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PAPER – II MATHEMATICS - 2022

Version Code	**B1**	**Question Booklet Serial Number:**	9120933

Time: 150 Minutes	**Number of Questions : 120**	**Maximum Marks : 480**

Name of the Candidate	
Roll Number	
Signature of the Candidate	

INSTRUCTIONS TO CANDIDATES

1. **Please ensure that the VERSION CODE shown at the top of this Question Booklet is same as that shown in the OMR Answer Sheet issued to you.** If you have received a Question Booklet with a different Version code, please get it replaced with a Question Booklet with the same Version Code as that of OMR Answer Sheet from the Invigilator. **THIS IS VERY IMPORTANT.**
2. Please fill the items such as Name, Roll Number and Signature in the columns given above. Please also write Question Booklet Serial Number given at the top of this page against item 3 in the OMR Answer Sheet.
3. This Question Booklet contains 120 questions. For each question five answers are suggested and given against (A), (B), (C), (D) and (E) of which only one will be the '**Most Appropriate Answer**'. Mark the bubble containing the letter corresponding to the 'Most Appropriate Answer' in the OMR Answer Sheet, by using either **Blue or Black Ball Point Pen only.**
4. **Negative Marking**: In order to discourage wild guessing the score will be subjected to penalization formula based on the number of right answers actually marked and the number of wrong answer marked. Each correct answer will be awarded FOUR marks. ONE mark will be deducted for each incorrect answer. More than one answer marked against a question will be deemed as incorrect answer and will be negatively marked.
5. Please read the instructions in the OMR Answer Sheet for marking the answers. Candidates are advised to strictly follow the instruction contained in the OMR Answer Sheet.

IMMEDIATELY AFTER OPENING THE QUESTION BOOKLET, THE CANDIDATE SHOULD VERIFY WHETHER THE QUESTION BOOKLET CONTAINS ALL THE 120 QUESTIONS IN SERIAL ORDER. IF NOT, REQUEST FOR REPLACEMENT.

DO NOT OPEN THE SEAL UNTIL THE INVIGILATOR ASKS YOU TO DO SO.

PLEASE ENSURE THAT THIS QUESTION BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120 PRINTED PAGES 32.

1. Let $A=\{1,2,3,4,5\}$ and let $B=\{1,2,3,4\}$. If the relation $R:A\to B$ is given by $(a,\ b)\in R$ if and only if $a+b$ is even, then $n(R)$ is equal to

(A) 10 (B) 16 (C) 20 (D) 12 (E) 6

2. The domain of the function $f(x)=\left(x^2-2x-63\right)^{3/2}$, $x\in\mathbb{R}$ is

(A) $(-\infty,-6]\cup[9,\infty)$
(B) $(-\infty,-9]\cup(7,\infty)$
(C) $(-\infty,-7]\cup[7,\infty)$
(D) $(-\infty,-5]\cup[9,\infty)$
(E) $(-\infty,-7]\cup[9,\infty)$

3. Let $A=\{x\in\mathbb{Z}:-1\le x<4\}$ and let $B=\{x\in\mathbb{Z}:0<\frac{x}{2}\le 3\}$. Then $A\cap B$ is equal to

(A) $\{1,\ 2,\ 3\}$
(B) $\{2,\ 3\}$
(C) $\{1,\ 2,\ 3,\ 4\}$
(D) $\{2,\ 3,\ 4\}$
(E) $\{0,\ 1,\ 2,\ 3\}$

4. Let $f(x)=\begin{cases} x+2, & \text{for } x<1 \\ 4x-1, & \text{for } 1\le x\le 3 \\ x^2+5, & \text{for } x>3 \end{cases}$. Then

(A) $f(x)$ is not continuous at $x=-1$
(B) $f(x)$ is continuous at $x=1$
(C) $f(x)$ is continuous at $x=3$
(D) $f(x)$ is not continuous at $x=5$
(E) $f(x)$ is not continuous at $x=2$

Space for rough work

5. Let $\odot$ be a binary operation on $\mathbb{Q}-\{0\}$ defined by $a \odot b = \frac{a}{b}$.

Then $1\odot(2\odot(3\odot 4))$ is equal to

(A) $\frac{3}{2}$ (B) $\frac{8}{3}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$ (E) $\frac{3}{8}$

6. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=\cos x$. Then

(A) f is one - one and odd
(B) f is odd but not one - one
(C) f is even and onto
(D) f is one - one and even
(E) f is even but not onto

7. If $n(A\cup B)=97$, $n(A\cap B)=23$ and $n(A-B)=39$, then $n(B)$ is equal to

(A) 52 (B) 55 (C) 58 (D) 62 (E) 65

8. The principal argument of the complex number $z=\frac{8+4i}{1+3i}$ is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{-3\pi}{4}$ (E) $\frac{\pi}{6}$

