Mathematics (Science)
SCORING KEY

Set: A

Maximum Score: 80

Qn. No.	Answer Key / Value Points	Sub score	Total score
1.	For showing R is reflexive For showing R is symmetric For showing R is transitive	1 1 1	
2.	$IA = A$ For row transformation $A^{-1} = \begin{bmatrix} \frac{5}{13} & \frac{-3}{13} \\ \frac{1}{13} & \frac{2}{13} \end{bmatrix}$	1 1 1	3
3.	i) $AB = \begin{bmatrix} -10 & +2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}$ $CB = \begin{bmatrix} -20 & 4 & 42 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}$ (Obtaining by doubling the first row of AB)	2 1	3
4.	$f(x) = (x-3)^2 + 1$ $y = (x-3)^2 + 1$ $\sqrt{y-1} + 3 = x$ $f^{-1}(x) = \sqrt{x-1} + 3$	1 1 1	3
5.	$\tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left[\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\times\frac{2}{11}}\right]$ $= \tan^{-1}\left(\frac{3}{4}\right)$	2 1	3
6.	$2\tan^{-1}(\cos x) = \tan^{-1}(2\cos x)$ $\frac{2\cos x}{1-\cos^2 x} = 2\cos x$ $\sin^2 x = 1$ $x = \pm\frac{\pi}{2}$	1 1 1	3
7.	$3x - x^2 = -2 - 8$ $x^2 - 3x - 10 = 0$ $x = 5$ or -2	1 1 1	3
8.	i) $\lvert A \rvert = -28$ ii) $\lvert \text{adj } A \rvert = \lvert A \rvert^2$ $= (-28)^2 = 784$ iii) $\lvert 3A \rvert = 27 \times \lvert A \rvert$ $= 27 \times -28 = -756$	1 1 1	3

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9.	i) For showing commutative	1	
	For showing associative	1	
	ii) $e = \frac{1}{2}$	1	
	iii) $a^{-1} = \frac{1}{4a}, a \neq 0$	1	4
10.	i) $(fof)\ x = f(f(x))$		
	$= f\left(\frac{2x-3}{x-2}\right)$	1	
	$= \frac{2\left(\frac{2x-3}{x-2}\right)-3}{\left(\frac{2x-3}{x-2}\right)-2}$	1	
	$= x$	1	
	ii) $f(x) = \frac{x}{x+1}$	1	4
11.	$A^T = \begin{bmatrix} 1 & 2 & 1 \\ -3 & 0 & 2 \\ 1 & 4 & -2 \end{bmatrix}$	1	
	$P = \frac{1}{2}[A + A^T] = \frac{1}{2}\begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & 6 \\ 2 & 6 & -4 \end{bmatrix}$		1
	$Q = \frac{1}{2}[A - A^T] = \frac{1}{2}\begin{bmatrix} 0 & -5 & 0 \\ 5 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$	1	
	$A = P + Q$	1	4
12.	i) $A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$	2	
	ii) $A^T = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$	1	
	iii) A is symmetric since $A = A^T$	1	4
13.	$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$	1	
	$R_1 \rightarrow R_1 + R_2 + R_3$		

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	$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & 1 & 1\\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b}\\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1\end{vmatrix}$	1	
	$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ $\quad abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & 0 & 0\\ \frac{1}{b} & 1 & 0\\ \frac{1}{c} & 0 & 1\end{vmatrix}$	1	
	$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$	1	4
14.	i) $\frac{dx}{d\theta} = -2\sin\theta$	½	
	$\frac{dy}{d\theta} = 3\cos\theta$	½	
	$\frac{dy}{dx} = \frac{-3}{2}\cot\theta$	1	
	ii) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$		
	$= \frac{3}{2}\operatorname{cosec}^2\theta \times \frac{d\theta}{dx}$	1	
	$= \frac{+3}{2}\operatorname{cosec}^2\theta \times \frac{1}{-2\sin\theta} = \frac{-3}{4}\operatorname{cosec}^3\theta$	1	4
15.	$y_1 = 2\tan^{-1}x \cdot \frac{1}{1+x^2}$	1	
	$(1+x^2)\,y_1 = 2\tan^{-1}x$	1	
	$(1+x^2)\,y_2 + y_1\,2x = \frac{2}{1+x^2}$	1	
	$(1+x^2)\,y_2 + 2x\,(1+x^2)\,y_1 = 2$	1	4
16.	$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) \quad 0 < x < 1$	1	
	Put $x = \tan\theta$		
	$y = \sin^{-1}(\cos 2\theta)$	1	
	$= \sin^{-1}\left[\sin\left(\frac{\pi}{2}\right) - 2\theta\right]$	1	
	$= \frac{\pi}{2} - 2\cdot\tan^{-1}x$		
	$\frac{dy}{dx} = -2 \times \frac{1}{1+x^2}$	1	4
17.	i) Area of $\Delta^{le}\,ABC = \frac{1}{2}\begin{vmatrix}-2 & -3 & 1\\ 3 & 2 & 1\\ -1 & -8 & 1\end{vmatrix}$	2	
	$= -15$		
	Area $= 15$ sq.units		

