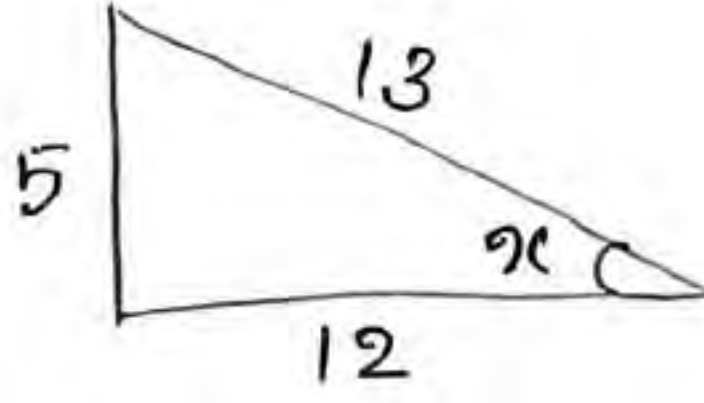


①
FIRST YEAR EXAMINATION - JUNE 2022
MATHEMATICS

Answer any 6 questions from 1 to 8.
Each carries 3 scores.

1	<p>(i) $A \cap A' = \phi$</p> <p>(ii) (a) $A = \{1, 2\}$</p> <p>(b) Subsets of A are $\{1\}, \{2\}, \{1, 2\}, \phi$.</p>	(1)	1	1	3
2	<p>(i) $25^\circ = 25 \times \frac{\pi}{180} = \frac{5\pi}{36}$ radian.</p> <p>(ii) $\tan x = \frac{5}{12}$, x in III quadrant</p> <p>$\sin x = \frac{-5}{13}$</p> <p>$\cos x = \frac{-12}{13}$</p> 	1	1	1	3
3	<p>(i) $\frac{4}{3}, x, \frac{3}{4}$ are in G.P</p> <p>$\therefore \frac{x}{4/3} = \frac{3/4}{x}$</p> <p>$x^2 = \frac{3}{4} \times \frac{4}{3} = 1$</p> <p>$x = \pm 1$</p>	$\frac{1}{2}$		$\frac{1}{2}$	

Prepared by Shiny. G. S, RRVGHSS, Kilimanoor

(2)

(ii) $\sqrt{3}, 3, 3\sqrt{3}, \dots$

$$a = \sqrt{3}$$

$$r = \frac{3}{\sqrt{3}} = \sqrt{3}$$

n^{th} term, $a_n = ar^{n-1}$
 $= \sqrt{3} (\sqrt{3})^{n-1}$
 $= \sqrt{3}^n$

4. Lines are $-\sqrt{3}x + y - 5 = 0$ — (1)

$$-x + \sqrt{3}y + 6 = 0$$
 — (2)

slope of (2), $m_1 = -\frac{a}{b} = -\frac{-1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

slope of (1), $m_2 = -\frac{a}{b} = -\frac{-\sqrt{3}}{1} = \sqrt{3}$.

If θ is the angle between the lines

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}} \right| = \left| \frac{3-1}{2\sqrt{3}} \right| = \left| \frac{1}{\sqrt{3}} \right|$$

$$\tan \theta = \pm \frac{1}{\sqrt{3}}$$

If $\tan \theta = \frac{1}{\sqrt{3}}$, $\theta = \underline{\underline{\hat{\pi}/6}}$

If $\tan \theta = -\frac{1}{\sqrt{3}}$, $\theta = \underline{\underline{\pi - \hat{\pi}/6 = 5\hat{\pi}/6}}$

(3)

5 (i)

$$y^2 = 8x$$

Parabola is of the form $y^2 = 4ax$

$$\therefore 4a = 8$$

$$a = 2$$

Focus of $y^2 = 4ax$ is $(a, 0)$

$$\therefore \text{Focus} = (2, 0)$$

(ii)

$$x^2 + y^2 + 6x - 4y - 3 = 0$$

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 3 + 9 + 4$$

$$(x+3)^2 + (y-2)^2 = 16$$

$$(x+3)^2 + (y-2)^2 = 4^2$$

$$\text{centre} = (-3, 2)$$

$$\text{radius} = 4$$

6.

