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COMMON QUARTERLY EXAMINATION 2018

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SUBJECT: BUSINESS MATHEMATICS & STATISTICS

ANSWER KEY

STD: XI **MARKS : 90**

SECTION-I

1.	a	-125	11.	a	25
2.	c	If $AX=B$ then $X=B^{-1}A$	12.	b	$3x+2y+8=0$
3.	a	$k^3\|A\|$	13.	b	0
4.	a	0	14.	b	$-1/2$
5.	a	2^6-2	15.	c	0
6.	b	16	16.	d	$3/5$
7.	c	9	17.	b.	0
8.	b.	13	18.	b.	$-1/x^2$
9.	c	$(x-3)^2+(y+4)^2=16$	19.	d	$f(x)=\sin x+\cos x$
10.	c	5	20.	c	$1/2$

SECTION - 2

21) $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & \lambda & 4 \\ 9 & 7 & 11 \end{bmatrix}$

Since A has no inverse, $|A| = 0$

$$1\begin{vmatrix} \lambda & 4 \\ 7 & 11 \end{vmatrix} - 1\begin{vmatrix} 2 & 4 \\ 9 & 11 \end{vmatrix} + 3\begin{vmatrix} 2 & \lambda \\ 9 & 7 \end{vmatrix} = 0$$

$$\lambda = 28/16$$

$$\lambda = 7/4$$

22) $\begin{vmatrix} x & x+2 \\ x-2 & x \end{vmatrix} = x^2 - (x+2)(x-2)$

$$= x^2 - (x^2 - 4)$$

$$= 4$$

23) $\frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$

$$\frac{1}{9!}\left(1 + \frac{1}{10}\right) = \frac{1}{9!}\left(\frac{n}{10 \times 11}\right)$$

$$\frac{11}{10} = \frac{n}{10 \times 11}$$

$$\boxed{n = 121}$$

24) MISSISSIPPI

There are 11 letters in the word MISSISSIPPI.

I occurs 4 times

S occurs 4 times

P occurs 2 times

The required no. of permutation $= \frac{11!}{4!\,4!\,2!}$.

25) $3x^2 - 5xy - 2y^2 + 17x + y + 10 = 0$

$a = 3 \quad h = -5/2 \quad b = -2$

$$\theta = \tan^{-1}\left[\left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|\right]$$

$$= \tan^{-1}\left[\left|\frac{2\sqrt{\frac{25}{4} + 6}}{3-2}\right|\right]$$

$$\theta = \tan^{-1}(7).$$

26) $(-2, 2)$, $x^2 + y^2 - 4x + 4y - 8 = 0$.

$$xx_1 + yy_1 - 4\left(\frac{x+x_1}{2}\right) + 4\left(\frac{y+y_1}{2}\right) - 8 = 0$$

$$x(-2) + y(-2) - \cancel{4}^{2}\left(\frac{x-2}{\cancel{2}}\right) + \cancel{4}^{2}\left(\frac{y-2}{\cancel{2}}\right) - 8 = 0$$

$$-2x - 2y - 2x + 4 + 2y - 4 - 8 = 0$$

$$-4x - 8 = 0$$

$$\boxed{x + 2 = 0}$$

27) Let $\tan^{-1}\left(\frac{1}{8}\right) = \theta$

$$\tan\theta = \frac{1}{8}$$

$$\tan\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{8}\right)\right) = \tan\left(\pi/4 - \theta\right)$$

$$= \frac{\tan\pi/4 - \tan\theta}{1 + \tan\pi/4 \tan\theta}$$

$$= \frac{1 - 1/8}{1 + 1/8} = 7/9$$

28) $\cos(-105^\circ) = \cos 105^\circ$

$$= \cos(60^\circ + 45^\circ)$$

$$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}.$$

29) $\lim_{x \to a} \frac{x^9 - a^9}{x - a} = \lim_{x \to 3} (x + 6)$

$$9a^{9-1} = 9$$

$$a^8 = 1$$

$$\boxed{a = \pm 1}$$

30) $f = \{(1,1), (2,5)\}$

$f(1) = 1 \quad f(2) = 5$

$f(x) = ax + b \Rightarrow a + b = 1$

$2a + b = 5$

$-a = -4 \Rightarrow a = 4$

$4 + b = 1$

$b = 1 - 4 = -3 \qquad a = 4, \; b = -3$

SECTION-III

(31) $AB = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

$AB^{-1} = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -61 & 87 \\ 47 & -67 \end{bmatrix}$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$ $B^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{2}\begin{bmatrix} -61 & 87 \\ 47 & -67 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$

