HIGHER SECONDARY
FIRST TERMINAL SECOND YEAR EXAMINATION 2018-19
MATHEMATICS (SCIENCE)

Maximum : 80 Scores
Time 2 ½ hours
Cool off time : 15 minutes

HSE II

Answer any six from questions 1 to 7. Each question carries 3 score.

1. Construct a 3×2 matrix $A=\left[a_{ij}\right]$ whose elements are given by $a_{ij}=\frac{(i+j)^2}{2}$ (3)

$$a_{ij}=\frac{(i+j)^2}{2}$$

$$a_{11}=\frac{(1+1)^2}{2}=2;\ a_{12}=\frac{(1+2)^2}{2}=4.5$$

$$a_{21}\quad \frac{(2\quad 1)^2}{2}\quad 4.5;\ a_{22}\quad \frac{(2\quad 2)^2}{2}\quad 8$$

$$a_{31}\quad \frac{(3\quad 1)^2}{2}\quad 8;\ a_{32}\quad \frac{(3\quad 2)^2}{2}\quad 12.5$$

$$\therefore A=\begin{bmatrix}2 & 4.5\\ 4.5 & 8\\ 8 & 12.5\end{bmatrix}$$

2. Show that the relation R on Z defined by $R=\left\{(a,b):|a-b|\ is\ \ even\right\}$ is an equivalence relation. (3)

i. $If\ |a-a|=0,\ is\ even\ \forall a\in R\Rightarrow R$ is reflexive.

ii. $If\ |a-b|$ is even $\Rightarrow |b-a|=|-(a-b)|=|a-b|\ \ is\ even\ \forall a,b\in R\Rightarrow R$ is symmetric.

iii. $If\ \ |a-b|$ is even and $if\ \ |b-c|$ is even $\Rightarrow$

$|(a-b)+(b-c)|=|a-c|\ is\ also\, even\ \forall a,b,c\in R\Rightarrow R$ is transitive.

Hence R is an equivalence relation.

3. If $A=\begin{bmatrix}1 & 3 & 2\\ 2 & -3 & 0\\ 5 & -2 & 1\end{bmatrix}, B=\begin{bmatrix}5 & -1\\ -6 & 2\\ 7 & 4\end{bmatrix}$

a) $AB=\begin{bmatrix}1 & 3 & 2\\ 2 & -3 & 0\\ 5 & -2 & 1\end{bmatrix}\begin{bmatrix}5 & -1\\ -6 & 2\\ 7 & 4\end{bmatrix}=\begin{bmatrix}5+-18+14 & -1+6+8\\ 10+18+0 & -2-6+0\\ 25+12+7 & -5-4+4\end{bmatrix}=\begin{bmatrix}1 & 13\\ 28 & -8\\ 44 & -5\end{bmatrix}$ (2)

b) Here multiplying all the elements of 2^{nd} row by 2, hence in the product matrix, multiplying all elements by 2, we have $AB = \begin{bmatrix} 1 & 13 \\ 56 & -16 \\ 44 & -5 \end{bmatrix}$ (1)

4. a) b) $|\sin x|$ (1)

b) Since modulus function and trigonometric functions are continuous everywhere, then the composite function $|\sin x|$ is continuous everywhere. (1)

c) It is clear from the graph that $|\sin x|$ is not differentiable at $x = n\pi,$ where n is an integer. (1)

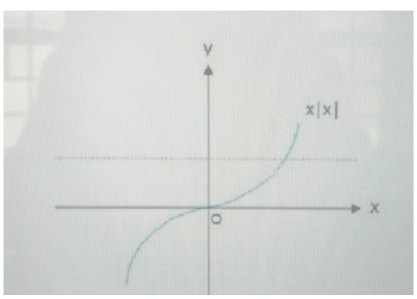
5. Consider the matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

a) $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = 2(0-20) - -3(-42-4) + 5(30-0) = -40 - 138 + 150 = -28$ (1)

b) $|adjA| = |A|^{n-1} = (-28)^{3-1} = (-28)^2 = 784$ (1)

c) $|2A| = 2^3 |A| = 8(-28) = -224$ (1)

