standard Deviation , $\sigma = \sqrt{variance}$

Coefficient of variation, CV = $\frac{\sigma}{\bar{x}} \times 100$

Note: If CV is more, the series is more variable and less consistent or less stable.

If CV is less, the series is more consistent or stable and less variable

MATHEMATICAL REASONING

- A sentence is called mathematically acceptable statement if it is either true or false but not both.
- A statement which is both true and false simultaneously is known as a **paradox**. Paradoxes are not statements.

Negation of a statement

The denial of a statement is called the negation of the statement.

If p is a statement, then the negation of p is also a statement and is denoted by $\sim p$ and read as '*not p*'

- If p is true then $\sim p$ is false.
- If p is false then $\sim p$ is true.

✓ If p is a statement, then $\sim p$ is formed by inserting the phrases "**It is not the case that**" or "**It is false that**" before p. or insert the word '**not**' at proper place in the statement p.

Compound and Simple statement

- A **compound statement** is a statement which is made up of two or more statements. Each statement is called a component statement.
- A statement which cannot be broken into component statement is called a **simple statement.**

Conjunction: If two simple statements p and q are connected by the word 'and', the resulting compound statement is called a conjunction of p and q , and is written as $p \wedge q$

Disjunction: If two simple statements p and q are connected by the word 'or', the resulting compound statement is called disjunction of p and q is written as $p \vee q$.

Quantifiers: These are the phrases like, "there exist" and "for all".

Conditional statement (if-then)

Let p and q be two statements then the compound statement of the form **'If p then q'** is called a conditional statement.

The different forms of if p then q.

✓ $p \Rightarrow q$

✓ p is a sufficient condition for q.

✓ p only if q

✓ q is a necessary condition for p

✓ $\sim q$ implies $\sim p$

Contrapositive : The contrapositive of the statement, "if p, then q" is " **if $\sim q$, then $\sim p$**".

Eg : " If a triangle is equilateral, then it is isosceles"

"If a triangle is not isosceles, then it is not equilateral"

Converse: The converse of a given statement "if p, then q" is "**if q, then p**".

Eg: "if a number n is even, then n^2 is even"

if a number n^2 is even, then n is even

Validating statements

I. Direct method

II. Contrapositive method

III. Contradiction method.

Verify by the method of contradiction that $\sqrt{2}$ is irrational.

Let p: $\sqrt{2}$ is irrational.

Assume that $\sqrt{2}$ is not irrational.

ie, $\sqrt{2}$ is rational.

ie, $\sqrt{2} = \frac{a}{b}$, where $a, b \epsilon Z$ and have no common factor.

$\therefore 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2$ ……………..(1)

ie, 2 divides a

$\Rightarrow$ a is a multiple of 2

$\therefore$ equation (1) becomes

$\Rightarrow (2k)^2 = 2b^2$

$\Rightarrow 4k^2 = 2b^2$

$\Rightarrow b^2 = 2k^2$

$\Rightarrow$ 2 divides b

or b is a multiple of 2, which contradicts the assumption that a and b have no common factor.

$\therefore \sqrt{2}$ is irrational.

PROBABILITY

Random experiment: A random experiment is a process in which the outcome cannot be predicted with certainty in probability.

Outcome: A possible result of a random experiment is called its outcome. Example: In an experiment of throwing a die, the outcomes are 1, 2, 3, 4, 5, or 6.

Equally likely outcomes: The outcomes of a random experiment are called equally likely, if all outcomes have equal preferences.

Sample point: Each outcome of the random experiment is also called a sample point.

Sample space: The set of all possible outcomes of a random experiment is called the sample space and is denoted by the symbol S.

Sample space of some important random experiments:

a) Tossing a coin

sample space $S = \{H, T\}$

b) **Throwing a die**

sample space $S = \{1,2,3,4,5,6\}$

c) **Tossing a coin two times / Two coins are tossed simultaneously**

Sample space = $S = \{HH, HT, TH, TT\}$

d) **Tossing a coin three times / Three coins are tossed simultaneously**

$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

e) **Rolling a pair of dice simultaneously / Throwing a die two times**

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

f) **A coin is tossed until a head comes up**

$S = \{H, TH, TTH, TTTH, \ldots\}$

- 2^n is the number of elements in the sample space of tossing n coins
- 6^n is the number of elements in the sample space of throwing n dice

Event: Subset of sample space is called event.