(32) $I - B = \begin{bmatrix} 0.50 & -0.25 \\ -0.40 & 0.33 \end{bmatrix}$ $B = \begin{bmatrix} 0.50 & 0.25 \\ 0.40 & 0.67 \end{bmatrix}$

The main diagonal elements of $I-B$ is +ve.

$|I-B| = 0.065 > 0$

Hawkin's Simon Condition are satisfied.

$\therefore$ The given system is viable.

(33) There are 15 players in a group, we have to select 11 players from the group.

i) No. of ways $= 15C_{11} = 1365$ ways.

ii) No. of ways $= 14C_{10} = 1001$ ways.

iii) No. of ways $= 14C_{11} = 364$ ways.

(34) HOPE

Arrange in Alphabet order E H O P

E _ _ _	3!	= 6
H E _ _	2!	= 2
H O E _	1!	= 1
H O P E	1	= 1
		10

Rank of Word HOPE is 10

(35) Given the lines are concurrent.

$$\begin{vmatrix} 3 & -5 & -11 \\ 5 & 3 & -7 \\ 1 & k & 0 \end{vmatrix} = 0$$

$34k = 68 \Rightarrow k = 2$

(36) Let m_1 and m_2 be the slope of the pair of straight lines.

$ax^2 + 2hxy + by^2 = 0$.

$m_1 + m_2 = \frac{-2h}{b}$, $m_1 m_2 = a/b$.

Given $m_2 = 2m_1$.

$m_1 = -\frac{2h}{3b}$, $2m_1^2 = a/b$.

$2\left(\frac{-2h}{3b}\right)^2 = a/b$

$8h^2 = 9ab$.

(37) $\tan\alpha = 1/3$ $\tan\beta = 1/7$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2 \times 1/3}{1 - 1/9} = 3/4$

$\tan(2\alpha+\beta) = \frac{\tan 2\alpha + \tan\beta}{1 - \tan 2\alpha \tan\beta} = \frac{\frac{3}{4} + \frac{1}{7}}{1 - \left(\frac{3}{4}\right)\left(\frac{1}{7}\right)}$

$= \frac{25/28}{25/28} = 1 = \tan \pi/4$

$\boxed{2\alpha + \beta = \pi/4}$

(38)

$(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

$= \left(-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right)^2 + \left(2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right)^2$

$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right)\left[\sin^2\left(\frac{\alpha+\beta}{2}\right) + \cos^2\left(\frac{\alpha+\beta}{2}\right)\right]$

$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right)$

(39) $f(x) = \frac{x+1}{x-1}$

$f(f(x)) = \frac{f(x)+1}{f(x)-1} = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$

$= \frac{2x}{2}$

$f(f(x)) = x$.

(40) $x = \log t \quad y = \sin t$

$$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{1/t} = t\cos t.$$

SECTION-IV

(41) Let x, y and z be the
(a) Cost of onion, wheat and Rice per kg.

$$4x + 3y + 2z = 320$$
$$2x + 4y + 6z = 560$$
$$6x + 2y + 3z = 380$$

$X = A^{-1}B. \quad A^{-1} = \frac{1}{|A|} \text{adj} A$

$|A| = 50$

$$A^{-1} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$X = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 320 \\ 560 \\ 380 \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ 60 \end{bmatrix}$$

Cost of 1kg onion ₹. 20
Cost of 1kg wheat ₹. 40
Cost of 1kg Rice ₹. 60

(b) $B = \begin{bmatrix} 3/7 & 2/13 \\ 4/7 & 6/13 \end{bmatrix}$

$$I - B = \begin{bmatrix} 4/7 & -2/13 \\ -4/7 & 7/13 \end{bmatrix}$$

$|I-B| = 20/91$ The given System is viable.