6. a)



(1)

b) Yes. In the graph, the line parallel to the x axis intersects the graph only one point. (1)

c) No. Every y element does not have pre-image. (1)

7. $f(x) = x^2 + 2, g(x) = 2x + 3$

$fog = f[g(x)] = f(2x+3) = (2x+3)^2 + 2 = 4x^2 + 12x + 9 + 2 = 4x^2 + 12x + 11$

$gof = g[f(x)] = g(x^2+2) = 2(x^2+2) + 3 = 2x^2 + 4 + 3 = 2x^2 + 7$ (3)

Answer any eight from questions 8 to 17. Each question carries 4 score.

8. Let $f(x)=4x^2+12x+15$

Let y y be the arbitrary element of range f. Then $y=4x^2+12x+15$

$y=4x^2+12x+15$, for some $x\in N$.

Now $y=4x^2+12x+9+6=(2x+3)^2+6\Rightarrow y-6=(2x+3)^2$

$$\Rightarrow 2x+3=\sqrt{y-6}\Rightarrow 2x=\sqrt{y-6}-3\Rightarrow x=\frac{\sqrt{y-6}-3}{2},\ as\ \ y\ge 6$$

Let us define $g:S\to N$ by $g(y)=\dfrac{\sqrt{y-6}-3}{2}$

$$Now\ \ gof(x)=g\left[f(x)\right]=g\left((2x+3)^2+6\right)=\frac{\sqrt{(2x+3)^2+6-6}-3}{2}=\frac{2x+3-3}{2}=x$$

and

$$fog(y)=f\left[g(y)\right]=f\left(\frac{\left(\sqrt{y-6}-3\right)}{2}\right)=\left(2\times\frac{\left(\sqrt{y-6}\right)-3}{2}+3\right)^2+6=\left(\sqrt{y-6}\right)^2+6=y-6+6=y$$

$gof=I_N$ and $fog=I_S$. This implies that *f* is invertible with $f^{-1}=g$ (4)

9. $A=\begin{bmatrix}1&2&3\\3&2&-1\end{bmatrix};\ B=\begin{bmatrix}1&-2\\3&2\\2&-1\end{bmatrix}$

a) $AB=\begin{bmatrix}1&2&3\\3&2&-1\end{bmatrix}\begin{bmatrix}1&-2\\3&2\\2&-1\end{bmatrix}=\begin{bmatrix}1+6+6&-2+4-3\\3+6-2&-6+4+1\end{bmatrix}=\begin{bmatrix}13&-1\\7&-1\end{bmatrix}$ (1)

b) $B'=\begin{bmatrix}1&2&3\\-2&2&-1\end{bmatrix}$; $A'=\begin{bmatrix}1&3\\2&2\\3&-1\end{bmatrix}$ (1)

c) $(AB)'=\begin{bmatrix}13&7\\-1&-1\end{bmatrix}$...........(1)

$B'=\begin{bmatrix}1&2&3\\-2&2&-1\end{bmatrix}\begin{bmatrix}1&3\\2&2\\3&-1\end{bmatrix}=\begin{bmatrix}1+6+6&3+6-2\\-2+4+-3&-6+4+1\end{bmatrix}=\begin{bmatrix}13&7\\-1&-1\end{bmatrix}$ ....(2)

From (1) and (2), $(AB)'=B'A'$. Hence proved (2)

$\therefore$ P is symmetric

Now, $Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0 & -2 & 1\\ 2 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}=Q$

Q is skew symmetric.

Now $P+Q=\frac{1}{2}\begin{bmatrix}6 & 0 & 3\\ 0 & 4 & 6\\ 3 & 6 & -4\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0 & -2 & 1\\ 2 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix}6 & -2 & 4\\ 2 & 4 & 6\\ 2 & 6 & -4\end{bmatrix}=\begin{bmatrix}3 & -1 & 2\\ 1 & 2 & 3\\ 1 & 3 & -2\end{bmatrix}$ = A, the square matrix.