Let $A(-2, 4, 7)$ and $B(3, -5, 8)$

Let the ratio be $k:1$

Since yz plane divide the line segment, then x -coordinate = 0.

$$\therefore \frac{m x_2 + n x_1}{m+n} = 0$$

$$\frac{k \cdot 3 + 1 \cdot (-2)}{k+1} = 0$$

$$3k - 2 = 0 \Rightarrow k = \frac{2}{3}$$

\therefore Ratio is $\frac{2}{3} : 1$ or $\underline{\underline{2:3}}$

(4)

7. (i) $\lim_{x \rightarrow 2} x^2 - 4 = 2^2 - 4 = \underline{\underline{0}}$

(ii) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{x^2 - 2^2}{x - 2}$

$$= 2 \cdot 2^{2-1}$$

$$= 2 \cdot 2 = \underline{\underline{4}}$$

(iii) $\lim_{x \rightarrow 0} \frac{\sin 4x}{x} = \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \cdot 4$

$$= 1 \cdot 4 = \underline{\underline{4}}$$

8.

$P : \sqrt{3}$ is irrational.

Assume that $\sqrt{3}$ is rational

i.e., $\sqrt{3} = \frac{a}{b}$, where a and b have no common factors.

$$a = \sqrt{3}b \Rightarrow a^2 = 3b^2$$

$$\Rightarrow 3 \text{ divides } a$$

i.e., $a = 3c \Rightarrow a^2 = 9c^2$

$$\therefore 3b^2 = 9c^2$$

$$b^2 = 3c^2 \Rightarrow 3 \text{ divides } b.$$

i.e., 3 is a common factor of a and b which contradicts our assumption that a and b have no common factor.

\therefore Our assumption is wrong.

$\therefore \sqrt{3}$ is not rational

$\therefore \sqrt{3}$ is irrational.

Answer any 6 questions from 9 to 17
Each carries 4 scores.

9 (i)

(a) $(-4, 5]$

(ii)

$U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{2, 3, 4, 6\}$

$B = \{3, 4, 5\}$

$A \cup B = \{2, 3, 4, 5, 6\}$

$(A \cup B)' = \{1, 7\}$

$A' = \{1, 5, 7\}$, $B' = \{1, 2, 6, 7\}$

$A' \cap B' = \{1, 7\}$

$\therefore (A \cup B)' = A' \cap B'$ verified.

1

1

1

1

4

10 (i)

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$R = \{(x, y) : 2x - y = 0, x, y \in A\}$

