FIRST TERMINAL EXAMINATION – 2019

MATHAMATICS (SCIENCE)

(Chapters 1 to 4)

Questions : -

Questions 1 to 7 carry 3 marks each. Answer any six questions. (6 × 3 = 18)

1. Express the matrix A = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. (3)

2. Using the operation table of the binary operation $*$ on {1,2,3,6} answer the following :

$*$	1	2	3	6
1	1	3	2	1
2	3	2	6	2
3	2	6	3	3
6	1	2	3	6

(i) Is $*$ commutative ? Justify (1)

(ii) Find the identity element of $*$ (1)

(iii) Find the elements which have inverse. (1)

3. (i) Which of the following is the value of $\sin^{-1}\frac{5}{4}$?

(a) $\frac{\pi}{5}$ (b) 1.8 (c) $\frac{\pi}{7}$ (d) Does not exist (1)

(ii) If $\tan^{-1}(-x) + \cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{2}$, then find the value of x. (2)

4. Let A = $\begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 6 \\ 7 & 1 & 2 \end{bmatrix}$

(i) Find |A| (2)

(ii) Hence evaluate |3A| (1)

5. Prove that $\tan^{-1}\frac{63}{16} = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$ (3)

6. Consider the set A ={1,2,3} .

(i) Write an equivalence relation containing the element (1,2). (2)

(ii) How many equivalence relations are possible which contain the element (1,2) ? (1)

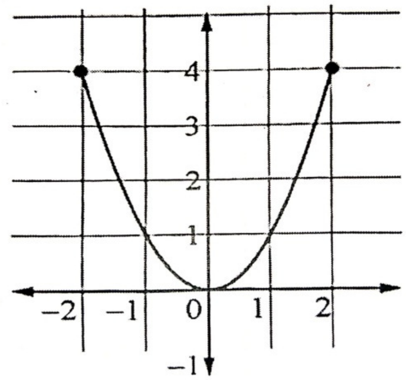
7. Let A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 3 & 4 \\ -2 & 3 & 1 \end{bmatrix}$

(i) Find 3A. (1)

(ii) Hence evaluate 3A – B . (2)

Answer any 8 questions from 8 to 17. Each carries 4 scores.

8. The figure shows the graph of a function f(x)



(i) Write the domain and range of f(x) (2)

(ii) How can restrict the domain of f(x) to make it invertible in this range? (2)

9. (i) Find the value of $\sin^{-1}\left(\sin\left(\frac{3\pi}{5}\right)\right)$ (1)

(ii) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $-\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. (2)

10. Consider B = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and AB = $\begin{bmatrix} 3 & 5 & 4 \\ 3 & 5 & 4 \end{bmatrix}$

(i) Find the order B and A (2)

(ii) Find A. (2)

11. (i) Evaluate $\begin{vmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{vmatrix}$ (1)

(ii) Using properties of determinants show that

$\begin{vmatrix} x+1 & x+2 & x+3 \\ x+4 & x+5 & x+6 \\ x+7 & x+8 & x+9 \end{vmatrix} = 0$ (3)

12. Show that the function f(x) = $\frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$ is inverse of itself. (4)

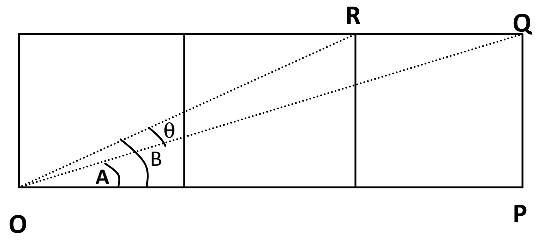
13. Using elementary transformations, find the inverse of the matrix A = $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ (4)

14. Given , $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

Find the values of x, y, z, w. (4)

15. If a triangle have area 35 sq units with vertices (2,-6), (5,4), (k,4). Then find the value of k. (4)

16. The figure given has 3 identical squares with ∠POQ = A, ∠POR = B, ∠QOR = θ,



(i) Which of the following is the value of tan A ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{2}{3}$ (1)

(ii) Find ∠B (1)

(iii) Find the angle θ (2)

17. Given adj A = $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

(i) Find matrix A (2)

(ii) Find A^{-1} (2)