Consider the random experiment of tossing a coin 3 times.

The sample space,

$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

Some events are given below

Exactly two heads: $\{HHT, HTH, THH\}$

Atleast two heads: $\{HHH, HHT, HTH, THH\}$

Atmost two heads: $\{HHT, HTH, THH, TTH, THT, HTT, TTT\}$

No heads: $\{TTT\}$

More than two heads: $\{HHH\}$

Types of events:

Impossible event: An event that has no chance of occurring is called an Impossible Event. Null set is an impossible event. The probability of an impossible event is always zero.

Sure event: A sure event is an event, which always happens. The probability of sure event is 1. Sample space S is sure event.

Simple event: Any event consisting of a single point of the sample space is known as a **simple event** in probability. For example, if $S = \{56,78,96,54,89\}$ and $E = \{78\}$ then E is a simple event.

Compound event: If an event has more than one sample point, it is called a compound event. For example, if $S = \{56, 78, 96, 54, 89\}$, E = {78, 56, 89} then, E represent compound events.

Mutually exclusive events:

Two events are said to be mutually exclusive if they cannot occur at the same time or simultaneously. In other words, **mutually exclusive events** are called disjoint events. ie, $A \cap B = \emptyset$

Consider a random experiment of throwing a die, sample space, $S = \{1,2,3,4,5,6\}$. $A = \{1,3,5\}$ and $B = \{2,4,6\}$ are mutually exclusive, since $A \cap B = \emptyset$

Exhaustive events:

If $E_1, E_2, E_3, \ldots E_n$ are n events of a sample space S, and if, $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$ then $E_1, E_2, E_3, \ldots E_n$ are called exhaustive events.

Consider a random experiment of throwing a die, sample space, $S = \{1,2,3,4,5,6\}$. $A = \{1,2,3\}$ and $B = \{3,4\}$, $C = \{4,5,6\}$ are exhaustive, since $A \cup B \cup C = S$

Algebra of events:

Complementary event ⇒ not A = $A^I = S - A$

The event A or B ⇒ $A \cup B$

The event A and B ⇒ $A \cap B$

The event A but not B ⇒ $A - B$ or $A \cap B^I$

Neither A nor B ⇒ $(A \cup B)^I$ or $A^I \cap B^I$

Not A and not B ⇒ $A^I \cap B^I$

Exactly one of A and B ⇒ $(A \cap B^I) \cup (A^I \cap B)$

Probability of equally likely outcomes

Probability = $\dfrac{Number\ of\ favourable\ outcomes}{Number\ of\ all\ outcomes}$

- The probability of a sure event is 1.
- The probability of an impossible event is 0.
- The probability of any event lies between 0 and 1.

Probability of an event A or B

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- If A and B are mutually exclusive $P(A \cap B) = \emptyset$

✓ $\boldsymbol{P(A\ or\ B) = P(A \cup B)}$

✓ $\boldsymbol{P(A\ and\ B) = P(A \cap B)}$

✓ $\boldsymbol{P(not\ A) = P(A^I) = 1 - P(A)}$

✓ $\boldsymbol{P(A\ and\ not\ B) = P(A - B) = P(A) - P(A \cap B)}$

✓ $\boldsymbol{P(not\ A\ and\ not\ B) = P(neither\ A\ nor\ B) = P(A^I \cap B^I) = P(A \cup B)^I = 1 - P(A \cup B)}$

✓ $\boldsymbol{P(not\ A\ or\ not\ B) = P(A^I \cup B^I) = P(A \cap B)^I = 1 - P(A \cap B)}$

✓ $\boldsymbol{P(both\ will\ not\ qualify) = P(not A\ and\ not B)}$

Basic concept on drawing a card:

- In a pack or deck of 52 playing cards
- They are divided into 4 suits of 13 cards each
- Spades ♠ hearts ♥, diamonds ♦, clubs ♣.
- Cards of spades and clubs are black cards (26).
- Cards of hearts and diamonds are red cards (26).
- The card in each suit, are ace, king, queen, jack or knaves, 10, 9, 8, 7, 6, 5, 4, 3 and 2.
- King, queen and jack (or knaves) are face cards. So, there are 12 face cards in the deck of 52 playing cards.