9. The minimum value of $|z+1|+|z-2|$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4 (E) 0

Space for rough work

10. If $z = \frac{(3+i)(7-i)^2}{3-i}$, then the value of $|z|$ is equal to

(A) 48 (B) $\sqrt{50}$ (C) 50 (D) $\sqrt{500}$ (E) $\sqrt{48}$

11. The value of $\left[\frac{5i}{(1-i)(2-i)(3-i)}\right]^{50}$ is equal to

(A) $\left(\frac{1}{2}\right)^{25}$ (B) $\left(\frac{1}{2}\right)^{50}$ (C) $-\left(\frac{1}{2}\right)^{25}$ (D) $-\left(\frac{1}{2}\right)^{50}$ (E) $\left(\frac{1}{10}\right)^{50}$

12. If $z^4 = 7-5i$, then $\text{Im}\left((\bar{z})^4\right)$ is equal to

(A) 5 (B) 7 (C) −7 (D) −5 (E) 0

13. The modulus of $\left(\frac{1+i}{1-i}\right)^{75} - \left(\frac{1-i}{1+i}\right)^{75}$ is

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 4 (E) 16

Space for rough work

14. If z_1 and z_2 are two different complex numbers with $|z_2| = 1$, then $\left|\frac{1-\bar{z}_1 z_2}{z_1 - z_2}\right|$ is equal to

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$ (E) 1

15. If $-1+7i$, $-1+xi$ and $3+3i$ are the three vertices of an isosceles triangle which is right angled at $-1+xi$, then the value of x is equal to

(A) –1 (B) 3 (C) –3 (D) 7 (E) –7

16. The sum of the first 24 terms of the series $9+13+17+\cdots$ is equal to

(A) 1212 (B) 1200 (C) 1440 (D) 1320 (E) 1230

17. In an A.P. there are 18 terms and the last three terms of the A.P. are 67, 72, 77. Then the first term of the A.P. is

(A) –7 (B) 9 (C) –9 (D) –8 (E) 7

18. If the first term of a G.P. is 3 and the sum of second and third terms is 60, then the common ratio of the G.P. is

(A) 4 or – 3 (B) 4 only (C) 4 or 5 (D) 4 or – 5 (E) –5 only

19. If n^{th} term of a series is $n+(-1)^{n-1}$, $n = 1, 2, 3, \ldots$, then the sum of first 40 terms of the series is

(A) 810 (B) 820 (C) 821 (D) 819 (E) 780

Space for rough work

20. The 11[th] term of the geometric series $\sum_{r=0}^{20} 2\times(-2)^r$ is equal to

(A) – 4096 (B) 1024 (C) 2048 (D) 1048 (E) – 2024

21. Let S_n be the sum of the first n terms of the series $a_1 + a_2 + \cdots a_n + \cdots$. If $S_n = n^2 + 4n$, then the n^{th} term a_n is

(A) $2n+3$ (B) $2n-1$ (C) $2n+5$ (D) $2n-3$ (E) $2n$

22. Let $t_n = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2$ for $n = 1, 2, 3, \cdots$. Then t_{10} is equal to

(A) $\frac{7}{600}$ (B) $\frac{231}{100}$ (C) $\frac{209}{600}$ (D) $\frac{11}{200}$ (E) $\frac{77}{200}$

23. The number of arrangements containing all the seven letter of the word ALRIGHT that begins with LG is

(A) 720 (B) 120 (C) 600 (D) 540 (E) 760

24. The number of numbers greater than 6000 that can be formed from the digits 3, 5, 6, 7 and 9 (no digit is repeated in a number) is equal to

(A) 264 (B) 720 (C) 192 (D) 132 (E) 544

Space for rough work

25. The number of subsets containing exactly 4 elements of the set $\{2, 4, 6, 8, 10, 12, 14, 16, 18\}$ is equal to

(A) 126 (B) 63 (C) 189 (D) 58 (E) 94

26. If $^{11}P_r = 7920$ and $^{11}C_r = 330$, then the value of r is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

27. In the binomial expansion of $\left(x - 2y^2\right)^9$, the coefficient of x^6y^6 is equal to

(A) −672 (B) 672 (C) 336 (D) −336 (E) −512

28. Let $(3+x)^{10} = a_0 + a_1(1+x) + a_2(1+x)^2 + \cdots a_{10}(1+x)^{10}$, where $a_1, a_2, \cdots a_{10}$ are constants. Then the value of $a_0 + a_1 + a_2 + \cdots a_{10}$ is equal to

(A) 2^{20} (B) 2^{10} (C) 3^{10} (D) 2^{11} (E) 2^{15}

29. If $^nC_5 + {^nC_6} = {^{51}C_6}$, then the value of n is equal to

(A) 49 (B) 50 (C) 45 (D) 46 (E) 51

30. Let $A = \begin{bmatrix} 3 & 4 \\ 1 & -2 \end{bmatrix}$ and let $AB = \begin{bmatrix} -5 & 41 \\ 5 & -13 \end{bmatrix}$. Then $|B^T| =$

(A) $\frac{1}{14}$ (B) 14 (C) 10 (D) −10 (E) −14

23

Space for rough work

31. Let $A=\begin{vmatrix} 2 & 1 & -2 \\ 1 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix}$ and let $B=|A|\operatorname{adj}(A)$. Then $|B|=$