(4)

12. Let $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x} & If\ x\neq\dfrac{\pi}{2}\\ 3 & if\ x=\dfrac{\pi}{2}\end{cases}$

$\lim\limits_{x\to\frac{\pi}{2}} f(x)=\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{k\times\cos x}{\pi-2x}\right)=\frac{k\cos\frac{\pi}{2}}{\pi-2\frac{\pi}{2}}=\frac{0}{0}$ (indeterminate form)

Put $x=\frac{\pi}{2}+h$

$As\ x\to\frac{\pi}{2};h\to 0$

$\lim\limits_{h\to 0}\left[\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}\right]=\lim\limits_{h\to 0}\frac{k(-\sin h)}{\pi-\pi-2h}=\lim\limits_{h\to 0}\frac{-k}{-2}\frac{\sin h}{h}=\frac{k}{2}\times 1=\frac{k}{2}$

$Now,\ f\left(\frac{\pi}{2}\right)=3$

f is continuous at $\frac{\pi}{2}$

$\lim\limits_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right)\Rightarrow\frac{k}{2}=3\Rightarrow k=6$ (4)

13. a) The question is wrong. Correct question is : The value of $\tan^{-1}\tan\left(\frac{3\pi}{4}\right)=$ ………… (1)

$\tan^{-1}\tan\left(\frac{3\pi}{4}\right)=\tan^{-1}\tan\left(\pi-\frac{\pi}{4}\right)=\tan^{-1}\tan\left(\pi-\frac{\pi}{4}\right)$

$=\tan^{-1}\left[-\tan\left(\frac{\pi}{4}\right)\right]=\tan^{-1}\tan\left(-\frac{\pi}{4}\right)=-\frac{\pi}{4}$

b) $\text{LHS} = 2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left[\frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}\right] + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left[\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right] = \tan^{-1}\left[\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right] = \tan^{-1}\left[\frac{31}{17}\right]$ (3)

$= \tan^{-1}\left(\frac{31}{17}\right) = \text{RHS}$

14. a) Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

$LHS = A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = RHS$ (2)

b) $A^2 - 4A + I = 0 \Rightarrow A - 4 + A^{-1} = 0$ || xing by A^{-1}

Hence, $A^{-1} = 4 - A = 4I - A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ (2)

15. a) $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$x^2 - 36 = 36 - 36 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

Ans: A (1)

b) $LHS = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$R_1 \to R_1; R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$

$$LHS=\begin{vmatrix}1 & a & bc\\ 0 & b-a & c(a-b)\\ 0 & c-a & b(a-c)\end{vmatrix}$$

$$=1\times(b-a)\times(c-a)\begin{vmatrix}1 & -c\\ 1 & -b\end{vmatrix} \qquad (3)$$

$$=(b-a)(c-a)(-b--c)$$

$$=(b-a)(c-a)(c-b)=(a-b)(b-c)(c-a)=RHS$$

16. a) $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$...............(1)

$$3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}...............(2)$$

$$(1)\times 3-(2)\times 2\Rightarrow$$

$$6X+9Y=\begin{bmatrix}6 & 9\\ 12 & 0\end{bmatrix}$$

$$6X+4Y=\begin{bmatrix}4 & -4\\ -2 & 10\end{bmatrix}$$

$$5Y=\begin{bmatrix}2 & 13\\ 14 & -10\end{bmatrix}$$

$$Y=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$$

$$in\ (1)\Rightarrow 2X=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}-3Y$$

$$X=\frac{1}{2}\left(\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}-3\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}\right)$$

$$=\frac{1}{2}\left(\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}-\begin{bmatrix}\frac{6}{5} & \frac{39}{5}\\ \frac{42}{5} & -6\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}\frac{4}{5} & \frac{-24}{5}\\ \frac{-22}{5} & 6\end{bmatrix}$$

$$\therefore\ X=\begin{bmatrix}\frac{2}{5} & -\frac{12}{5}\\ -\frac{11}{5} & 3\end{bmatrix}$$

(4)

17. $A=\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$

Let A = IA

$$\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix}1 & 2\\0 & -5\end{bmatrix}=\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}A$$

$R_2 \to \frac{-1}{5}R_2$

$$\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ \frac{2}{5} & \frac{-1}{5}\end{bmatrix}A$$

$R_1 \to R_1 - 2R_2$ (4)

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & \frac{-1}{5}\end{bmatrix}A$$

$$A^{-1}=\frac{1}{5}\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$$

Answer any eight from questions 18 to 24. Each question carries 6 score.