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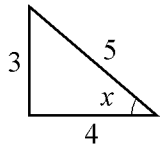
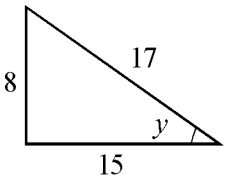
	ii) Base line is fixex as AB for the third point. Choose any value for x randomly, and find the y co-ordinate for that x co-ordinate accordingly. (or vice versa) For example if we choose x = 1 then $= \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ 1 & y & 1 \end{vmatrix} = \pm 15$ Solve and find the value of y.	2	
18.	Let $x = \sin^{-1}\left(\frac{3}{5}\right) \Rightarrow \sin x = \frac{3}{5}$	½	
	$y = \sin^{-1}\left(\frac{8}{17}\right) \Rightarrow \sin y = \frac{8}{17}$	½	
	$\cos(x - y) = \cos x \cos y + \sin x \sin y$	1	
	$\sin x = \frac{3}{5}$ 		
	$\cos x = \frac{4}{5}$	½	
	$\sin y = \frac{8}{17}$ 		
	$\cos y = \frac{15}{17}$	½	
	$\cos(x - y) = \frac{84}{85}$		
	$x - y = \cos^{-1}\left(\frac{84}{85}\right)$	1	4
	$\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$		
19.	i) $f(x) = 2x - 3$ one-one if $f(x_1) = f(x) \Rightarrow x_1 = x_2$ $2x_1 - 3 = 2x_2 - 3$ $x_1 = x_2$	1	
	for showing onto	1	
	$y = 2x - 3$ $y + 3 = 2x$, $\frac{y+3}{2} = x$		
	$f^{-1}(x) = \frac{x+3}{2}$	1	
	ii) Option (C)	1	
	iii) $f(x) = \begin{cases} \lvert x \rvert & x \le 1 \\ 2 - x & x > 1 \end{cases}$		
	[Any other corrected answer can be given full credit]	2	6

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20.	$A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$	1	
	$A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$	1	
	$A^3 - 23A - 40\mathrm{I} = 0$	1	
	$A^{-1}A^3 - 23A^{-1}A - 40A^{-1}\mathrm{I} = 0$	1	
	ii) $A^{-1} = \frac{A^2 - 23\mathrm{I}}{40}$	1	
	$A^{-1} = \begin{bmatrix} -4 & 4 & 8 \\ 1 & -11 & 8 \\ 14 & 6 & -8 \end{bmatrix}$	1	6
21.	$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$	1	
	$AX = B$	½	
	$X = A^{-1}B$	½	
	$A^{-1} = \begin{bmatrix} \frac{7}{4} & \frac{1}{4} & \frac{-3}{4} \\ \frac{-19}{4} & \frac{-1}{4} & \frac{11}{4} \\ \frac{-11}{4} & \frac{-1}{4} & \frac{7}{4} \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$	3	
	$X = \begin{bmatrix} \frac{7}{4} & \frac{1}{4} & \frac{-3}{4} \\ \frac{-19}{4} & \frac{-1}{4} & \frac{11}{4} \\ \frac{-11}{4} & \frac{-1}{4} & \frac{7}{4} \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$		
	$x = 2, y = 1, z = 3$	1	6
22.	i) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{4}\right)\right]$	1	
	$= \sin^{-1}\left(\sin\frac{\pi}{4}\right)$		
	$= \frac{\pi}{4}$		
	ii) Option (b)	1	
	iii) Put $x = \sin\theta$	1	
	$\sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$		
	$= \sin^{-1}(\sin 2\theta)$	1	
	$= 2\theta$		
	$= 2\sin^{-1}x$	1	6

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23.	i) $2x + \frac{xdy}{dx} + y + 2y \cdot \frac{dy}{dx} = 0$	1	
	$\frac{dy}{dx}(x + 2y) = -(2x + y)$		
	$\frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$	1	
	ii) $u = x^{\sin x}$ $\quad v = (\sin x)^{\cos x}$	1	
	$y = u + v$		
	$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$	1	
	$\log u = \sin x \log x$		
	$\frac{1}{u}\frac{du}{dx} = \frac{\sin x}{x} \log x \cos x$		
	$\frac{dv}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right)$	1	
	$\log v = \cos x (\log \sin x)$		
	$\frac{1}{v}\frac{dv}{dx} = \frac{\cos x}{\sin x} \cdot \cos x + \log(\sin x) - \sin x$		
	$\frac{dv}{dx} = (\sin x)^{\cos x} \cdot \left[\frac{\cos^2 x}{\sin x} - \sin x \log(\cos x)\right]$	1	6
	$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$		
24.	i) $f(x) = (x-2)^2$, $x \in [1, 4]$		
	$f(x)$ is continuous in $[1, 4]$	1	
	$f(x)$ is differentiable in $(2, 4)$		
	There exists $c \in (1, 4)$ so that		
	$f'(c) = \frac{f(4)-f(1)}{4-1}$	1	
	$2(c-2) = \frac{4-1}{4-1} = 1$		
	$c = \frac{5}{2} \in (1, 4)$	1	
	MVT verified		
	ii) Clearly $c = \frac{5}{2}$ from first part will be the x-coordinate to the point of contact of tangent and the curve		
	$y = (x-2)^2$, $x = \frac{5}{2}$		
	$y = \frac{5}{2} - 2$	1	
	$y = \frac{1}{4}$		
	Point $\left(\frac{5}{2}, \frac{1}{4}\right)$		

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	iii) Tangent parallel to x axis $\Rightarrow f'(c) = 0$ $2(c-2) = u$ $c = 2$ $y = (x-2)^2$ $y = 0$ Point (2, 0)	1 1	 6
25.	i) A = {1, 2, 3, 6} $a * b$ = HCF of a and b	3	
	ii) $e = 6$	1	
	iii) [write any commutative binary with 3 as identity]	2	6

*	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

*	1	2	3	6
1	6	3	1	2
2	3	2	2	1
3	1	2	3	6
6	2	1	6	1