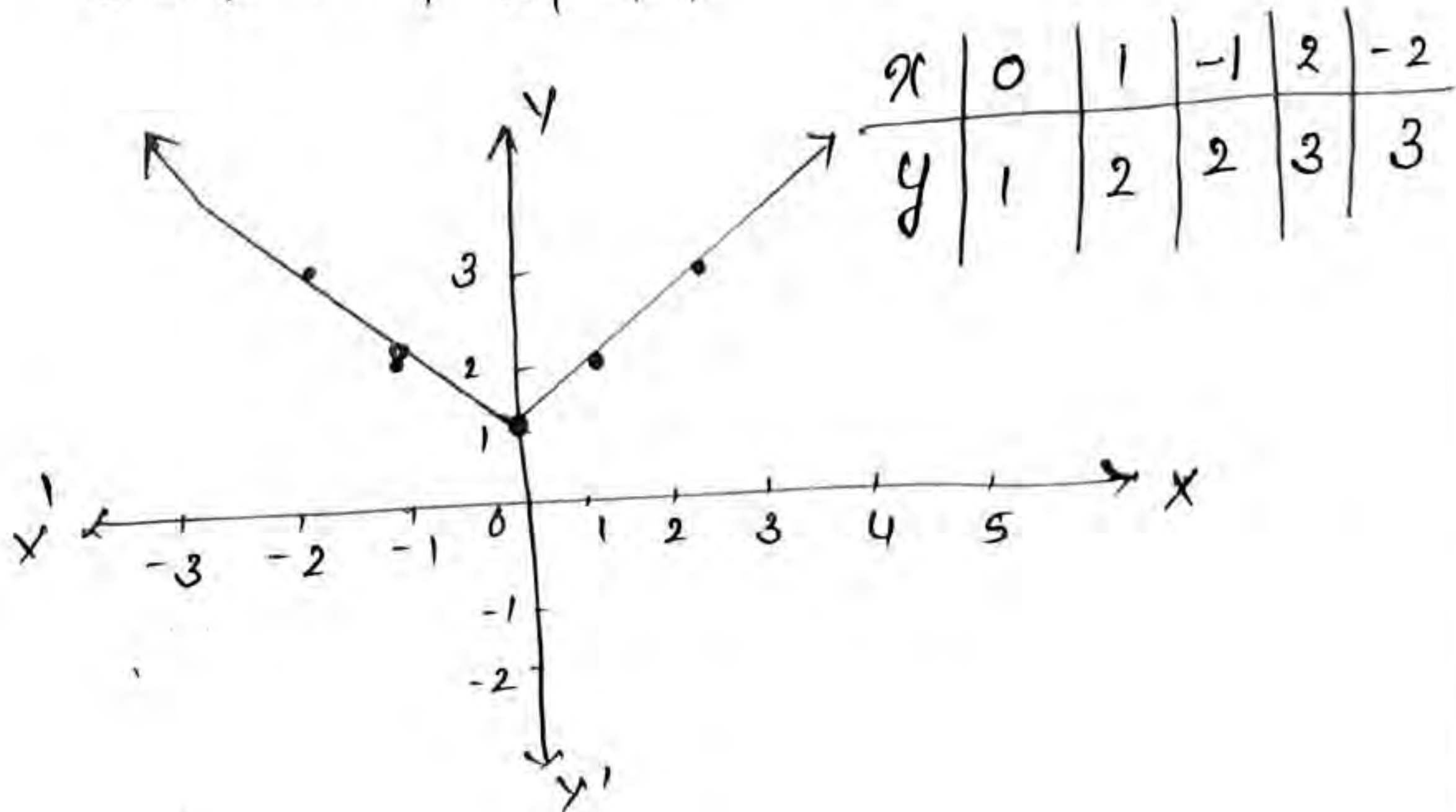
$R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$

Domain = $\{1, 2, 3, 4\}$

Range = $\{2, 4, 6, 8\}$

(ii)

$f(x) = |x| + 1$



2

4

(6)

11 (i) $P(n) : \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

$P(1) : \frac{1}{2} = 1 - \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

$\therefore P(1)$ is true

(ii) Assume that $P(k)$ is true.

$P(k) : \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$

To P.T $P(k+1)$ is true.

$P(k+1) :$

LHS = $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}}$

= $1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$

= $1 - \frac{1}{2^k} \left(1 - \frac{1}{2}\right)$

= $1 - \frac{1}{2^k} \cdot \frac{1}{2}$

= $1 - \frac{1}{2^{k+1}} = \text{RHS}$

$\therefore P(k+1)$ is true.

Hence by principle of mathematical induction $P(n)$ is true for all $n \in \mathbb{N}$.

12. (i) $n C_9 = n C_8$, then $n = 17$

(ii) $n P_r = \frac{n!}{(n-r)!}$

(iii)

⑦
MATHEMATICS

$$n = 11$$

M repeats 2 times

A repeats 2 times

T repeats 2 times

$$\text{Number of permutations} = \frac{11!}{2! 2! 2!}$$

$$= 4989600.$$

13.

$$(x + 9y)^{10}$$

(i) number of terms = 11

(ii) General term,

$$\begin{aligned} T_{r+1} &= n C_r a^{n-r} b^r & n &= 10 \\ &= 10 C_r (x)^{10-r} (9y)^r & a &= x \\ &= 10 C_r 9^r (x)^{10-r} y^r & b &= 9y. \end{aligned}$$

(iii) 5th term,

$$\begin{aligned} T_5 &= T_{4+1} = 10 C_4 9^4 (x)^{10-4} y^4 \\ &= 10 C_4 9^4 x^6 y^4 \\ &= \underline{\underline{1377810 x^6 y^4}}. \end{aligned}$$

14.