(A) 256 (B) 64 (C) 512 (D) 1024 (E) 128

32. The values of x satisfying the equation $\begin{vmatrix} x & 4 & 0 \\ 2 & 2 & -x \\ 1 & 1 & 1 \end{vmatrix}=0$ are

(A) 2, −4 (B) 1, 2 (C) −1, 2 (D) −1, −2 (E) −2, 4

33. If $A=\begin{bmatrix} 2 & 0 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 5 \\ 7 & -2 \\ 6 & 6 \end{bmatrix}$, then $AB=$

(A) $\begin{bmatrix} 42 & 46 \end{bmatrix}$

(B) $\begin{bmatrix} 42 \\ 46 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & 10 \\ 0 & 0 \\ 36 & 36 \end{bmatrix}$

(D) $\begin{bmatrix} 17 & 19 \end{bmatrix}$

(E) $\begin{bmatrix} 2 & 12 \\ 14 & -4 \end{bmatrix}$

34. If A is non-singular matrix and if $A^{-1}=\frac{1}{2}\begin{bmatrix} -10 & -4 \\ 2 & 1 \end{bmatrix}$, then $\operatorname{adj}(A)=$

(A) $\begin{bmatrix} -1 & -4 \\ 2 & 10 \end{bmatrix}$

(B) $\begin{bmatrix} 10 & 4 \\ -2 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4 \\ -2 & -10 \end{bmatrix}$

(D) $\begin{bmatrix} -10 & -4 \\ 2 & 1 \end{bmatrix}$

(E) $\begin{bmatrix} -1 & -4 \\ 10 & 2 \end{bmatrix}$

Space for rough work

35. $\begin{vmatrix} \sin\alpha & \cos(\alpha+\theta) & \cos\alpha \\ \sin\beta & \cos(\beta+\theta) & \cos\beta \\ \sin\gamma & \cos(\gamma+\theta) & \cos\gamma \end{vmatrix} =$

(A) -1 (B) 1 (C) 2 (D) 4 (E) 0

36. The solution set of the inequality $-2 \le \frac{3x+2}{2} < 7$ is

(A) $\{x : 3 \le x < 4\}$ (B) $\{x : -2 \le x < 3\}$ (C) $\{x : -2 \le x < 4\}$

(D) $\{x : 0 \le x < 6\}$ (E) $\{x : -2 \le x < 6\}$

37. The set of all x satisfying the inequality $|3x+4| \le 7$ is

(A) $\left[-1, \frac{11}{3}\right]$ (B) $\left[\frac{4}{3}, \frac{7}{3}\right]$ (C) $\left[\frac{-11}{3}, 1\right]$

(D) $\left[\frac{-4}{3}, \frac{7}{3}\right]$ (E) $\left[\frac{-4}{3}, \frac{11}{3}\right]$

38. If the solution set of the inequality $|a+3x| \le 6$ is $\left[\frac{-8}{3}, \frac{4}{3}\right]$, then the value of a is equal to

(A) -1 (B) -2 (C) 4 (D) -4 (E) 2

Space for rough work

39. Consider the following statements :

(i) For every positive real number x , $x-10$ is positive.

(ii) Let n be a natural number. If n^2 is even, then n is even.

(iii) If a natural number is odd, then its square is also odd.

Then

(A) (i) False, (ii) True and (iii) True

(B) (i) False, (ii) False and (iii) True

(C) (i) True, (ii) False and (iii) True

(D) (i) True, (ii) True and (iii) True

(E) (i) False, (ii) True and (iii) False

40. If $\cos\theta = \frac{5}{11}$ and $\tan\theta < 0$, then the value of $\sin\theta$ is equal to

(A) $\frac{8\sqrt{6}}{11}$ (B) $\frac{-8\sqrt{6}}{11}$ (C) $\frac{4\sqrt{6}}{11}$ (D) $\frac{-4\sqrt{6}}{11}$ (E) $\frac{6}{11}$

41. If α and β are two acute angles of a right triangle, then

$(\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2 =$

(A) $1+\sin 2\alpha$ (B) $2(1+\sin 2\alpha)$ (C) $1+\cos 2\alpha$

(D) $2(1+2\cos 2\alpha)$ (E) $2+\sin 2\alpha$

42. The range of the function $f(x) = 2\sin(3x)+1$ is equal to

(A) $[-1, 1]$ (B) $\left[\frac{-1}{3}, \frac{1}{3}\right]$ (C) $[-2, 1]$ (D) $[-1, 2]$ (E) $[-1, 3]$

43. The period of the function $g(x) = 5\cot\left(\frac{\pi}{3}x+\frac{\pi}{6}\right)+2$ is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Space for rough work

44. If $\theta \in (-\pi, 0)$ and $\cos\theta = \frac{-12}{13}$, then $\sin\left(\frac{\theta}{2}\right) =$

(A) $\frac{-5\sqrt{26}}{26}$ (B) $\frac{5\sqrt{26}}{26}$ (C) $\frac{-5\sqrt{13}}{13}$ (D) $\frac{5\sqrt{13}}{13}$ (E) $\frac{-5\sqrt{13}}{26}$

