Common Quarterly Exam - Sep- 2019

12- Business maths & Statistics Key

PART-I

Choose the best answer :-

1. (b) n
2. (a) a unique Solution (ஒரேஒரு தீர்வு உண்டு)
3. (c) 6
4. (c) 0.4
5. (d) $\log(e^x+1)+c$
6. (a) 5040
7. (b) $\frac{7!}{5^8}$
8. (b) 0
9. (a) 2
10. (b) $\frac{9}{2}$ units.
11. (b) $\frac{8}{3}$ sq.units
12. (b) $35x+\frac{7x}{2}-x^2$
13. (d) $e^{\int p\,dy}$
14. $\frac{d^2y}{dx^2}-y=0$
15. (c) $y^2dx+(x^2-xy-y^2)dy=0$
16. (a) $A+Be^x$
17. (a) $\Delta^{m+n} f(x)$
18. (a) $y_2-2y_1+y_0$
19. (c) $f(a)-f(a-h)$
20. (b) $2x+3$

PART-II

Two Mark Questions Key:-

21. $|A| = \begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix}_{2\times2} = 8-8=0$

$\therefore \rho(A) \neq 2$

Consider a first order minor $|1| \neq 0$

$\therefore \rho(A)=1$

22. $T = \begin{array}{c c} & \begin{array}{cc} S & H \end{array} \\ \begin{array}{c} S \\ H \end{array} & \begin{pmatrix} 4/5 & 1/5 \\ 1/3 & 2/3 \end{pmatrix} \end{array}$ where $S+H=1$

At equilibrium,

$(S \quad H)T = (S \quad H)$

$(S \quad H)\begin{pmatrix} 4/5 & 1/5 \\ 1/3 & 2/3 \end{pmatrix} = (S \quad H)$

$\frac{4}{5}S + \frac{1}{3}H = S \Rightarrow S = 5/8 \quad \therefore H = \frac{3}{8}$

23. $\int_0^{\pi/2} \cos^2 x\,dx = \frac{1}{2}\int_0^{\pi/2}(1+\cos 2x)dx$

$= \frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\pi/2}$

$= \frac{1}{2}\left[\frac{\pi}{2}+0\right]$

$= \frac{\pi}{4}$

24. $\int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + c$

$= (x-1)e^x + c$

25. Area $= \int_1^4 y\,dx$

$= \int_1^4 (4x+3)dx = [2x^2+3x]_1^4$

$= 32+12-2-3$

$A = 39$ sq.units.

(26). $R = \int (2x^2+6x-5)\,dx + k$

$= \left(\frac{2x^3}{3} + \frac{6x^2}{2} - 5x\right) + k$

$R = \frac{2x^3}{3} + 3x^2 - 5x + k$

Since $R=0$, when $x=0$, $k=0$

$R = \frac{2x^3}{3} + 3x^2 - 5x$

Demand function $p = \frac{R}{x}$

$p = \frac{2x^2}{3} + 3x - 5$

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(27) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\int \frac{\sec^2 x}{\tan x}\,dx + \int \frac{\sec^2 y}{\tan y}\,dy = \int 0$

$\log \tan x + \log \tan y = \log C$

$\log(\tan x \cdot \tan y) = \log C$

$\tan x \tan y = c$

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(28) $\Delta^2 e^x = \Delta[\Delta e^x]$

$= \Delta[e^{x+h} - e^x]$

$= \Delta[e^x(e^h - 1)]$

$= (e^h - 1)\Delta e^x$

$= (e^h - 1)(e^h - 1)e^x$

$\Delta^2 e^x = (e^h - 1)^2 e^x$

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(29). $\Delta^4 U_0 = (E-1)^4 U_0$