18.

$AX = B$

Where $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1+3) = --1(2--3)+1(2-1)$

$= 1(4)+1(5)+1(1)$

$= 4+5+1 = 10$

$A_{11} = 4 \quad A_{12} = -5 \quad A_{13} = 1$

$A_{21} = +2 \quad A_{22} = 0 \quad A_{23} = -2$

$A_{31} = 2 \quad A_{32} = +5 \quad A_{33} = 3$

$adjA = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 1b+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$

$\therefore x = 2; y = -1; z = 1$ (6)

19. a) $\sin\left(\sin^{-1} x + \cos^{-1} x\right) = \sin\frac{\pi}{2} = 1$ (1)

b) $\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{2\pi}{5}\right)\right) = \sin^{-1}\left(\sin\frac{2\pi}{5}\right) = \frac{2\pi}{5} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (2)

c) $\sin\left(\tan^{-1} x\right) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{\sqrt{1+x^2}}$ (1)

d) $LHS = 2\sin^{-1}\left(\frac{3}{5}\right) = 2\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right) = \tan^{-1}\left(\frac{\frac{6}{4}}{1-\frac{9}{16}}\right)$

$$= \tan^{-1}\left(\frac{\frac{6}{4}}{\frac{16-9}{16}}\right) = \tan^{-1}\left(\frac{6}{4}\times\frac{16}{7}\right) = \tan^{-1}\left(\frac{24}{7}\right) = RHS \qquad (2)$$

20. a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \Rightarrow \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \in [0, \pi]$ (1)

b) $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$ $\quad \| \; put \;\; x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$\cos^{-1}\sin 2\theta = \cos^{-1}\cos\left(\frac{\pi}{2} - 2\theta\right) = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1} x$ (3)

c) $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right) = \frac{\pi}{2} - 2\tan^{-1} x$

$\frac{dy}{dx} = 0 - 2\times\frac{1}{1+x^2} = -\frac{2}{1+x^2}$ (2)

21.

a) $y = \cos(x^3).\sin^2(x^5)$ (2)

$\frac{dy}{dx} = \cos(x^3).\cos(x^5).5x^4 + \sin(x^5).-\sin(x^3).3x^2$

$= 5x^4(x^5)\cos(x^3) - 3x^2\sin(x^5).\sin(x^3)$

b) $x^2 + y^2 = 100$ (2)

diff wrt x

$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow 2y\frac{dy}{dx} = -2x$$

$$\therefore \frac{dy}{dx} = \frac{-2}{2y} = -\frac{x}{y}$$

c) $y = (\log x)^x + x^{\log x}$ (2)

$put\, u = (\log x)^x ; v = x^{\log x}$

$Then\; y = u + v$

And $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$...............(1)

Let $u = (\log x)^x$

$\log u = x\log(\log x)$

$\frac{1}{u}.\frac{du}{dx} = x.\frac{1}{\log x}.\frac{1}{x} + \log(\log x).$

$$\therefore \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right]$$

$$Now, \quad V = x \log^x$$

$$\log V = \log x^{\log x} = \log x . \log x = (\log x)^2$$

$$\frac{1}{V} . \frac{dV}{dx} = V . \frac{2\log x}{x} = x^{\log x} . \frac{2\log x}{x}$$

$$= x^{\log x - 1} . 2\log x$$

$$(1) \Rightarrow$$

$$\frac{dy}{dx} \quad (\log x)^x \quad \frac{1}{\log x} \quad \log(\log x) \quad x^{\log x - 1} . 2\log x$$