45. The solutions of the equation $\cos\theta = 2 - 3\sin\left(\frac{\theta}{2}\right)$ in the interval $0 \le \theta \le \pi$ are

(A) $\frac{\pi}{4}, \pi$ (B) $\frac{\pi}{3}, \frac{\pi}{2}$ (C) $\frac{\pi}{3}, \pi$ (D) $\frac{\pi}{6}, \frac{\pi}{2}$ (E) $\frac{\pi}{6}, \pi$

46. The value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ is equal to

(A) $\frac{7\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) $\frac{5\pi}{6}$

47. The value of $\tan\left(\sin^{-1}\left(\frac{7}{25}\right)\right)$ is equal to

(A) $\frac{18}{25}$ (B) $\frac{24}{25}$ (C) $\frac{7}{24}$ (D) $\frac{3}{4}$ (E) $\frac{7}{18}$

48. $\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{200}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{200}\right)\right) =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) 1 (E) 0

Space for rough work

49. The equation of the straight line parallel to $y = -3x$ and passing through the point $(3, -2)$ is

(A) $y = -3x + 7$ (B) $y = -3x + 9$ (C) $y = -3x - 11$

(D) $y = -3x - 7$ (E) $y = -3x + 11$

50. The intercepts of a line with coordinate axes are equal. If the line passes through (2, 3), then its equation is

(A) $2x + 3y = 5$ (B) $x + y = 5$ (C) $5x + 5y = 1$

(D) $x + y = 6$ (E) $3x + 2y = 5$

51. If the line $y = mx + c$ is perpendicular to $y = 1 + x$ and passes through the point (1, 2), then the value of c is equal to

(A) 1 (B) –1 (C) –3 (D) 3 (E) 0

52. Let $A(-1, 2)$, $B(1, 3)$ and $C(a, b)$ be collinear. If B divides AC such that $BC = 8\,AB$, then the coordinates of C are

(A) $\left(\frac{5}{4}, \frac{25}{8}\right)$ (B) (17, 9) (C) (17, 11) (D) $\left(\frac{5}{4}, \frac{5}{8}\right)$ (E) (1, 11)

53. If the lines $2x - 3y + 5 = 0$, $9x - 5y + 14 = 0$ and $3x - 7y + \lambda = 0$ are concurrent, then the value of λ is equal to

(A) 7 (B) 8 (C) 10 (D) 9 (E) 6

Space for rough work

54. The points of intersection of the line $y = x + 2$ and the circle $(x - 2)^2 + y^2 = 16$ are

(A) (–2, 0), (2, 4) (B) (–2, 4), (2, 0) (C) (4, 0), (4, 2)

(D) (4, 6), (4, – 2) (E) (4, 0), (4, – 2)

55. The three vertices of a triangle are (0, 0), (3, 1) and (1, 3). If this triangle is inscribed in a circle, then the equation of the circle is

(A) $2x^2+2y^2-2x-6y=0$ (B) $x^2+y^2-3x-y=0$

(C) $x^2+y^2-x-3y=0$ (D) $2x^2+2y^2-6x-2y=0$

(E) $2x^2+2y^2-5x-5y=0$

56. The equation of the circle touching the x-axis at (5, 0) and the line $y = 10$ is

(A) $x^2+y^2-10x-10y+25=0$ (B) $x^2+y^2-10x-10y-25=0$

(C) $x^2+y^2-5x-5y-5=0$ (D) $x^2+y^2-5x-5y+5=0$

(E) $x^2+y^2+10x+10y-25=0$

57. If the radius of the circle $x^2+y^2+ax+by+3=0$ is 2, then the point (a, b) lies on the circle

(A) $x^2+y^2=7$ (B) $x^2+y^2=4$ (C) $x^2+y^2=14$

(D) $x^2+y^2=28$ (E) $x^2+y^2=1$

58. If the line $2x-3y+c=0$ passes through the focus of the parabola $x^2=-8y$, then the value of c is equal to

(A) 4 (B) –6 (C) 6 (D) –4 (E) 2

Space for rough work

59. The centre of the ellipse $x^2+7y^2-14x+28y+49=0$ is

(A) (7, 0) (B) (7, −4) (C) (7, −2) (D) (−7, 4) (E) (−7, 2)

60. The end points of the major axis of an ellipse are (2, 4) and (2, −8). If the distance between foci of this ellipse is 4, then the equation of the ellipse is

(A) $\frac{(x-2)^2}{32}+\frac{(y+2)^2}{36}=1$ (B) $\frac{(x-4)^2}{32}+\frac{(y+2)^2}{36}=1$

(C) $\frac{(x-2)^2}{36}+\frac{(y+2)^2}{32}=1$ (D) $\frac{(x-2)^2}{32}+\frac{(y-4)^2}{36}=1$

(E) $\frac{(x-2)^2}{36}+\frac{(y-4)^2}{32}=1$

61. If (−1, 0) and (3, 0) are foci of an ellipse and the length of the major axis is 6, then the length of the minor axis is

