ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION 2021 - 2022

PART-~~I~~/II/III

SUBJECT: Mathematics, Comaerce

CODE NO: ~~SY557~~ SAY 757 VERSION: P

80 SCORES 2½ HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1.		$A' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$	1	1
2.		(B) $\pi/2$	1	1
3.		(A) $6x$	1	1
4		(C) $1/2$	1	1
5		1	1	1
6.		$x+y+z = 5$	1	1
7.		(C) $k^3\|A\|$	1	1
8.		(D) $1/x$.	1	1
9.		(B) $(0, 1, 0)$	1	1
10.		(B) $P(F)$	1	1
11.		$(1,2) \in R \Rightarrow (2,1) \in R \therefore R$ is symmetric $(1,1) \notin R$ R is not reflexive $(1,2), (2,1) \in R$ but $(1,1) \notin R$ Not transitive	1 1	2
12.		For singular matrix, $\|A\| = 0$ ie $\begin{vmatrix} 2 & 2 \\ 2 & x \end{vmatrix} = 0 \Rightarrow x - 4 = 0$ $\Rightarrow x = 4$	1 1	2

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13.		Marginal revenue, $\frac{dR}{dx} = 26x + 26$	1	
		$\left.\frac{dR}{dx}\right\|_{x=5} = 26\times5 + 26 = 156$ rupees	1	2
14.		put $1+x^2 = t$	1	
		$2x\,dx = dt$		
		$\int \frac{dt}{t} = \log\|t\| + C$		
		$= \log\|1+x^2\| + C$	1	2
15.		$\bar{r} = \bar{a} + \lambda\bar{b}$	1	
		$\bar{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})$		
		$\bar{r} = \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})$	1	2
16.		$\cos^{-1}\left(\frac{1}{2}\right) + 2.\sin^{-1}\left(\frac{1}{2}\right)$		
		$= \frac{\pi}{3} + 2.\frac{\pi}{6}3$	1	
		$= 2\pi/3$	1	2
17.		$A = IA$		
		$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$	1/2	
		$R_1 \to R_1 - R_2$		
		$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$	1/2	
		$R_2 \to R_2 - R_1$		
		$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$	1/2	
		$A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$	1/2	2

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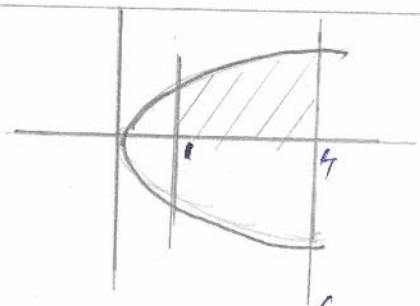
18.		$\int_0^1 x^2\,dx = \left(\frac{x^3}{3}\right)_0^1$	1	
		$= \frac{1}{3}[1-0] = \frac{1}{3}$	1	2
19.		Let $x_1, x_2 \in R$ $f(x_1) = f(x_2) \Rightarrow 4x_1+3 = 4x_2+3$ $\Rightarrow 4x_1 = 4x_2$ $\Rightarrow x_1 = x_2$ f is one-one.	2	
		Let $y \in$ Range of f, such that $y = f(x)$ $y = 4x+3$ $x = \frac{y-3}{4} \in R$ $f(x) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right)+3 = y$ f is onto	2	4
20.	a)	$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
	b)	$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\cdot\frac{2}{11}}\right)$	2	
		$= \tan^{-1}\frac{15}{20} = \tan^{-1}\left(\frac{3}{4}\right)$	1	4.
21.	a)	$f'(x) = 4x-3.$	1	
	b)	For max. or min, $f'(x) = 0$ $\Rightarrow 4x-3 = 0 \Rightarrow x = 3/4$	1	
		Interval | Sign $f'(x)$ | Nature of function		
		$(-\infty, 3/4)$ | $-ve$ | f is strictly decreasing	1	
		$(3/4, \infty)$ | $+ve$ | f is strictly increasing	1	4

