Reg. No. : $\qquad$

Name : $\qquad$

## SAY / IMPROVEMENT EXAMINATION, JULY - 2022

## Part - III <br> Time : $2^{1 ⁄ 2}$ Hours <br> MATHEMATICS (SCIENCE) Cool-off time : 15 Minutes

Maximum : 80 Scores

## General Instructions to Candidates:

- 15 minutes is given as cool off time in addition to $2 \frac{1}{2}$ hours of exam time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.














## PART - I

## A. Answer any 4 questions from 1 to 6. Each carries 1 score.

1. Let R be the relation in the set of natural numbers N given by $R=\{(a, b): a=b-2, b>6\}$. Choose the correct answer.
(a) $(2,4) \in R$
(b) $(3,8) \in \mathrm{R}$
(c) $(6,8) \in \mathrm{R}$
(d) $(8,7) \in R$
2. Value of $\tan ^{-1}\left(2 \sin \frac{\pi}{3}\right)=$ $\qquad$
(a) $\frac{\pi}{3}$
(b) $\sqrt{3}$
(c) $\frac{\pi}{6}$
(d) $\frac{\pi}{4}$
3. Slope of the tangent to the curve $\mathrm{y}=x^{3}$ at the point $(1,1)$ is $\qquad$ .
(a) 1
(b) 3
(c) 6
(d) 2
4. Degree of the differential equation $x y \frac{\mathrm{~d}^{2} \mathrm{y}}{\mathrm{d} x^{2}}+x\left(\frac{\mathrm{dy}}{\mathrm{d} x}\right)^{2}-\mathrm{y} \cdot \frac{\mathrm{dy}}{\mathrm{d} x}=0$ is $\qquad$ -.
5. Direction ratios of the vector $\vec{a}=\hat{i}-2 \hat{j}+\hat{k}$ is $\qquad$ .
6. Cartesian equation of the line that passes through the origin and the point $(5,-2,3)$ is
$\qquad$ .

## PART－I

## 



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(4 \times 1=4)
$$



（a）$(2,4) \in R$
（b）$\quad(3,8) \in \mathrm{R}$
（c）$(6,8) \in \mathrm{R}$
（d）$(8,7) \in \mathrm{R}$

2． $\tan ^{-1}\left(2 \sin \frac{\pi}{3}\right)$ कுளை விய $=$ $\qquad$
（a）$\frac{\pi}{3}$
（b）$\sqrt{3}$
（c）$\frac{\pi}{6}$
（d）$\frac{\pi}{4}$

$\qquad$ （ロロஸ゙．
（a） 1
（b） 3
（c） 6
（d） 2
 $\qquad$ ．
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 กัロவอณృ $\qquad$ ®円ஸ゙．
B. Answer all questions from 7 to 10. Each carries 1 score.
7. Principal value of $\cos ^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\qquad$ .
(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{6}$
8. $\frac{\mathrm{d}}{\mathrm{d} x}(\log 2 x)=$ $\qquad$ .
(a) $\frac{1}{x}$
(b) $\frac{1}{2 x}$
(c) $2 \log x$
(d) $\quad \log 2$
9. Magnitude of the vector $\vec{a}=2 \hat{i}+\hat{j}-2 \hat{k}$ is $\qquad$ .
10. Direction cosines of $x$ axis is $\qquad$ .

PART - II
A. Answer any 3 questions from 11 to 15 . Each carries 2 scores.
11. If $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}, \mathrm{f}(x)=\cos x$ and $\mathrm{g}: \mathrm{R} \rightarrow \mathrm{R}, \mathrm{g}(x)=3 x^{2}$, find fog.
12. Construct a $2 \times 2$ matrix $A=\left[a_{i j}\right]$ whose elements are given by $a_{i j}=i+2 j$.
13. Find the rate of change of the area of a circle with respect to its radius when $\mathrm{r}=5 \mathrm{~cm}$.