(A) $\sqrt{5}$ (B) 5 (C) 10 (D) $2\sqrt{5}$ (E) 3

62. The eccentricity of the hyperbola $\frac{(x-3)^2}{9}-\frac{4(y-1)^2}{45}=1$ is equal to

(A) $\frac{3}{\sqrt{5}}$ (B) $\frac{5}{3}$ (C) $\frac{5}{\sqrt{3}}$ (D) $\frac{5}{2}$ (E) $\frac{3}{2}$

63. If $\vec{a}\times\vec{b}=7\hat{i}+9\hat{j}+10\hat{k}$ and $\vec{a}\cdot\vec{b}=-20$, then $|\vec{a}|^2\,|\vec{b}|^2=$

(A) 530 (B) 580 (C) 400 (D) 630 (E) 560

Space for rough work

64. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} + \lambda\hat{k}$. If $\vec{a} \cdot \vec{b} = 4$, then the value of λ is equal to

(A) 3 (B) –3 (C) –6 (D) 6 (E) 0

65. If $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{10}$, $|\vec{a} - \vec{b}| = \sqrt{24}$ and θ is angle between $\vec{a}$ and $\vec{b}$, then $\cos\theta =$

(A) $\frac{\sqrt{35}}{70}$ (B) $\frac{\sqrt{6}}{12}$ (C) $\frac{\sqrt{15}}{60}$ (D) $\frac{\sqrt{210}}{35}$ (E) 0

66. If $|\vec{a}| = 10$ and $|\vec{b}| = 5$, then the value of $(\vec{a} + 2\vec{b}) \cdot (\vec{a} - 2\vec{b})$ is equal to

(A) 32 (B) 16 (C) 8 (D) 4 (E) 0

67. If $\vec{a} = \hat{i} - 3\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then the value of $(\vec{a} \times \vec{b}) \cdot \vec{b}$ is equal to

(A) 3 (B) –3 (C) 7 (D) –7 (E) 0

68. If $\vec{a}$ and $\vec{b}$ are position vectors of the points $(\alpha, 3, 0)$ and (1, 0, 0) respectively and if the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$, then the value of α is equal to

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

69. If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$, then a unit vector in the direction of $\vec{a} + \vec{b}$ is

(A) $\frac{1}{6}(3\hat{i} + 6\hat{j} - 2\hat{k})$ (B) $\frac{1}{\sqrt{70}}(3\hat{i} + 6\hat{j} - 5\hat{k})$

(C) $\frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$ (D) $\frac{1}{\sqrt{50}}(3\hat{i} + 6\hat{j} - 3\hat{k})$

(E) $\frac{1}{\sqrt{6}}(\hat{i} + 2\hat{j} - \hat{k})$

Space for rough work

70. If $|\vec{u}|=3$, $|\vec{v}|=2$ and $|\vec{u} \times \vec{v}| = 3$, then the angle between $\vec{u}$ and $\vec{v}$ is equal to

(A) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (B) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ (D) $\frac{\pi}{2}$ (E) 0

71. The equation of the plane passing through the point $(-1,-2,-3)$ and perpendicular to the x-axis is

(A) $x = -1$ (B) $y = -2$ (C) $z = -3$

(D) $2x + 3y = 5$ (E) $x + y + z = 6$

72. Let L_1 be the line joining (0, 0, 0) and (1, 2, 3) and L_2 be the line joining (2, 3, 4) and (3, 4, 5). The point of intersection of L_1 and L_2 is

(A) (0, 0, 0) (B) (1, 2, 3) (C) (2, 3, 4) (D) (3, 4, 5) (E) (1, 1, 1)

73. The equation of the line through the point $(1, -1, 1)$ and parallel to the line joining the points $(-2, 2, 0)$ and $(-1, 1, 1)$ is

(A) $\frac{x-1}{-3} = \frac{y-1}{-1} = z-1$ (B) $1-x = 1+y = 1-z$

(C) $x+1 = -(y-1) = z-1$ (D) $\frac{x-1}{-1} = \frac{y+1}{2} = \frac{z-1}{1}$

(E) $x+2 = y-2 = z$

Space for rough work

74. If the points (1, 0, 0), (0, 3, 0) and (0, 0, 2) lie on a plane, then the unit normal vector $\hat{n}$ to the plane is

(A) $\frac{1}{\sqrt{14}}\left(\hat{i}+3\hat{j}+2\hat{k}\right)$

(B) $\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$

(C) $\frac{1}{\sqrt{14}}\left(2\hat{i}+3\hat{j}+\hat{k}\right)$

(D) $\frac{1}{7}\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$

(E) $\frac{1}{7}\left(6\hat{i}+2\hat{j}+3\hat{k}\right)$

75. The equation of the plane through the point (1, –5, 3) and having a normal vector $\bar{n}=2\hat{i}-2\hat{j}-\hat{k}$ is

(A) $2x+2y+z=9$

(B) $2x-2y-z=11$

(C) $2x+2y-z=9$

(D) $2x-2y-z=9$

(E) $2x-2y-z=13$

76. If θ is angle between the lines $\frac{x}{1}=\frac{y+1}{2}=\frac{z-1}{3}$ and $\frac{x+1}{3}=\frac{y}{2}=\frac{z}{1}$, then $\cos\theta=$