Answer any 5 questions from 18 to 24. Each carries 6 scores

18. Solve the system of linear equations by matrix method, 3x – 2y + 3z = 8

2x + y – z = 1

4x – 3y + 2z = 4 (6)

19. Let f(x) = $\frac{x-1}{x-3}$, x≠ 3 and g(x) = $\frac{x-3}{x-1}$, x≠ 1 be two functions defined on R.

(i) Find fog(x) , x≠ 0 (2)

(ii) Find $f^{-1}(x)$ and $g^{-1}(x)$, x≠ 1 (2)

(iii) Find $(gof)^{-1}$ (x) (2)

20. Let A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

(i) Verify that $A^3 - 6A^2 + 9A - 4I = 0$ (4)

(ii) Hence find A^{-1} (2)

21. (i) Simplify $\tan(\cos^{-1} x)$ and hence evaluate $\tan\left(\cos^{-1}\frac{8}{17}\right)$ (3)

(ii) Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$ (3)

22. (i) Construct a 3x3 matrix A = $[a_{ij}]$ such that a_{ij}= 2i – 3j . Hence if B = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ show that AB≠BA (3)

(ii) Find x if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$. (3)

23. Consider the set A = {-1, 1}

(i) Define a binary operation on A. (2)

(ii) Check that the above binary operation is commutative and associative. (2)

(iii) How many binary operations are possible on A? (2)

24. (i) Let $\begin{vmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 4 & 3 \end{vmatrix} = 3$, then what is the value of $\begin{vmatrix} 6 & 7 & 6 \\ 2 & 0 & 1 \\ 3 & 4 & 3 \end{vmatrix}$?

(a) 3 (b) 6 (c) 0 (d) 18 (1)

(ii) Using properties of determinant show that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

(5)

Answers : -

1. Given , A = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

Symmetric Part, P = $\frac{1}{2}(A + A^T)$

$$\therefore P = \frac{1}{2}\left(\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix}$$

Skew symmetric Part, Q = $\frac{1}{2}(A - A^T)$

$$\therefore Q = \frac{1}{2}\left(\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$$

We have , A = P + Q

$$ie,\ \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$$

2. (i) Yes, $*$ is commutative

$a*b = b*a$, for all a and b.

[table is symmetric with respect to the main diagonal]

(ii) e= 6 [$6* a = a*6 = 6$, for all a]

(iii) $2*3 = 3*2 = 6 \Rightarrow$

Inverse of 2 = 3 , inverse of 3 = 2

$6*6=6 \Rightarrow$

Inverse of 6 = 6

3. (i) (d) does not exist $\left(\text{Since, } \frac{5}{4} \notin [-1, 1]\right)$

(ii) We have, $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Given, $\tan^{-1}(-x) + \cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{2}$

$\therefore \tan^{-1}(-x) = \frac{\pi}{2} - \cos^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{2} - \frac{2\pi}{3}$

$= -\frac{\pi}{6}$

$\therefore -x = \tan\left(-\frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

$\therefore x = \frac{1}{\sqrt{3}}$

4. (i) Given, $A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 6 \\ 7 & 1 & 2 \end{bmatrix}$

$|A| = 3(10-6) - 2(-2-42) + 1(-1-35)$

$= 3(4) - 2(-44) + 1(-36) = 12 + 88 - 36$

$= 64$

(ii) $|3A| = 3^3 |A| = 27|A| = 27x64 = 1728$

5. RHS $= \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$

Let, $x = \sin^{-1}\left(\frac{5}{13}\right)$ and $y = \cos^{-1}\left(\frac{3}{5}\right)$

$\therefore \sin x = \frac{5}{13}$ and $\cos y = \frac{3}{5}$

$\cos x = \sqrt{1 - \sin^2 x}$ and $\sin y = \sqrt{1 - \sin^2 y}$

$\cos x = \sqrt{1 - \left(\frac{5}{13}\right)^2}$ and $\sin y = \sqrt{1 - \left(\frac{3}{5}\right)^2}$

$\cos x = \sqrt{1 - \frac{25}{169}}$ and $\sin y = \sqrt{1 - \frac{9}{25}}$

$\cos x = \sqrt{\frac{144}{169}}$ and $\sin y = \sqrt{\frac{16}{25}}$

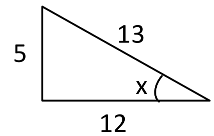
$\cos x = \frac{12}{13}$ and $\sin y = \frac{4}{5}$