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(4 \times 1=4)
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(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{6}$
8. $\frac{\mathrm{d}}{\mathrm{d} x}(\log 2 x)=$ $\qquad$ .
(a) $\frac{1}{x}$
(b) $\frac{1}{2 x}$
(c) $2 \log x$
(d) $\quad \log 2$
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## PART - II



$(3 \times 2=6)$
11. $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}, \mathrm{f}(x)=\cos x$





14. Find the projection of the vector $\hat{i}+3 \hat{j}+7 \hat{k}$ on the vector $7 \hat{i}-\hat{j}+8 \hat{k}$.
15. Find a vector perpendicular to each of the vectors $\vec{a}=3 \hat{i}+2 \hat{j}+2 \hat{k}$ and $\overrightarrow{\mathrm{b}}=\hat{\mathrm{i}}+2 \hat{\mathrm{j}}-2 \hat{\mathrm{k}}$.
B. Answer any 2 questions from 16 to 18. Each carries 2 scores.
16. Find the identity element of the binary operation defined on the set of all rational numbers Q by $\mathrm{a} * \mathrm{~b}=\frac{\mathrm{ab}}{2}$.
17. Find the Cartesian equation of the plane that passes through the point $(1,0,2)$ and the normal to the plane is $\hat{i}+\hat{j}-\hat{k}$.
18. A random variable X has the following probability distribution :

| X | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | 0 | k | 2 k | 2 k |

Find the value of $k$.

PART - III
A. Answer any 3 questions from 19 to 23. Each carries 4 scores.
19. Let $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}$ be given by $\mathrm{f}(x)=\frac{2 x+1}{3}$
(i) Show that f is invertible.
(ii) Find the inverse of f.






( $2 \times 2=4$ )







| $X$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $P(X)$ | 0 | $k$ | $2 k$ | $2 k$ |



## PART - III

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( $3 \times 4=12$ )



20. Find the intervals in which the function f given by $\mathrm{f}(x)=2 x^{3}-3 x^{2}-36 x+7$ is
(a) increasing
(b) decreasing
21. Find the area of the region bounded by the curve $y^{2}=9 x$, the lines $x=2, x=4$ and the X - axis in the first quadrant.
22. Find the general solution of the differential equation $\frac{d y}{d x}+\frac{y}{x}=x^{2}$.
23. Find the shortest distance between the lines whose vector equations are
$\vec{r}=\hat{i}+2 \hat{j}+3 \hat{k}+\lambda(\hat{i}-3 \hat{j}+2 \hat{k})$
$\vec{r}=4 \hat{i}+5 \hat{j}+6 \hat{k}+\mu(2 \hat{i}+3 \hat{j}+\hat{k})$
B. Answer any 1 question from 24 to 25. Carries 4 scores.
24. Find the area of the triangle with vertices $(2,7),(1,1)$ and $(10,8)$.
25. Find the area of the region bounded by the two parabolas $\mathrm{y}=x^{2}$ and $\mathrm{y}^{2}=x$.
PART - IV

## A. Answer any 3 questions from 26 to 29. Each carries 6 scores.

26. Prove that
(i) $\tan ^{-1} \frac{4}{3}+\tan ^{-1} \frac{1}{7}=\tan ^{-1} \frac{31}{17}$
(ii) Write $\tan ^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}, x<\pi$ in the simplest form.







27. $\vec{r}=\hat{i}+2 \hat{j}+3 \hat{k}+\lambda(\hat{i}-3 \hat{j}+2 \hat{k})$
$\vec{r}=4 \hat{i}+5 \hat{j}+6 \hat{k}+\mu(2 \hat{i}+3 \hat{j}+\hat{k})$

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$(1 \times 4=4)$



## PART - IV





27. (i) Find the value of k so that the function $\mathrm{f}(x)=\left\{\begin{array}{ll}\mathrm{k} x^{2} & \text { if } x \leq 2 \\ 3 & \text { if } x>2\end{array}\right.$ is continuous at

$$
\begin{equation*}
x=2 . \tag{3}
\end{equation*}
$$

(ii) Find $\frac{d y}{d x}$ if $x^{2}+x y+y^{2}=100$.
(3)
28. Find :
(i) $\int \frac{(\log x)^{2}}{x} \mathrm{~d} x$.
(3)
(ii) $\int \frac{x}{(x+1)(x+2)} \mathrm{d} x$.
29. Solve the Linear Programming Problem graphically :

