ANSWER KEY

Second YEAR HIGHER SECONDARY EXAMINATION March 2022

PART-I/II/III

SUBJECT: Mathematics .Science (80).

CODE NO: ~~SX 556~~ SAY 756 VERSION: Q

80 SCORES 2½ HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
A.		Part I		
1		(c) (6, 8)	1	
2		(a) $\frac{\pi}{3}$	1	
3		(b) 3	1	
(4)		1	1	
5		1, -2, 1	1	
6.		$\frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$	1	
B				
7.		$\frac{\pi}{6}$.	1	
8		$\frac{1}{x}$.	1	
9		3.	1	
10		1, 0, 0.	1	
		Part II		
11.		$fog(x) = f(g(x))$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$= f(3x^2)$	1/2	2
		$= \cos(3x^2)$	1	
12		$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$	1	2
		$= \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$	1	
13.		$A = \pi r^2$.	1/2	2.
		$\frac{dA}{dr} = 2\pi r$	1	
		$= 10\pi \text{ cm}^2/\text{cm}$	1/2	
14		Projection of $\vec{a}$ on $\vec{b}$		2
		$= \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$	1/2	
		$= \frac{(\hat{i}+3\hat{j}+7\hat{k})\cdot(7\hat{i}-\hat{j}+8\hat{k})}{\sqrt{7^2+1^2+8^2}}$	1/2	
		$= \frac{60}{\sqrt{114}}$	1	
15		$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 1 & 2 & -2 \end{vmatrix}$	1	2
		$= \hat{i}\begin{vmatrix} 2 & 2 \\ 2 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix}$	1/2	
		$= -8\hat{i} + 8\hat{j} + 4\hat{k}$.	1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
16.		$a * e = a = e * a$.	1	
		$a * e = \frac{ae}{2} = a$.	½	2.
		$\therefore e = 2$.	½	
17.		$A(x-x_1) + B(y-y_1) + C(z-z_1) = 0$	½	
		$1(x-1) + 1(y-0) + 1(z-2) = 0$.	½ 1	2
		$x + y - 2 + 1 = 0$	½	
18		$\sum P(x_i) = 1$.	½	
		$k + 2k + 2k = 1$.	1	2
		$5k = 1$	½	
		$k = 1/5$		
A 19	(i)	Part III $f(x_1) = f(x_2)$	½	
		$\frac{2x_1+1}{3} = \frac{2x_2+1}{3}$	1	
		$x_1 = x_2$		
		$\therefore f$ is 1-1		
		Let $y = \frac{2x+1}{3}$	½	4
		$x = \frac{3y-1}{2} \in R$.	½	
		$\therefore f$ is onto.	1	
		$\therefore f$ is bijective		
		$\therefore f$ is invertible.		
	(ii)	$f^{-1}(x) = \frac{3x-1}{2}$.	1	