8 + 88 + 888 + ... to n terms

$$= 8 (1 + 11 + 111 + \dots \text{ to } n \text{ terms})$$

$$= \frac{8}{9} (9 + 99 + 999 + \dots \text{ to } n \text{ terms})$$

$$= \frac{8}{9} ((10-1) + (100-1) + (1000-1) + \dots \text{ to } n \text{ terms})$$

(8)

$$= \frac{8}{9} (10 + 10^2 + 10^3 + \dots \text{ to } n \text{ terms} - n)$$

$$= \frac{8}{9} \left[10 \left(\frac{10^n - 1}{10 - 1} \right) - n \right]$$

$$= \frac{8}{9} \left(\frac{10^{n+1} - 10 - 9n}{9} \right)$$

$$= \frac{8}{81} (10^{n+1} - 9n - 10)$$

15. (i)

$$x - 7y + 5 = 0. \quad a=1, \quad b=-7$$

$$\text{slope} = -\frac{a}{b} = -\frac{1}{-7} = \frac{1}{7}$$

(ii)

Lines are perpendicular.

$$\therefore \text{slope of line} = \frac{-1}{\frac{1}{7}} = -7$$

Line passes through (3, 0)

\therefore Equation of line passing through (3, 0) with slope -7 is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -7(x - 3)$$

$$y = -7x + 21$$

$$7x + y - 21 = 0$$

16.

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

Ellipse is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(9)

$$a^2 = 36 \Rightarrow a = 6$$

$$b^2 = 16 \Rightarrow b = 4.$$

$$a^2 = b^2 + c^2 \Rightarrow 36 = 16 + c^2$$

$$\Rightarrow c^2 = 36 - 16 = 20$$

$$\Rightarrow c = \sqrt{20} = 2\sqrt{5}.$$

$$\text{foci} = (\pm c, 0) = (\pm 2\sqrt{5}, 0)$$

$$\text{vertices} = (\pm a, 0) = (\pm 6, 0)$$

$$\text{length of major axis} = 2a = 2 \times 6 = 12$$

$$\text{length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 16}{6} = \frac{16}{3}$$

17.

$$P(A) = 0.54$$

$$P(B) = 0.69$$

$$P(A \cap B) = 0.35$$

$$(i) P(A \cup B) = P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= 0.54 + 0.69 - 0.35$$

$$= 0.88.$$

$$(ii) P(\text{not } A \text{ and not } B) = P(A' \cap B')$$

$$= P(A \cup B)'$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.88 = \underline{\underline{0.12}}$$

Answer any 3 questions from 18 to 22
Each carries 6 score.

18. (i)

$$\frac{\cos 9^\circ - \cos 5^\circ}{\sin 17^\circ - \sin 3^\circ}$$

(10)

$$= \frac{-2 \sin\left(\frac{9\pi+5\pi}{2}\right) \sin\left(\frac{9\pi-5\pi}{2}\right)}{2 \cos\left(\frac{17\pi+3\pi}{2}\right) \sin\left(\frac{17\pi-3\pi}{2}\right)}$$

$$= \frac{-\sin(7\pi) \sin(2\pi)}{\cos(10\pi) \sin(7\pi)}$$

$$= \frac{-\sin 2\pi}{\cos 10\pi}$$

(ii)

$$\sin \pi = -\frac{\sqrt{3}}{2}$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$\sin \pi$ is -ve

\therefore principal solutions lies in III and IV quadrant.

$$\sin(\pi + \frac{\pi}{3}) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\text{or, } \sin 4\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\sin(2\pi - \frac{\pi}{3}) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\sin 5\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

\therefore Principal solutions are $4\frac{\pi}{3}$ and $5\frac{\pi}{3}$

$$\sin 4\frac{\pi}{3} \quad \sin \pi = \sin 4\frac{\pi}{3}$$

$$x = (-1)^n n\pi \quad x = n\pi + (-1)^n \frac{\pi}{3}$$

General solution is $x = n\pi + (-1)^n \frac{4\pi}{3}$

6

(11)