(A) $\frac{5}{9}$ (B) $\frac{5}{8}$ (C) $\frac{5}{6}$ (D) $\frac{5}{7}$ (E) $\frac{6}{7}$

Space for rough work

77. The distance from the point (2, 2, 2) to the plane $2x - y + 3z = 5$ is equal to

(A) $\frac{3\sqrt{7}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{3\sqrt{14}}{7}$ (D) $\frac{3\sqrt{14}}{14}$ (E) $\frac{\sqrt{3}}{3}$

78. The angle between the planes $x=\sqrt{3}$ and $z = \sqrt{2}$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) 0

79. Three fair dice are rolled simultaneously. Let a, b, c be the numbers on the top of the dice. Then the probability that $\min(a, b, c) = 6$ is

(A) $\frac{1}{216}$ (B) $\frac{1}{36}$ (C) $\frac{1}{6}$ (D) $\frac{11}{216}$ (E) $\frac{5}{6}$

80. If A and B are two events such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, then $P(A|(A \cup B))$ is equal to

(A) $\frac{6}{7}$ (B) $\frac{5}{6}$ (C) $\frac{5}{7}$ (D) $\frac{4}{7}$ (E) $\frac{1}{2}$

Space for rough work

81. There are 37 men and 33 women at a party. If a prize is given to one person chosen at random, then the probability that the prize goes to a woman is

(A) $\frac{33}{70}$ (B) $\frac{32}{70}$ (C) $\frac{33}{80}$ (D) $\frac{37}{70}$ (E) $\frac{37}{80}$

82. A fair coin is tossed twice. Given that the first toss resulted in head, then the probability that the second toss also, would result in head is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$ (E) $\frac{5}{8}$

83. The coefficient of variation (C.V.) and the mean of a distribution are respectively 75 and 44. Then the standard deviation of the distribution is

(A) 30 (B) 31 (C) 32 (D) 33 (E) 35

84. There are 4 red, 3 blue and 3 yellow marbles in an urn. If three marbles are drawn simultaneously, then the probability that the number of yellow marbles will be less than 2 is equal to

(A) $\frac{97}{120}$ (B) $\frac{49}{60}$ (C) $\frac{47}{60}$ (D) $\frac{59}{60}$ (E) $\frac{39}{60}$

Space for rough work

85. In a box there are four marbles and each of them is marked with distinct number from the set $\{1, 2, 5, 10\}$. If one marble is randomly selected four times with replacement and the number on it noted, then the probability that the sum of numbers equals 18 is

(A) $\frac{1}{64}$ (B) $\frac{3}{16}$ (C) $\frac{5}{32}$ (D) $\frac{3}{32}$ (E) $\frac{1}{32}$

86. $\lim_{t\to 0}\left(\frac{(2t-3)(t-2)}{t}-\frac{3(t+2)}{t}\right)$ is equal to

(A) 10 (B) -10 (C) -7 (D) 7 (E) 5

87. If $f(x)=\begin{cases} x^2\sin\left(\frac{\pi}{6}x\right) & \text{for } x\le -3 \\ x\cos\left(\frac{\pi}{3}x\right) & \text{for } x>-3 \end{cases}$, then the value of $\lim_{x\to -3^+} f(x)$ is equal to

(A) 3 (B) -3 (C) 9 (D) -9 (E) 0

88. $\lim_{x\to 0}\frac{\log(1+x)+1-e^x}{4x^2-9x}$ is equal to

(A) $\frac{-1}{9}$ (B) $\frac{1}{9}$ (C) $\frac{-1}{18}$ (D) $\frac{1}{18}$ (E) 0

Space for rough work

89. $\lim\limits_{t \to 0} \dfrac{\sin(t^2)}{t\sin(5t)}$ is equal to

(A) 5 (B) 25 (C) $\frac{1}{25}$ (D) $\frac{1}{5}$ (E) 0

90. Let $f(x) = \begin{cases} 3x+6, & \text{if } x \geq c \\ x^2-3x-1, & \text{if } x < c \end{cases}$, where $x \in \mathbb{R}$ and c is a constant. The values of c for which f is continuous on $\mathbb{R}$ are

(A) −7, 1 (B) 1, 3 (C) −1, 7 (D) −1, 6 (E) 2, −3

91. If $\lim\limits_{x \to -2} \dfrac{3x^2+ax-2}{x^2-x-6}$ is a finite number, then the value of a is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

92. If $x = \sqrt{10^{\cos^{-1}\theta}}$ and $y = \sqrt{10^{\sin^{-1}\theta}}$, then $\frac{dy}{dx}$ is equal to

(A) xy (B) $\frac{x}{y}$ (C) $\frac{y}{x}$ (D) $\frac{-x}{y}$ (E) $\frac{-y}{x}$

Space for rough work

93. If $y = e^{3\log(2x+1)}$, then $\frac{dy}{dx} =$

(A) $6e^{3\log(2x+1)}$ (B) $6\frac{e^{3\log(2x+1)}}{2x+1}$ (C) $\frac{e^{3\log(2x+1)}}{2x+1}$