Minimise

$$
z=-3 x+4 y
$$

subject to

$$
\begin{aligned}
& x+2 y \leq 8 \\
& 3 x+2 y \leq 12 \\
& x \geq 0 ; y \geq 0
\end{aligned}
$$

B. Answer any 2 questions from 30 to 32. Each carries $\mathbf{6}$ scores.
30. (i) If $y=5 \cos x-3 \sin x$ prove that $\frac{\mathrm{d}^{2} y}{d x^{2}}+y=0$.
(ii) Find $\frac{\mathrm{dy}}{\mathrm{d} x}$ if $\mathrm{y}=x^{\sin x}$.
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(3)
28. (i) $\int \frac{(\log x)^{2}}{x} \mathrm{~d} x$.
(3)




Minimise

$$
z=-3 x+4 y
$$

subject to

$$
\begin{aligned}
& x+2 y \leq 8 \\
& 3 x+2 y \leq 12 \\
& x \geq 0 ; y \geq 0
\end{aligned}
$$




$$
(2 \times 6=12)
$$


31. (i) Evaluate $\int_{0}^{5} x \mathrm{~d} x$ as the limit of a sum.
(4)
(ii) Find $\int \sec x(\sec x+\tan x) \mathrm{d} x$.
(2)
32. (i) Verify that the function $\mathrm{y}=\mathrm{e}^{x}+1$ is a solution of the differential equation

$$
\begin{equation*}
\frac{\mathrm{d}^{2} \mathrm{y}}{\mathrm{~d} x^{2}}-\frac{\mathrm{dy}}{\mathrm{~d} x}=0 \tag{2}
\end{equation*}
$$

(ii) Solve the differential equation :

$$
\begin{equation*}
\frac{\mathrm{dy}}{\mathrm{~d} x}=\frac{x+\mathrm{y}}{\mathrm{y}} \tag{4}
\end{equation*}
$$

PART - V

Answer any 2 questions from 33 to 35. Each carries 8 scores.
33. (i) Let $\mathrm{A}=\left[\begin{array}{ll}2 & 4 \\ 3 & 2\end{array}\right], \mathrm{B}=\left[\begin{array}{cc}1 & 3 \\ -2 & 5\end{array}\right]$ find $3 \mathrm{~A}-\mathrm{B}$.
(ii) If $A=\left[\begin{array}{ll}3 & -2 \\ 4 & -2\end{array}\right]$ and $I=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$, find $k$ so that $A^{2}=k A-2 I$.
(iii) Express the matrix $\left[\begin{array}{cc}3 & 5 \\ 1 & -1\end{array}\right]$ as the sum of a symmetric matrix and a skew symmetric matrix.
34. (i) Using properties of determinants prove that $\left|\begin{array}{lll}1 & a & a^{2} \\ 1 & b & b^{2} \\ 1 & c & c^{2}\end{array}\right|=(a-b)(b-c)(c-a)$
(3)

(ii) $\int \sec x(\sec x+\tan x) \mathrm{d} x$ बऽОறு|க.
(2)




PART - V



$$
(2 \times 8=16)
$$





(3)


(ii) Solve the following system of equations by matrix method:

$$
\begin{align*}
& 3 x-2 y+3 z=8 \\
& 2 x+y-z=1 \\
& 4 x-3 y+2 z=4 \tag{5}
\end{align*}
$$

35. (i) $A$ and $B$ are two events associated with a random experiment. If $P(A)=0.8$, $\mathrm{P}(\mathrm{B})=0.5$ and $\mathrm{P}(\mathrm{B} / \mathrm{A})=0.4$ find
(a) $P(A \cap B)$
(b) $\mathrm{P}(\mathrm{A} / \mathrm{B})$
(c) $P(A \cup B)$
(4)
(ii) A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from first bag.


$3 x-2 y+3 z=8$
$2 x+y-z=1$
$4 x-3 y+2 z=4$


(a) $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$
(b) $\quad \mathrm{P}(\mathrm{A} / \mathrm{B})$

(4)