19 (i)

$$z = -1 + i\sqrt{3} \quad a = -1, \quad b = \sqrt{3}$$

$$r = |z| = \sqrt{a^2 + b^2} \\ = \sqrt{1 + 3} = 2.$$

$$\arg z, \theta =$$

$$-1 + i\sqrt{3} = r(\cos\theta + i\sin\theta) \\ = 2(\cos\theta + i\sin\theta)$$

$$-1 = 2\cos\theta, \quad \sqrt{3} = 2\sin\theta$$

$$\cos\theta = -\frac{1}{2} \quad \sin\theta = \frac{\sqrt{3}}{2}$$

θ lies in II quadrant,

$$\theta = \pi - \frac{\pi}{3} = 2\frac{\pi}{3}$$

Polar form of $-1 + i\sqrt{3} = 2(\cos 2\frac{\pi}{3} + i\sin 2\frac{\pi}{3})$

(ii)

$$\sqrt{5}x^2 + x + \sqrt{5} = 0$$

$$a = \sqrt{5}, \quad b = 1, \quad c = \sqrt{5}.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 - 20}}{2\sqrt{5}}$$

$$= \frac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}$$

20.
(i)

$$3 \frac{(x-2)}{5} \leq 5 \left(\frac{2-x}{3} \right)$$

$$9x - 18 \leq 50 - 25x$$

$$25x + 9x \leq 50 + 18$$

$$34x \leq 68$$

$$x \leq 2$$

$$x \in \underline{\underline{(-\infty, 2]}}$$

(ii)