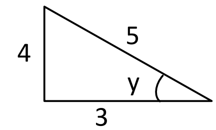
$\tan x = \frac{\sin x}{\cos x}$ and $\tan y = \frac{\sin y}{\cos y}$

$\tan x = \frac{\frac{5}{13}}{\frac{12}{13}}$ and $\tan y = \frac{\frac{4}{5}}{\frac{3}{5}}$

$\tan x = \frac{5}{12}$ and $\tan y = \frac{4}{3}$

$\therefore x = \tan^{-1}\left(\frac{5}{12}\right)$ and $y = \tan^{-1}\left(\frac{4}{3}\right)$





$\therefore \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{4}{3}\right)$

$= \tan^{-1}\left(\frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} x \frac{4}{3}}\right)$

$= \tan^{-1}\left(\frac{\frac{15+48}{36}}{1 - \frac{20}{36}}\right)$ $\left[\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= \tan^{-1}\left(\frac{\frac{63}{36}}{\frac{16}{36}}\right) = \tan^{-1} \frac{63}{16}$ = LHS

6. (i) R= {(1,1),(2,2),(3,3),(1,2),(2,1)}

(ii) 2 relations are possible

R_1 = {(1,1),(2,2),(3,3),(1,2),(2,1)}

R_2 = {(1,1),(2,2),(3,3),(1,2),(2,1),(3,2),(2,3),
(3,1),(1,3)}}

7. (i) $3A = 3\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 3 \\ 12 & -3 & 9 \end{bmatrix}$

(ii) $3A - B = \begin{bmatrix} 9 & 6 & 3 \\ 12 & -3 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 4 \\ -2 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 3 & -1 \\ 14 & -6 & 8 \end{bmatrix}$

8. (i) Domain = [–2, 2], Range = [0,4]

(ii) [–2, 0] or [0,2]

9. (i) $\sin\frac{3\pi}{5} = \sin\left(\pi - \frac{2\pi}{5}\right) = \sin\frac{2\pi}{5}$

$\therefore \sin^{-1}(\sin\frac{3\pi}{5}) = \sin^{-1}(\sin\frac{2\pi}{5})$

$= \frac{2\pi}{5} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$

$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$

$[\text{Cos } 2x = \text{Cos}^2 x - \text{Sin}^2 x]$

$[\text{Sin } 2x = 2 \text{ Sin } x \text{ Cos } x]$

$[\text{Cos}^2 x + \text{Sin}^2 x = 1]$

$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right]$

$= \tan^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right]$

Dividing both Nr. and Dr. by $\cos\frac{x}{2}$, we get,

$= \tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$

$= \tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ $\left[\mathrm{Tan}\left(\frac{\pi}{4}+x\right)=\frac{1+\mathrm{Tan}\,x}{1-\mathrm{Tan}\,x}\right]$

$= \frac{\pi}{4}+\frac{x}{2}$

10. (i) Order of B = 2x3

Order of A = 2x2 $[A_{2x2}\ B_{2x3} = AB_{2x3}]$

(ii) Let A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

We have, AB = $\begin{bmatrix} 3 & 5 & 4 \\ 3 & 5 & 4 \end{bmatrix}$

ie, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 4 \\ 3 & 5 & 4 \end{bmatrix}$

ie, $\begin{bmatrix} a+2b & 2a+3b & 3a+b \\ c+2d & 2c+3d & 3c+d \end{bmatrix} = \begin{bmatrix} 3 & 5 & 4 \\ 3 & 5 & 4 \end{bmatrix}$

we have, a + 2b = 3 → ⟨1⟩

and 2a + 3b = 5 → ⟨2⟩

⟨1⟩ x 2 ⇒ 2a + 4b = 6 →⟨3⟩

⟨2⟩ − ⟨3⟩ ⇒ −b = −1 ∴ b = 1

⟨1⟩ ⇒ a+ 2 = 3 ∴ a = 1

Similarly, solving c + 2d = 3 and 2c + 3d = 5

We get c = 1, d = 1

∴ A = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

11. (i) $\begin{vmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{vmatrix} = \begin{vmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 3 & 3 & 3 \end{vmatrix}$ Applying $R_3 \to R_3 - R_2$