(D) $\frac{e^{3\log(2x+1)}}{3(2x+1)}$ (E) $(2x+1)e^{3\log(2x+1)}$

94. If $x\sin y + y\sin x = \pi$, then $\frac{dy}{dx}$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is equal to

(A) 1 (B) $\frac{\pi}{2}$ (C) -1 (D) $\frac{-\pi}{2}$ (E) 0

95. Let $f(x) = \begin{cases} \tan x, & \text{if } 0 \le x \le \frac{\pi}{4} \\ ax+b, & \text{if } \frac{\pi}{4} < x < \frac{\pi}{2} \end{cases}$. If $f(x)$ is differentiable at $x = \frac{\pi}{4}$, then the values of a and b are respectively

(A) 2, $\frac{2-\pi}{2}$ (B) 2, $\frac{4-\pi}{4}$ (C) 1, $\frac{-\pi}{4}$ (D) 2, $\frac{-\pi}{4}$ (E) 2, $1-\pi$

96. $\frac{d}{dx}\left(\frac{1}{x}\frac{d^2}{dx^2}\left(\frac{1}{x^3}\right)\right) =$

(A) $-36x^{-7}$ (B) $36x^{-7}$ (C) $72x^{-6}$ (D) $72x^{-7}$ (E) $-72x^{-7}$

Space for rough work

97. Air is blown into a spherical balloon. If its diameter d is increasing at the rate of 3 cm/min, then the rate at which the volume of the balloon is increasing when $d = 10$ cm, is

(A) $120\pi\ cm^3/min$
(B) $150\pi\ cm^3/min$
(C) $100\pi\ cm^3/min$
(D) $180\pi\ cm^3/min$
(E) $210\pi\ cm^3/min$

98. The equation of tangent to the circle $(x-5)^2 + y^2 = 25$ at (2, 4) is

(A) $3x - 4y + 10 = 0$
(B) $x + y = 6$
(C) $2x - y = 0$
(D) $3x - 2y + 2 = 0$
(E) $3x - 4y - 10 = 0$

99. If x and y are both non-negative and if $x + y = \pi$, then the maximum value of $5 \sin x \sin y$ is equal to

(A) 1 (B) $\sqrt{5}$ (C) 5 (D) –5 (E) 0

100. The normal to the curve $y = \sqrt{x}$ at the point (25, 5) intersects the y-axis at

(A) (0, 245)
(B) (0, 255)
(C) (255, 0)
(D) (245, 0)
(E) (0, 100)

Space for rough work

101. The function $f(x)=x^5e^{-x}$ is increasing in the interval

(A) $(5, \infty)$ (B) $(4, \infty)$ (C) $(-4, \infty)$ (D) $(-\infty, 5)$ (E) $(-5, \infty)$

102. If $x+13y=40$ is normal to the curve $y=5x^2+\alpha x+\beta$ at the point (1, 3), then the value of $\alpha\beta$ is equal to

(A) 15 (B) –6 (C) 6 (D) 13 (E) –15

103. Let $f(x)=\cos x$ for $0\le x\le\frac{\pi}{3}$. Then the value of c which satisfies the conclusion of the Mean Value Theorem for the function f on $\left[0, \frac{\pi}{3}\right]$ is equal to

(A) $\sin^{-1}\left(\frac{3}{2\pi}\right)$ (B) $\sin^{-1}\left(\frac{1}{3\pi}\right)$ (C) $\sin^{-1}\left(\frac{\pi}{12}\right)$

(D) $\sin^{-1}\left(\frac{1}{6\pi}\right)$ (E) $\sin^{-1}\left(\frac{\pi}{4}\right)$

104. $\int \frac{e^{\frac{1}{\sqrt{t}}}}{t\sqrt{t}}\,dt =$

(A) $\frac{1}{2}e^{\frac{1}{\sqrt{t}}}+C$ (B) $\frac{-1}{2}e^{\frac{1}{\sqrt{t}}}+C$ (C) $2e^{\frac{1}{\sqrt{t}}}+C$

(D) $-2e^{\frac{1}{\sqrt{t}}}+C$ (E) $e^{\frac{1}{\sqrt{t}}}+C$

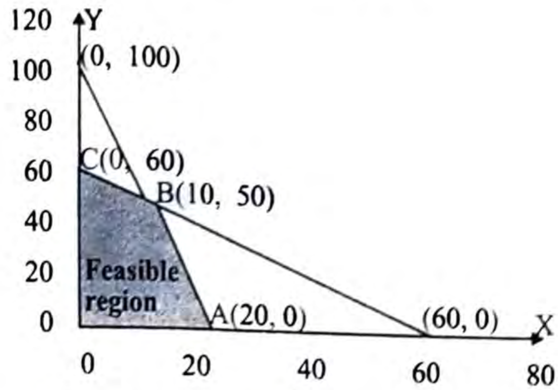
Space for rough work

105. $\int \frac{\sin^{25} x}{\cos^{27} x} dx$ is equal to

(A) $\frac{\sin^{26}(x)}{26} + C$ (B) $\frac{\cos^{26}(x)}{26} + C$ (C) $\tan^{26}(x) + C$