$$x + 3y \leq 9, \quad 2x + y \leq 12, \quad x \geq 0, \quad y \geq 0$$

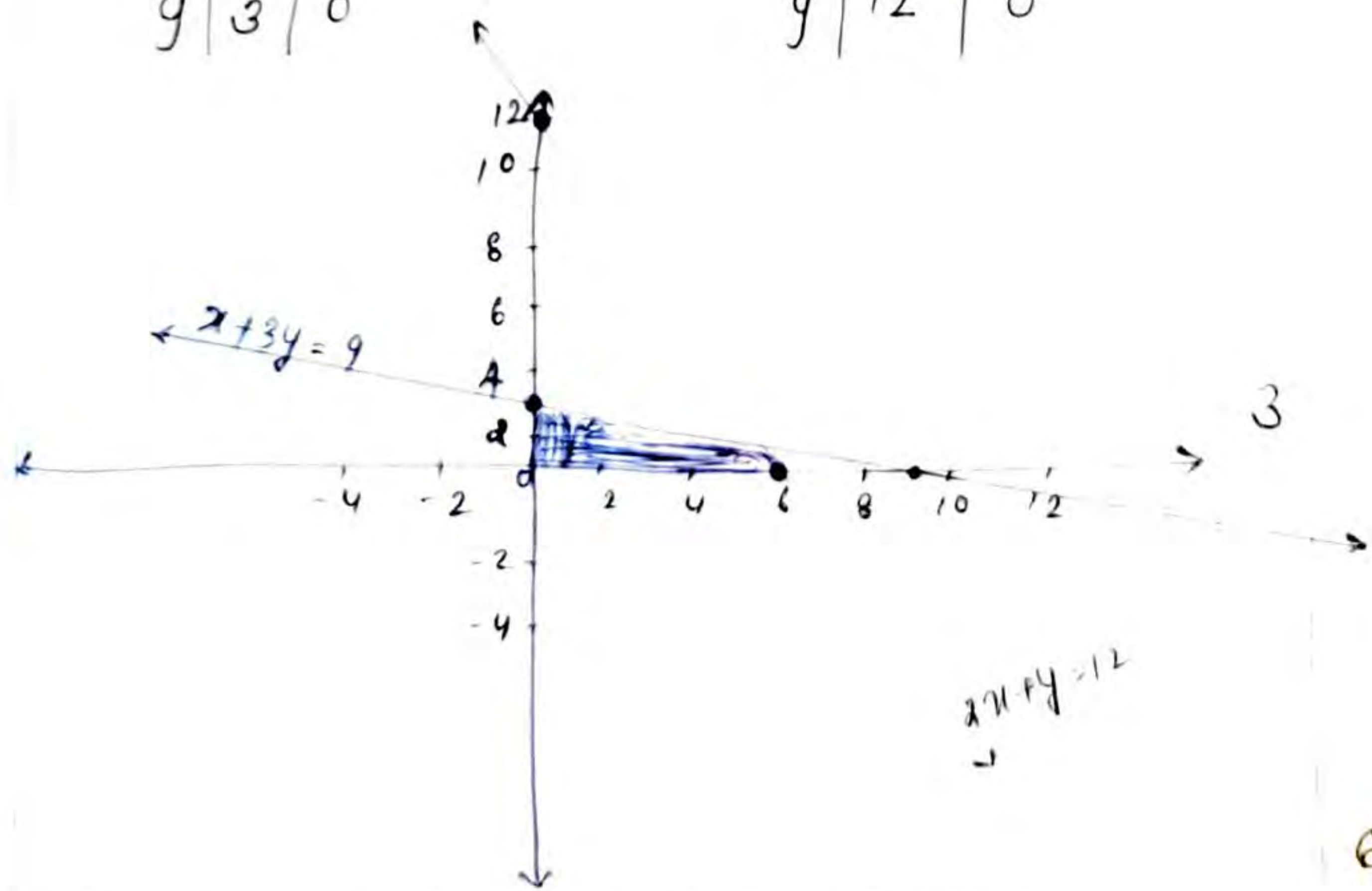
Consider the lines

$$x + 3y = 9$$

and $2x + y = 12$

x	0	9
y	3	0

x	0	6
y	12	0



Take (0,0), $x + 3y \leq 9 \Rightarrow 0 \leq 9$ is True
 $2x + y \leq 12 \Rightarrow 0 \leq 12$ is True

Shaded region is the solution region

21. (i)

$$f(x) = \cos x \quad f(x+h) = \cos(x+h)$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[-2 \sin\left(\frac{x+h+x}{2}\right) \sin\left(\frac{x+h-x}{2}\right) \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[-2 \sin\left(\frac{2x+h}{2}\right) \sin \frac{h}{2} \right] \\ &= \lim_{h \rightarrow 0} -2 \sin\left(\frac{2x+h}{2}\right) \frac{\sin \frac{h}{2}}{2 \cdot \frac{h}{2}} \\ &= -\sin\left(\frac{2x+0}{2}\right) \cdot 1 \\ &= -\sin x \end{aligned}$$

$$\therefore \frac{d}{dx} (\cos x) = -\sin x.$$

(ii)

$$\begin{aligned} &\frac{d}{dx} \left(\frac{x^2}{3x-1} \right) \\ &= \frac{(3x-1) \frac{d}{dx} (x^2) - x^2 \cdot \frac{d}{dx} (3x-1)}{(3x-1)^2} \\ &= \frac{(3x-1) 2x - x^2 \cdot 3}{(3x-1)^2} \\ &= \frac{6x^2 - 2x - 3x^2}{(3x-1)^2} \\ &= \frac{3x^2 - 2x}{(3x-1)^2} = \frac{x(3x-2)}{(3x-1)^2} \end{aligned}$$

22.

x_i	b_i	$x_i b_i$	$b_i x_i^2$
5	6	30	150
15	8	120	1800
25	14	350	8750
35	16	560	19600
45	4	180	8100
55	2	110	6050
	Σb_i = 50	$\Sigma x_i b_i$ = 1350	$\Sigma b_i x_i^2$ = 44450

(3)

(i) Mean, $\bar{x} = \frac{\Sigma x_i b_i}{\Sigma b_i} = \frac{1350}{50} = \underline{\underline{27}}$

1

(ii) Variance, $\sigma^2 = \frac{\Sigma b_i x_i^2}{\Sigma b_i} - (\bar{x})^2$
 $= \frac{44450}{50} - (27)^2$
 $= 889 - 729$
 $= \underline{\underline{160}}$

1

(iii) S.D, $\sigma = \sqrt{\text{var}} = \sqrt{160}$
 $= \underline{\underline{12.65}}$

1

Prepared by Shonij G.S
 RRVCHSS
 Kollamanoor