$= \begin{vmatrix} 3 & 4 & 5 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix}$ applying $R_2 \to R_2 - R_1$

= 0, R_2 and R_3 are identical

(ii) $\begin{vmatrix} x+1 & x+2 & x+3 \\ x+4 & x+5 & x+6 \\ x+7 & x+8 & x+9 \end{vmatrix}$

$= \begin{vmatrix} x+1 & x+2 & x+3 \\ x+4 & x+5 & x+6 \\ 3 & 3 & 3 \end{vmatrix}$ Applying $R_3 \to R_3 - R_2$

$= \begin{vmatrix} x+1 & x+2 & x+3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix}$ applying $R_2 \to R_2 - R_1$

= 0 [R_2 and R_3 are identical]

12. Given, f(x) = $\frac{4x+3}{6x-4}$

Let, y = $\frac{4x+3}{6x-4}$

ie,, y(6x – 4) = 4x + 3

ie, 6xy – 4y = 4x + 3

∴ 6xy – 4x = 4y + 3

x(6y – 4) = 4y + 3

∴ x = $\frac{4y+3}{6y-4}$

ie, $f^{-1}(y) = \frac{4y+3}{6y-4}$ or $f^{-1}(x) = \frac{4x+3}{6x-4}$

Clearly, $f^{-1} = f$

Or

Show that fof(x) = x which implies, $f^{-1} = f$

13. Let A = $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

We have A = IA

ie, $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A$

Applying, $R_1 \to \frac{R_1}{2}$,

$\begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}A$

Applying, $R_2 \to R_2 - 7R_1$,

$\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix}A$

Applying, $R_2 \to 2R_2$,

$\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -7 & 2 \end{bmatrix}A$

Applying, $R_1 \to R_1 - \frac{1}{2}R_2$,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}A$

ie, I = BA ⇒

B = $A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

14. Given, $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

ie, $\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$

Equating corresponding elements,

3x = x + 4 ⇒ 2x = 4 ⇒ x = 2

3y = 6 + x + y ⇒ 2y = 6+2 = 8 ⇒ y = 4

$3w = 2w + 3 \Rightarrow w = 3$

$3z = -1 + z + w \Rightarrow 2z = -1 + 3 = 2 \Rightarrow z = 1$

15. Let , $(x_1, y_1) = (2, -6)$

$(x_2, y_2) = (5, 4)$

$(x_3, y_3) = (k, 4)$

We have, Area of Δ = 35

$ie, \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 35$

$ie, \frac{1}{2}\{2(4-4) + 6(5-k) + 1(20-4k)\} = \pm 35$

$ie, 2(4-4) + 6(5-k) + 1(20-4k) = \pm 70$

$ie, 2(0) + 30 - 6k + 20 - 4k = \pm 70$

$ie, 50 - 10k = \pm 70$

$ie, 50 - 10k = 70 \Rightarrow 10k = -20 \quad \therefore k = -2$

$50 - 10k = -70 \Rightarrow 10k = 120 \quad \therefore k = 12$

k = – 2, 12

16. (i) Suppose length of a side of each square=x

$\therefore \tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{x}{3x} = \frac{1}{3}$

(ii) We have, $\tan B = \frac{x}{2x} = \frac{1}{2}$

$\therefore B = \tan^{-1}\frac{1}{2}$

(iii) We have, $\tan A = \frac{1}{3}$ and $\tan(A+\theta) = \frac{1}{2}$

$\therefore A = \tan^{-1}\frac{1}{3}$ and $A+\theta = \tan^{-1}\frac{1}{2}$

$\therefore \theta = \tan^{-1}\frac{1}{2} - A$

$= \tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{3}$

$= \tan^{-1}\left(\frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{2}\cdot\frac{1}{3}}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{6}}{1+\frac{1}{6}}\right) = \tan^{-1}\left(\frac{\frac{1}{6}}{\frac{7}{6}}\right)$

$= \tan^{-1}\frac{1}{7}$

17. (a) Given $\text{Adj } A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