$$(I-B)^{-1} = \frac{1}{|I-B|} \text{adj}(I-B)$$

$$= \frac{91}{20}\begin{bmatrix} 7/13 & 2/13 \\ 4/7 & 4/7 \end{bmatrix}$$

$$X = (I-B)^{-1}D$$

$$= \frac{91}{20}\begin{bmatrix} 7/13 & 2/13 \\ 4/7 & 4/7 \end{bmatrix}\begin{bmatrix} 12 \\ 18 \end{bmatrix} = \begin{pmatrix} 42 \\ 78 \end{pmatrix}$$

The Gross output of $X = 42$ and $Y = 78$.

(42)
(a) $P(n): 1^2 + 2^2 \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Step: 1 put $n = 1$

L.H.S = 1 R.H.S = 1

$P(1)$ is true.

Step: 2 Assume that $P(k)$ is true.

$$P(k): 1^2 + 2^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Step: 3: To prove $P(k+1)$ is true.

$$1^2 + 2^2 + \ldots + k^2 + (k+1)^2 = P(k) + (k+1)^2$$
$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$P(k+1)$ is true whenever $p(k)$ is true.

$\therefore P(n)$ is true for all $n \in N$.

(b) $$\frac{x+4}{(x^2-4)(x+1)} = \frac{x+4}{(x+2)(x-2)(x+1)}$$
$$= \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+1}$$

$$x + 4 = A(x+2)(x+1) + B(x-2)(x+1) + C(x-2)(x+2)$$

put $x = -2 \Rightarrow B = 1/2$

put $x = 2 \Rightarrow A = 1/2$

put $x = -1 \Rightarrow C = -1$

$$\frac{x+4}{(x^2-4)(x+1)} = \frac{1}{2(x-2)} + \frac{1}{2(x+2)} - \frac{1}{x+1}.$$

43 (a) Equation of Circle be

$$x^2+y^2+2gx+2fy+c=0.$$

The Circle passes through the point (0,0)

$$\boxed{c=0}$$

The Circle passes through the point (1,2)

$$2g+4f+c=-5$$

The circle passes through the point (2,0)

$$4g+c=-4$$

Solve these eqns, we get

$$g=-1,\ f=-3/4,\ c=0.$$

Circle Eqn. is

$$\boxed{2x^2+2y^2-4x-3y=0}.$$

43 (b) $y^2-8y-8x+24=0$

$$(y-4)^2=8(x-1)$$

$Y^2=8X$. It is parabola open Right.

$x=X+1,\ y=Y+4.$

$\boxed{a=2}$

	X, Y	$x=X+1,\ y=Y+4$
Axis	$Y=0$	$y=4$
Vertex	$V(0,0)$	$V(1,4)$
Focus	$F(2,0)$	$F(3,4)$
Eqn. of Directrix	$X=-2$	$x=-1$
Length of Latus Rectum	$4a=8$	8

44 (a) $4x^2+12xy+9y^2-6x-9y+2=0$

$a=4\quad b=9\quad h=6$

$h^2-ab=0.$

The given equation represents a pair of parallel straight lines.

$$4x^2+12xy+9y^2=(2x+3y)^2=z^2$$

$$4x^2+12xy+9y^2-6x-9y+2=0$$

$$(2x+3y)^2-3(2x+3y)+2=0$$

$$z^2-3z+2=0$$

$$(z-1)(z-2)=0$$

The seperate equations are

$2x+3y-1=0,\quad 2x+3y-2=0.$

(b)

$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$$=\sin 60^\circ\left(\sin 20^\circ \sin(60-20^\circ)\sin(60+20^\circ)\right)$$

$$=\frac{\sqrt3}{2}\sin 20^\circ\left(\sin^2 60^\circ-\sin^2 20^\circ\right)$$

$$=\frac{\sqrt3}{2}\sin 20^\circ\left(\frac{3}{4}-\sin^2 20^\circ\right)$$

$$=\frac{\sqrt3}{2}\left(\frac14\right)\left(3\sin 20^\circ-4\sin^3 20^\circ\right)$$