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
22	a)	$\log \lvert x+a \rvert + C$	1	
	b)	$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$		
		$1 = A(x+2) + B(x+1)$	1	
		when $x = -1$, $A = 1$		
		when $x = -2$ $B = -1$		
		$\int \frac{1}{(x+1)(x+2)} = \int \frac{1}{x+1} dx + \int \frac{-1}{x+2} dx$	1	
		$= \log \lvert x+1 \rvert - \log \lvert x+2 \rvert + C$		
		$= \log \lvert \frac{x+1}{x+2} \rvert + C$	1	4
23.	a)		1	
	b)	Req. Area $= \int_0^4 \sqrt{x}\, dx$	1	
		$= \left(\frac{x^{3/2}}{3/2} \right)_1^4 = \frac{2}{3}\left[4^{3/2} - 1^{3/2}\right]$		
		$= \frac{14}{3}$ Sq. unit.	2	4
24.		Equation of plane, $P_1 + k P_2 = 0$		
		$(3x - y + 2z - 4) + k(x + y + z - 2) = 0$ — ①	1	
		Given, $(2,2,1)$ is on ①		
		$2 + 3k = 0 \Rightarrow k = -2/3$	2	
		put $k = -2/3$ in ①		
		$7x - 5y + 4z - 8 = 0$ is the plane	1	4

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25		$L.H.S = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$		
		$R_1 \to R_1 - R_2$ $R_2 \to R_2 - C_3$		
		$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 1 & 1 \\ a+b & b+c \end{vmatrix}$	2 1	
		$= (a-b)(b-c)(c-a) = RHS$	1	4
26.	a)	$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$	1	
		$= \begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$	1 (2)	
	b)	$\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$		
		$AX = B$	1	
		$\lvert A \rvert = -11 \neq 0$	1	
		$A^{-1} = \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$	1	
		$X = A^{-1}B = \frac{-1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$		2+4
		$= \frac{-1}{11}\begin{bmatrix} -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$	1 (4)	=6
		$x = 3, y = -1$		
27	a)	$\lim_{x \to 2^-} kx^2 = \lim_{x \to 2^+} 3$	1	
		$4k = 3$	1	
		$k = 3/4$	1	

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	5).	$\frac{dy}{dx} = \frac{dy/da}{dx/da} =$	1/2	
		$\frac{dy}{da} = b \cdot \cos a$	1/2	
		$\frac{dx}{da} = a - \sin a$	1/2	
		$\frac{dy}{dx} = \frac{b\cos a}{-a\sin a} = -\frac{b}{a}\cot a$	1	
		$\left.\frac{dy}{dx}\right\|_{a=\pi/4} = -\frac{b}{a}\cot \pi/4 = -\frac{b}{a}$	1/2	6
28.	a)	$y = a\sin(x+b)$		
		$\frac{dy}{dx} = a\cos(x+b)$	1	
		$\frac{d^2y}{dx^2} = -a\sin(x+b) = -y$	1	
		$\frac{d^2y}{dx^2} + y = 0$	1	
	b)	$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$		
		Separate variable.		
		$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$ integrate w.r.to x	1	
		$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$	1	
		$\tan^{-1}y = \tan^{-1}x + C$	1	6

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29.	a)	$\bar{a}.\bar{b} = a_1a_2 + b_1b_2 + c_1c_2$	1	
		$= 1.7 + 3(-1) + 7.8$		
		$= 60$	1	
	b)	projection $\bar{a}$ on $\bar{b} = \frac{\bar{a}.\bar{b}}{\lvert b \rvert}$	1	
		$= 60/\sqrt{114}$	1	
	c)	$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$	1	
		$= \begin{vmatrix} i & j & k \\ 1 & 3 & 7 \\ 7 & -1 & 8 \end{vmatrix}$	1	
		$= 31i + 41j - 22k$		6
30	a)	$y = x^{\sin x}$. (Take logarithm on both side)	1	
		$\log y = \sin x \log x$		
		$\frac{1}{y}\frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \log x \cdot \cos x$.	1	
		$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + \log x \cdot \cos x\right]$		
		$= x^{\sin x}\left[\frac{\sin x}{x} + \log x \cdot \cos x\right]$	1	
	b.	$y = 5\cos x - 3\sin x$.	1	
		$\frac{dy}{dx} = -5\sin x - 3\cos x$.	1	
		$\frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -y$	1	6.
		$\frac{d^2y}{dx^2} + y = 0$		