22. a) $a * b = \frac{ab}{2}$

$$b * a = \frac{ba}{2} = \frac{ab}{2}$$

$\therefore *$ is commutative

Again $a * (b * c) = a * \left(\frac{bc}{2} \right) = \frac{a \frac{bc}{2}}{2} = \frac{abc}{4}$

$$(a * b) * c = \left(\frac{ab}{2} \right) * c = \frac{\frac{ab}{2} c}{2} = \frac{abc}{4}$$

$$\Rightarrow a * (b * c) = (a * b) * c$$

$\therefore *$ is associative. (2)

b) Consider the set $A = \{1,2,3,4,5\}$

i.

*	1	2	3	4	5
1	1	2	3	4	5
2	1	2	3	4	5
3	3	3	3	3	3
4	1	2	3	4	5
5	1	2	3	4	5

(2)

ii. The table has 25 cells, Fill one row and column of 3 as required 16 cells remain. Here repetition is allowed, so there are 5 raised to 16 (5^{16}) binary operations are there. (2)

23. a)

$$LHS = \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\frac{12}{13}$$

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\therefore LHS = \cos^{-1}\left[\frac{4}{5}\times\frac{12}{13} - \sqrt{1-\left(\frac{4}{5}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \sqrt{1-\frac{16}{25}}\sqrt{1-\frac{144}{169}}\right] = \cos^{-1}\left[\frac{48}{65} - \sqrt{\frac{9}{25}}\sqrt{\frac{25}{169}}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \frac{3}{5}\times\frac{5}{13}\right] = \cos^{-1}\left(\frac{48-15}{65}\right)$$

$$= \cos^{-1}\left(\frac{33}{65}\right) = RHS$$

(3)

b)

$$\tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right) = \frac{\pi}{4}$$

$$\frac{5x}{1-6x^2} = \tan\frac{\pi}{4} = 1$$

$$5x = 1-6x^2 \Rightarrow 6x^2+5x-1=0$$

$$6x^2+6x-x-1=0$$

$$6x(x+1)-1(x+1)=0$$

$$(x+1)(6x-1)=0$$

$$x=-1(or)6x=1 \Rightarrow x=\frac{1}{6}$$

But $x=-1$ is impossible.

$$\therefore x = \frac{1}{6}$$

(3)

24. a) Find $\frac{dy}{dx}$ *if* $x = a\left(\cos t + \log\tan\left(\frac{1}{2}\right)\right), y = a\sin t$ (3)

The question has a type mismatch. The correct question is:

Find $\frac{dy}{dx}$ *if* $x = a\left(\cos t + \log\tan\left(\frac{t}{2}\right)\right), y = a\sin t$

$$x = a\left(\cos t + \log\tan\frac{t}{2}\right); y = a\sin t$$

$$\frac{dx}{dt}=a\left(-\sin t+\frac{1}{\tan\frac{t}{2}}.\sec^2\frac{t}{2}.\frac{1}{2}\right)=a\left(\sin t+\frac{1}{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}}\times\frac{1}{\cos^2\frac{t}{2}}\times\frac{1}{2}\right)$$

$$=a\left(-\sin t+\frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right)=a\left(-\sin t+\frac{1}{\sin t}\right)$$

$$=a\left(\frac{-\sin^2 t+1}{\sin t}\right)=a\frac{\cos^2 t}{\sin t}=\frac{a\cos^2 t}{\sin t}$$

$$y=a\sin t$$

$$\frac{dy}{dx}=a\cos t$$

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=a\cos t\times\frac{\sin t}{a\cos^2 t}=\tan t$$

b) $y=\sin^{-1}x$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}.\frac{dy}{dx}=1$$

diff wrt x

$$\sqrt{1-x^2}.\frac{d^2y}{dx^2}+\frac{dy}{dx}.\frac{-x}{\sqrt{1-x^2}}=0$$

$$(1-x)^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0 \qquad \|\times ing\ by\ \sqrt{1-x^2}$$

Hence proved. (3)