$$=\frac{\sqrt3}{8}\sin 60^\circ=\frac{\sqrt3}{8}\left(\frac{\sqrt3}{2}\right)$$

$$=3/16.$$

45 (a)

$$\frac{\sin(180^\circ+A)\cos(90^\circ-A)\tan(270^\circ-A)}{\sec(540^\circ-A)\cos(360^\circ+A)\operatorname{cosec}(270^\circ+A)}$$

$$=\frac{(-\sin A)(\sin A)(\cot A)}{(-\sec A)(\cos A)(-\sec A)}$$

$$=\frac{-\sin A\ \sin A\ \frac{\cos A}{\sin A}}{\left(\frac{-1}{\cos A}\right)\cos A\left(-1/\cos A\right)}$$

$$=-\sin A\cos^2 A.$$

(b) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right).$

$\tan^{-1}(x+1)+\tan^{-1}(x-1)$

$$=\tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right)=\tan^{-1}\left(\frac{2x}{1-(x^2-1)}\right)$$

$$=\tan^{-1}\left(\frac{2x}{2-x^2}\right).$$

$$\tan^{-1}\left(\frac{2x}{2-x^2}\right)=\tan^{-1}(4/7)\Rightarrow\frac{2x}{2-x^2}=4/7$$

$$2x^2+7x-4=0\Rightarrow(2x-1)(x+4)=0$$

$\boxed{x=1/2}$ (or) $x=-4$ (not possible)

46 (a) $y = \left(\dfrac{(x-1)(x-2)}{(x-3)(x^2+x+1)}\right)^{1/2}$

$\log y = \frac{1}{2}\log\left(\dfrac{(x-1)(x-2)}{(x-3)(x^2+x+1)}\right)$

$\log y = \frac{1}{2}\left[\log(x-1)+\log(x-2)-\log(x-3) - \log(x^2+x+1)\right]$

Diff. w.r.t. to x.

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{2}\left[\dfrac{1}{x-1}+\dfrac{1}{x-2}-\dfrac{1}{x-3}-\dfrac{(2x+1)}{x^2+x+1}\right]$

$\dfrac{dy}{dx} = \dfrac{1}{2}\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x^2+x+1)}}\left[\dfrac{1}{x-1}+\dfrac{1}{x-2}-\dfrac{1}{x-3} - \dfrac{(2x+1)}{x^2+x+1}\right].$

(b) $f(x) = e^{3x}$ $f(x+h) = e^{3(x+h)}$

$\dfrac{d}{dx}(f(x)) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$\dfrac{d}{dx}(e^{3x}) = \lim_{h\to 0} \dfrac{e^{3x}\cdot e^{3h} - e^{3x}}{h}$

$= e^{3x} \lim_{h\to 0} \dfrac{e^{3h}-1}{\frac{3h}{3}}$

$= 3e^{3x} \lim_{h\to 0} \dfrac{e^{3h}-1}{3h} = 3e^{3x}(1)$

$= 3e^{3x}$

47 (a) $(1+x)^{2n}$

$T_{r+1} = nC_r x^{n-r} a^r$

Middle term:

$T_{n+1} = 2nC_n (1)^{2n-n}(x)^n = 2nC_n x^n.$

$= \dfrac{2n(2n-1)(2n-2)\cdots 4\cdot 3\cdot 2\cdot 1}{n!\,n!} x^n$

$= \dfrac{2^n\, n!\,(2n-1)(2n-3)\cdots 3\cdot 1}{n!\,n!} x^n$

$= \dfrac{1\cdot 3\cdot 5\cdots(2n-3)(2n-1)}{n!} 2^n x^n.$

(b) $A+B = 45^\circ$

$\tan(A+B) = 1$

$\tan A + \tan B = 1 - \tan A \tan B$

Adding 1 on both sides.

$1 + \tan A + \tan B + \tan A \tan B = 2$

$(1+\tan A)(1+\tan B) = 2$

put $A = B = 22\frac{1}{2}^\circ$

$(1+\tan 22\frac{1}{2}^\circ)^2 = 2$

$1 + \tan 22\frac{1}{2}^\circ = \pm\sqrt{2}$

$\tan 22\frac{1}{2}^\circ = \pm\sqrt{2} - 1.$

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