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31.		Let x and y be two numbers		
		$x + y = 24$, $y = 24 - x$	1	
		$P(x) = x.(24 - x)$		
		$= 24x - x^2$	1	
		For Loc. Max./Min.		
		$\frac{dP}{dx} = 0 \Rightarrow 24 - 2x = 0$	1	
		$x = 12$	1	
		$\frac{d^2P}{dx^2} = -2 < 0$	1	
		$\therefore x = 12$ is max.		
		$\therefore y = 24 - 12 = 12$	1	(6)
32.	a.	$\cos\theta = \left\| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}} \right\|$	1	
		$= \frac{2.3 + (-1)(-5) + 1.2}{\sqrt{2^2 + (-1)^2 + 1^2}\ \sqrt{3^2 + (-5)^2 + 2^2}}$		
		$= \frac{13}{\sqrt{228}}$	1	
	b).	$\bar{a}_1 = \hat{i} + \hat{j}$, $\bar{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$		
		$\bar{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$, $\bar{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$	1	
		$\bar{a}_2 - \bar{a}_1 = \hat{i} - \hat{k}$		
		$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$.	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$\bar{b}_1 \times \bar{b}_2 = \sqrt{9+1+49} = \sqrt{59}$	1	
		$d = \left\| \frac{(\bar{b}_1 \times \bar{b}_2) \cdot (\bar{a}_2 - \bar{a}_1)}{\|\bar{b}_1 \times \bar{b}_2\|} \right\| = \frac{10}{\sqrt{59}}$	1	6
33	a)	$\frac{1}{2}(A+A') + \frac{1}{2}(A-A') = A$	1	
		$A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$		
		$A+A' = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$	1	
		$A-A' = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$	1	(4)
		$\frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$	1	
	b)	$AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}\begin{bmatrix} 1 & 3 & -6 \end{bmatrix} = \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}$	1	
		$A' = \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$	1	
		$B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$	1	
		$B' \cdot A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}\begin{bmatrix} -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}$	1	(4)
		$= (AB)'$		8

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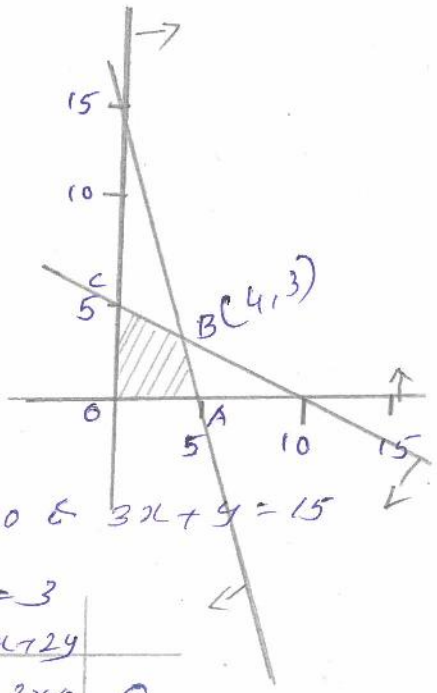
34.		$x+2y=10$ x | 0 | 10 y | 5 | 0 $3x+y=15$ x | 0 | 5 y | 15 | 0  Solve $x+2y=10$ & $3x+y=15$ $x=4$ and $y=3$ $Z=3x+2y$ O(0,0) | $3\times0+2\times0$ | 0 A(5,0) | $3\times5+2\times0$ | 15 B(4,3) | $3\times4+2\times3$ | 18 C(0,5) | $3\times0+2\times5$ | 10 Z is maximum at B(4,3) Maximum value = 18	1 1 4 1 1	8
35.	a)	$P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $P(A\cap B)=P(A)\cdot P(B)$ $=0.3\times0.4=0.12$ $P(A\cup B)=0.3+0.4-0.12$ $=0.58$ $P(A/B)=P(A\cap B)/P(B)=\frac{0.12}{0.4}=0.3$	1 1 1	

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	5)	Let B_1 the event that selected bag is 1st B_2 - The event that selected bag is 2nd.	1	
		$P(B1) = P(B2) = 1/2$		
		$P(R/B_1) = 4/8$ (Red from B_1)	1	
		$P(R/B_2) = 2/8$ (Red from B_2)	1	
		By Bayes theorem		
		Req. Probability,		
		$P(B_1/R) = \frac{P(B_1).P(R/B_1)}{P(B_1).P(R/B_1) + P(B_2).P(R/B_2)}$		
		$= \frac{\frac{1}{2}\cdot\frac{4}{8}}{\frac{1}{2}\cdot\frac{4}{8}+\frac{1}{2}\cdot\frac{2}{8}} = \frac{4}{4+2} = \frac{2}{3}$	2 (5)	8