(b) $|A| = \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = 6 - 4 = 2$

$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

18. Given, $3x - 2y + 3z = 8$

$2x + y - z = 1$

$4x - 3y + 2z = 4$

$ie, \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

ie, AX = B, where $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

We have , $X = A^{-1}B$

$A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix} = 3(2-3)+2(4+4)+3(-6-4)$

$= 3(-1) + 2(8) + 3(-10)$

$= -3 + 16 - 30$

$= -17$

To find adj A :–

Cofactor of 3, $A_{11} = \begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = 2 - 3 = -1$

Cofactor of –2, $A_{12} = -\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} = -(4+4) = -8$

Cofactor of 3, $A_{13} = \begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix} = -6 - 4 = -10$

Cofactor of 2, $A_{21} = -\begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = -(-4+9) = -5$

Cofactor of 1, $A_{22} = \begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix} = 6 - 12 = -6$

Cofactor of –1, $A_{23} = -\begin{vmatrix} 3 & -2 \\ 4 & -3 \end{vmatrix} = -(-9+8) = 1$

Cofactor of 4, $A_{31} = \begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix} = 2 - 3 = -1$

Cofactor of –3, $A_{32} = -\begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix} = -(-3-6) = 9$

Cofactor of 2, $A_{33} = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 3 + 4 = 7$

$\text{adj } A = \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}^T = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$= \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

$X = A^{-1}B = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

$= \frac{1}{-17}\begin{bmatrix} -8-5-4 \\ -64-6+36 \\ -80+1+28 \end{bmatrix} = \frac{1}{-17}\begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

ie, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\therefore x = 1, y = 2, z = 3$

19. (i) $fog(x) = f(g(x)) = f\left(\frac{x-3}{x-1}\right) = \frac{\left(\frac{x-3}{x-1}\right)-1}{\left(\frac{x-3}{x-1}\right)-3}$

$$= \frac{\frac{x-3-(x-1)}{x-1}}{\frac{x-3-3(x-1)}{x-1}} = \frac{-2}{-2x} = \frac{1}{x}$$

(ii) Given , f(x) = $\frac{x-1}{x-3}$

Take, y = $\frac{x-1}{x-3}$

ie, y(x – 3) = x – 1

ie, xy – 3y = x – 1

∴ xy – x = 3y – 1

x(y – 1) = 3y – 1

∴ x = $\frac{3y-1}{y-1}$ ⇒ $f^{-1}(y) = \frac{3y-1}{y-1}$ ⇒ $f^{-1}(x) = \frac{3x-1}{x-1}$

Given , g(x) = $\frac{x-3}{x-1}$

Take, y = $\frac{x-3}{x-1}$

ie, y(x – 1) = x – 3

ie, xy – y = x – 3

∴ xy – x = y – 3

x(y – 1) = y – 3

∴ x = $\frac{y-3}{y-1}$ ⇒ $g^{-1}(y) = \frac{y-3}{y-1}$ ⇒ $g^{-1}(x) = \frac{x-3}{x-1}$

(iii) $(gof)^{-1}$ (x) = f^{-1} o g^{-1} (x)

$= f^{-1}(g^{-1}(x))$

$= f^{-1}\left(\frac{x-3}{x-1}\right) = \frac{3\left(\frac{x-3}{x-1}\right)-1}{\left(\frac{x-3}{x-1}\right)-1}$

$= \frac{3(x-3)-(x-1)}{(x-3)-(x-1)} = \frac{2x-8}{-2} = 4 - x$

20. (i) Given, A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$

$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$

∴ $A^3 - 6A^2 + 9A - 4I$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

(ii) We have , $A^3 - 6A^2 + 9A - 4I = 0$

Multiplying both sides by A^{-1} we get,

$A^{-1}(A^3 - 6A^2 + 9A - 4I) = A^{-1} \cdot 0$

ie, $A^2 - 6A + 9I - 4A^{-1} = 0$

∴ $4A^{-1} = A^2 - 6A + 9I$

$A^{-1} = \frac{1}{4}(A^2 - 6A + 9I) =$

$= \frac{1}{4}\left(\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$

$= \frac{1}{4}\left(\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}\right)$