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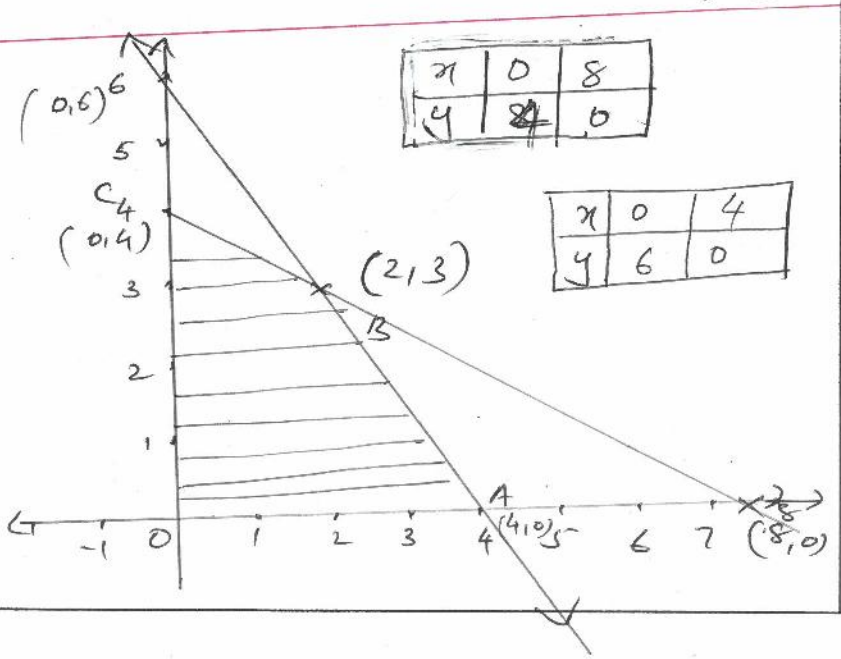
20.		$f(x) = 2x^3 - 3x^2 - 36x + 7$.		
		$f'(x) = 6x^2 - 6x - 36$.		
		$= 6(x^2 - x - 6)$.	1	
		$= 6(x-3)(x+2)$	1/2	
		$f'(x) = 0$.	1	
		$x = 3, -2$.	1/2	
		$(-\infty, -2), (-2, 3), (3, \infty)$		
		When $x \in (-\infty, -2)$, $f'(x) > 0$	1/2	4
		$f(x)$ is increasing		
		When $x \in (-2, 3)$		
		$f'(x) < 0$ $\therefore f(x)$ is decreasing.	1/2	
		When $x \in (3, \infty)$, $f'(x) > 0$	1/2	
		$\therefore f(x)$ is increasing.		
21.		$\text{Area} = \int_2^4 y\,dx$	1	
		$= \int_2^4 \sqrt{9x}\,dx$	1	4
		$= 3 \times \frac{2}{3}\left[x^{3/2}\right]_2^4$	1	
		$= (16 - 4\sqrt{2})$ Sq. units.	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
22		$P = \frac{1}{x}$; $Q = x^2$.	1/2	
		$\therefore I.F = e^{\int P dx} = e^{\int \frac{1}{x} dx}$.	1	
		$= e^{\log \lvert x \rvert} = x$.		
		General soln is.		
		$y \cdot IF = \int (Q \cdot IF)\, dx + C$	1	4
		$y x = \int x^3 dx + C$.	1	
		$= \frac{x^4}{4} + C$.	1/2	
23		$d = \left\lvert \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{\lvert b_1 \times b_2 \rvert} \right\rvert$.	1	
		$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$	1/2	
		$\vec{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$; $\vec{b_2} = 2\hat{i} + 3\hat{j} + k$		
		$\lvert \vec{b_1} \times \vec{b_2} \rvert = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$		4
		$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3-6)$	1	
		$= -9\hat{i} + 3\hat{j} + 9\hat{k}$		
		$\lvert \vec{b_1} \times \vec{b_2} \rvert = \sqrt{81+9+81} = \sqrt{171}$	1/2	
		$d = \left\lvert \frac{(3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (-9\hat{i} + 3\hat{j} + 9\hat{k})}{\sqrt{171}} \right\rvert$	1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$d = \frac{3}{\sqrt{19}}$	1/2	
B 24		$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$	1	4
		$= \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$	1	
		$= \frac{1}{2}\left[2\begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} - 7\begin{vmatrix} 1 & 1 \\ 10 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix}\right]$	1	
		$= \frac{47}{2}$ sq. units.	1	
25.		$A = \int_0^1 [f(x) - g(x)]\,dx.$	1	4
		$= \int_0^1 (\sqrt{x} - x^2)\,dx.$	1	
		$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$	1	
		$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ sq. units.	1	
		Part IV		
A ~~26~~ 26. (i)		$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$\text{LHS} = \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)$	1	
		$= \tan^{-1}\left(\frac{\frac{31}{21}}{\frac{17}{21}}\right)$	½	3
		$= \tan^{-1}\frac{31}{17}$	½	
	(ii)	$\tan^{-1}\sqrt{\frac{2\sin^2 x/2}{2\cos^2 x/2}}$	1	
		$= \tan^{-1}\tan x/2$	1	3
		$= x/2$	1	
2)	(i)	$f(2-) = f(2+)$	1	
		$f(2-) = \lim_{x\to 2} kx^2$; $f(2+) = \lim_{x\to 2} f(2) = 3$.	1	3
		$4k = 3$	½	
		$k = 3/4$	½	
	(ii)	$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0.$	1½	
		$\frac{dy}{dx}[x+2y] = -[2x+y]$	1	3
		$\frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
28	(1)	Put $\log x = t$.	½	
		$\frac{1}{x}dx = dt$.	½	
		$\int \frac{(\log x)^2}{x} dx = \int t^2 dt$	½	3
		$= \frac{t^3}{3} + c = \frac{(\log x)^3}{3} + c$	1 + ½	
	(2)	$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$	½	
		$x = A(x+2) + B(x+1)$.		
		$-2 = -B \quad \therefore B = 2$.	1	
		$-1 = A \quad \therefore A = -1$.		
		$\int \frac{x}{(x+1)(x+2)} dx = \int \frac{-1}{x+1} dx$	½	3
		$+ \int \frac{2}{x+2} dx$.		
		$= -\log\lvert x+1\rvert + 2\log\lvert x+2\rvert + c$.	1	
29.				