(D) $\frac{\tan^{26}(x)}{26} + C$ (E) $26\tan^{26}(x) + C$

106. The feasible region for a L.P.P. is shown in the figure below. Let $z = 50x + 15y$ be the objective function, then the maximum value of z is



(A) 900 (B) 1000 (C) 1250 (D) 1300 (E) 1520

Space for rough work

107. $\int \frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx =$

(A) $\frac{-1}{6}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(B) $\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(C) $\frac{-1}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(D) $\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(E) $\frac{-4}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

108. $\int\left(\tan^2(2x)-\cot^2(2x)\right)dx =$

(A) $\frac{-1}{2}(\tan 2x+\cot 2x)+C$

(B) $2(\tan 2x+\cot 2x)+C$

(C) $\frac{1}{2}(\tan 2x-\cot 2x)+C$

(D) $\frac{-1}{2}(\tan 2x-\cot 2x)+C$

(E) $\frac{1}{2}(\tan 2x+\cot 2x)+C$

109. $\int\sin^3 x\,dx+\int\cos^2 x\sin x\,dx =$

(A) $-\cos x+C$

(B) $-\sin x+C$

(C) $x-\cos x+C$

(D) $x-\sin x+C$

(E) $\cos x-\sin x+C$

Space for rough work

110. $\int \frac{dx}{x^2 - x} =$

(A) $\log \frac{|x|}{|x-1|} + C$

(B) $\frac{-1}{x^2} + \log|x-1| + C$

(C) $x \log|x-1| + C$

(D) $\log \frac{|x-1|}{|x|} + C$

(E) $-x \log|x-1| + C$

111. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cot x}{\sin x} dx$ is equal to

(A) $\frac{-1}{2}$

(B) $\frac{1}{2}$

(C) $\frac{-3}{2}$

(D) $\frac{3}{2}$

(E) 1

112. The area bounded by the curve $y = x(2-x)$ and the line $y = x$ is

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{5}{6}$

(E) $\frac{2}{3}$

Space for rough work

113. The value of $\int_{-1}^{2}(x-2|x|)\,dx$ is equal to

(A) $\frac{-1}{2}$ (B) $\frac{-3}{2}$ (C) $\frac{-5}{2}$ (D) $\frac{-7}{2}$ (E) $\frac{-9}{2}$

114. The value of $\int_{-10}^{10}\frac{x^{10}\sin x}{\sqrt{1+x^{10}}}\,dx$ is equal to

(A) $\frac{1}{100}$ (B) $\frac{-1}{100}$ (C) $\frac{1}{50}$ (D) $\frac{-1}{50}$ (E) 0

115. If $f(x)=\begin{cases}\cos x & \text{for } x\geq 0\\ 2x & \text{for } x<0\end{cases}$, then the value of $\int_{-2}^{\frac{\pi}{2}} f(x)\,dx$ is equal to

(A) 2 (B) -2 (C) -3 (D) 3 (E) 0

Space for rough work

116. The value of $\int_0^{\frac{\pi}{16}} \cos 6x \cos 2x \, dx$ is equal to

(A) $\frac{1+\sqrt{2}}{16}$
(B) $\frac{1+\sqrt{2}}{8}$
(C) $\frac{2+\sqrt{2}}{16}$
(D) $\frac{-1+\sqrt{2}}{16}$
(E) $\frac{-1+\sqrt{2}}{8}$

117. A particular solution of the differential equation $\frac{dy}{dx} = xy^2$ with $y(0)=1$ is

(A) $y=\frac{2-x^2}{2}$
(B) $y=\frac{2}{2-x^2}$
(C) $y=\frac{2}{x^2}-2$
(D) $y=\frac{x^2-2}{2}$
(E) $y=\frac{2}{x^2-2}$

118. The general solution of the differential equation $(x^2y^2+y)dx-(x-2x^3y)dy=0$ is

(A) $x^2y^2-\frac{y}{x}=C$
(B) $x^3y+\frac{x}{y}=C$
(C) $xy^2+\frac{y}{x}=C$
(D) $xy^2-\frac{y}{x}=C$
(E) $x^2y+\frac{y}{x}=C$

Space for rough work

119. The integrating factor of the differential equation $4xdy - e^{-2y}dy + dx = 0$ is

(A) e^{-2y} (B) e^{2x^2} (C) e^{4y} (D) e^{-4y} (E) x^4

120. Consider the linear programming problem:

Maximize $z = 10x + 5y$

subject to the constraints

$2x + 3y \leq 120$

$2x + y \leq 60$

$x, y \geq 0$.

Then the coordinates of the corner points of the feasible region are

(A) (0, 0), (30, 0), (0, 40) and (15, 30)

(B) (0, 0), (60, 0), (0, 40) and (15, 30)

(C) (0, 0), (30, 0), (0, 60) and (15, 30)

(D) (0, 0), (30, 0), (0, 40) and (30, 40)

(E) (0, 0), (60, 0), (0, 40) and (30, 40)

Space for rough work