$= \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

21. (i) Let $y = \cos^{-1} x$ ∴ $\cos y = x$

$\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-x^2}$

$\tan y = \frac{\sin y}{\cos y} = \frac{\sqrt{1-x^2}}{x}$

∴ $y = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

∴ $\tan(\cos^{-1} x) = \tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)$

$= \frac{\sqrt{1-x^2}}{x}$

∴ $\tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{\sqrt{1-\left(\frac{8}{17}\right)^2}}{\frac{8}{17}} = \frac{\sqrt{1-\frac{64}{289}}}{\frac{8}{17}} = \frac{\sqrt{\frac{225}{289}}}{\frac{8}{17}}$

$= \frac{\frac{15}{17}}{\frac{8}{17}} = \frac{15}{8}$

(ii) We have, $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$

$ie,\ 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}x$

$ie,\ \tan^{-1}\left(\frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2}\right) = \tan^{-1}x$

$\therefore \frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2} = x$ ie, $\frac{2\left(\frac{1-x}{1+x}\right)}{\frac{(1+x)^2-(1-x)^2}{(1+x)^2}} = x$

$ie,,\ \frac{2\left(\frac{1-x}{1+x}\right)}{\frac{4x}{(1+x)^2}} = x$ ie, $2\left(\frac{1-x}{1+x}\right)\frac{(1+x)^2}{4x} = x$

$ie,\ \frac{(1-x)(1+x)}{2} = x^2$ ie, $1 - x^2 = 2x^2$

$ie,\ 3x^2 = 1 \quad \therefore x^2 = 1/3 \quad \therefore x = \pm\frac{1}{\sqrt{3}}$

22. (i) Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Given, $a_{ij} = 2i - 3j$

$\therefore$ $a_{11} = 2(1) - 3(1) = 2 - 3 = -1$

$a_{12} = 2(1) - 3(2) = 2 - 6 = -4$

$a_{13} = 2(1) - 3(3) = 2 - 9 = -7$

$a_{21} = 2(2) - 3(1) = 4 - 3 = 1$

$a_{22} = 2(2) - 3(2) = 4 - 6 = -2$

$a_{23} = 2(2) - 3(3) = 4 - 9 = -5$

$a_{31} = 2(3) - 3(1) = 6 - 3 = 3$

$a_{32} = 2(3) - 3(2) = 6 - 6 = 0$

$a_{33} = 2(3) - 3(3) = 6 - 9 = -3$

$\therefore\ A = \begin{bmatrix} -1 & -4 & -7 \\ 1 & -2 & -5 \\ 3 & 0 & -3 \end{bmatrix}$

A is of order 3x3 and B is of order 3x1

AB is of order 3x1 . But BA is not defined

$\therefore$ AB $\neq$ BA

(i) Given, $[x \quad -5 \quad -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$ie,\ [x+0-2 \quad 0-10+0 \quad 2x-5-3]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$ie,\ [x-2 \quad -10 \quad 2x-8]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$ie,\ [x^2 - 2x - 40 + 2x - 8] = 0$

$ie,\ [x^2 - 48] = 0$

$ie,\ x^2 - 48 = 0$

$ie,\ x^2 = 48 \quad \therefore \quad x = \sqrt{48}$

23. (i) $a * b = ab$

(ii) $a * b = ab$ and $b * a = ba$

Clearly , $a * b = b * a$

$\therefore$ * is commutative

$a*(b*c) = a*(bc) = abc$

$(a*b)*c = (ab)*c = abc$

Clearly, $a*(b*c) = (a*b)*c$

$\therefore$ * is associative

(iii) Number of possible binary operations = $n^{(n^2)}$

$= 2^4 = 16$

24. (i) $\begin{vmatrix} 6 & 7 & 6 \\ 2 & 0 & 1 \\ 3 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 7 & 5 \\ 2 & 0 & 1 \\ 3 & 4 & 3 \end{vmatrix}$ Applying $R_1 \to R_1 - R_2$

$= \begin{vmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 4 & 3 \end{vmatrix} = 3$ [Applying $R_1 \to R_1 - R_3$]

Ans : (a) 3

(ii) LHS = $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

Taking out a from R_1 , b from R_2, and c from R_3

$= abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

Applying, $R_1 \to R_1 + R_2 + R_3$

$= abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

Applying, $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. { 1(1 – 0) – 0 + 0}

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ = RHS