x	0	8
y	8	0

x	0	4
y	6	0

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		shaded region OABC is the feasible region.		
		Corner Points | Value of Z (0,0) | 0. (4,0) | -12. (0,4) | 16 (2,3) | 6		
		Minimum Value of Z = -12 at (4,0).		
B. 30	(i)	$\frac{dy}{dx} = -5\sin x - 3\cos x$.	1	
		$\frac{d^2y}{dx^2} = -5\cos x + 3\sin x$.	1	3
		$= -(5\cos x - 3\sin x)$.	1	
		$= -y$.		
		$\therefore \frac{d^2y}{dx^2} + y = 0$.		
7	(ii)	$\log y = \sin x \log x$.	1	
		$\frac{1}{y}\cdot\frac{dy}{dx} = \sin x \times \frac{1}{x} + \log x \cos x$.	1 ½	3
		$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x\right]$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
31	(i)	$f(x) = x$, $a=0$ & $b=5$.		
		$nh = 5$		
		$\int_a^b f(x)dx = \lim_{h\to 0} h[f(a)+f(a+h)$	1	
		$+ \cdots + f(a+\overline{n-1}h)]$.		
		$\int_0^5 x\,dx = \lim_{h\to 0} h[f(0)+f(h)+\cdots$		
		$+ f(n-1)h)]$	1/2	
		$= \lim_{h\to 0} h[h + 2h + \cdots + n-1\,h]$	1/2	4
		$= \lim_{h\to 0} h^2[1+2+\cdots+n-1]$	1/2	
		$= \lim_{h\to 0} h^2 \frac{n(n-1)}{2}$	1/2	
		$= \lim_{h\to 0} nh\frac{(nh-h)}{2}$	1/2	
		$= \frac{5\times 5}{2} = \frac{25}{2}$		
	(ii)	$I = \int(\sec^2 x + \sec x\tan x)dx$	1/2	2
		$= \tan x + \sec x + c$.	1 1/2	
32	(i)	$\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^x$	1	
		$\frac{d^2y}{dx^2} = e^x$.	1/2	2
		$\therefore \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$.	1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
	(ii)	$\frac{dy}{dx} = \frac{x+y}{x}$.		
		Put $y = Vx$.		
		$V + x\frac{dV}{dx} = \frac{x+Vx}{x} = 1+V$.	1	
		$x\frac{dV}{dx} = 1$.		
		$dV = \frac{dx}{x}$.	1	4
		$\int dV = \int \frac{dx}{x}$.	½	
		$V = \log\lvert x\rvert + C$.	1	
		$\frac{y}{x} = \log\lvert x\rvert + C$.	½	
		$y = x\log\lvert x\rvert + Cx$.		
33	(i)	$3A - B = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$	1	2.
		$= \begin{bmatrix} 5 & 9 \\ 11 & 1 \end{bmatrix}$	1	
	(ii).	$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$		
		$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$.	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$kA = A^2 + 2I$		
		$k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$	1	3
		$k = 1$.	1	
	(iii).	$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$	1	
		$P = \frac{A + A^T}{2}$		
		$P = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$ $P^T = P$ is symmetric.	1	
		$Q = \frac{A - A^T}{2}$		3.
		$= \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$		
		$Q^T = -Q$. $\therefore Q$ is symmetric.	1.	
		$\therefore P + Q = A$. Hence the result.		

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34	(i)	$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$		
		$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$	1	
		$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$		
		$= 1 \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$	1	3.
		$= (b-a)(c-a) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$	1	
		$= (a-b)(b-c)(c-a)$		
	(ii)	$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1	
		$\lvert A \rvert = -17 \neq 0$.	1	
		$\text{adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	1	5
		$X = A^{-1}B = \frac{-1}{17} \begin{bmatrix} -1 & 5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1	
		$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix}$ $x = 1, y = 2$ $z = 3$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
35	(i)	$P(A \cap B) = P(B/A)\,P(A)$ $= 0.4 \times 0.8$ $= 0.32$.	2	
	(ii)	$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0.64$	1	4
	(iii)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= 0.98$	1	
	(iv)	Let E_1 and E_2 be the events of selecting the bags I and II respectively. A be the event of drawing a red ball		
		$P(E_1) = P(E_2) = 1/2$	1/2	
		$P(A/E_1) = \frac{4}{8}$	1/2	
		$P(A/E_2) = 2/8$	1/2	
		Using Bayes theorem.		
		$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$	1	4
		$= \frac{\frac{1}{2} \times \frac{4}{8}}{\frac{1}{2} \times \frac{4}{8} + \frac{1}{2} \times \frac{2}{8}}$	1	
		$= \frac{2}{3}$	2	