$= (E^4 - 4E^3 + 6E^2 - 4E + 1)U_0$

$= E^4 U_0 - 4E^3 U_0 + 6E^2 U_0 - 4EU_0 + U_0$

$= U_4 - 4U_3 + 6U_2 - 4U_1 + U_0$

$= 29 - 4(28) + 6(21) - 4(11) + 1$

$= 0$

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(30). $(D^2 + D)y = 0$

The A.E is $m^2 + m = 0$

$m(m+1) = 0$

$m = 0,\ m = -1$

$\therefore CF = Ae^{0x} + Be^{-1x}$

$CF = A + Be^{-x}$; $P.I = 0$

$\therefore y = A + Be^{-x}$

PART – III

Three mark Questions Key:

(31) $I = \int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x + \cos^7 x}\,dx$ — (1)

$I = \int_0^{\pi/2} \frac{\sin^7(\frac{\pi}{2} - x)}{\sin^7(\frac{\pi}{2} - x) + \cos^7(\frac{\pi}{2} - x)}\,dx$

$I = \int_0^{\pi/2} \frac{\cos^7 x}{\cos^7 x + \sin^7 x}\,dx$ — (2)

(1) + (2) ⇒

$2I = \int_0^{\pi/2} dx = (x)_0^{\pi/2} = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

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(32) $\frac{Ey}{Ex} = \frac{-7x}{(1-2x)(2+3x)}$

$\frac{x}{y}\frac{dy}{dx} = \frac{-7x}{(1-2x)(2+3x)}$

$\int \frac{dy}{y} = \int \frac{7\,dx}{(2x-1)(3x+2)}$

$\log y = 2\int \frac{1}{2x-1}\,dx - 3\int \frac{dx}{3x+2}$

$\log y = \frac{2}{2}\log(2x-1) - \frac{3}{3}\log(3x+2) + \log c$

$\log\left(\frac{y}{c}\right) = \log\left(\frac{2x-1}{3x+2}\right)$

$$\frac{y}{c} = \frac{2x-1}{3x+2}$$

$$y = c\left(\frac{2x-1}{3x+2}\right)$$

when $x=2$, $y=\frac{3}{8}$

then $c=1$

$$\therefore y = \frac{2x-1}{3x+2}$$

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(33). $y = ae^{4x} + be^{-x} \rightarrow ①$

$y' = 4ae^{4x} - be^{-x} \rightarrow ②$

$y'' = 16ae^{4x} + be^{-x} \rightarrow ③$

$y'' =$

$① + ② \Rightarrow y + y' = 5ae^{4x}$

$② + ③ \Rightarrow y' + y'' = 20ae^{4x}$

$y' + y'' = 4(y + y')$

$y'' - 3y' - 4y = 0$

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(34).

$$x\frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} = \frac{x+y}{x} \quad —①$$

put $y = vx$ then

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ in ①

$$\therefore ① \Rightarrow v + x\frac{dv}{dx} = \frac{x+vx}{x}$$

$$\Rightarrow x\frac{dv}{dx} = 1$$

$$\int dv = \int \frac{1}{x}\,dx$$

$v = \log x + \log c$

$e^{v} = cx$

$x = ce^{y/x}$

(35.) $x + y = 5$

$2x + y = 8$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1$$

$$\Delta x = \begin{vmatrix} 5 & 1 \\ 8 & 1 \end{vmatrix} = 5 - 8 = -3$$

$$\Delta y = \begin{vmatrix} 1 & 5 \\ 2 & 8 \end{vmatrix} = 8 - 10 = -2$$

$$x = \frac{\Delta x}{\Delta} = \frac{-3}{-1} = 3$$

$$y = \frac{\Delta y}{\Delta} = \frac{-2}{-1} = 2$$

(ஒருங்கமைவு உடையது ஒரே ஒரு தீர்வு உண்டு)

consistent and it has unique solution.

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(36). $\int_{-\pi/2}^{\pi/2} \cos x \, dx$

$f(x) = \cos x$

$f(-x) = \cos(-x) = \cos x = f(x)$

f is even.

$$\therefore \int_{-\pi/2}^{\pi/2} \cos x\,dx = 2\int_{0}^{\pi/2} \cos x\,dx$$

$$= 2(\sin x)_0^{\pi/2}$$

$$= 2[\sin \pi/2 - \sin 0]$$

$$= 2.$$

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(37). $A = \int_1^2 x\,dx$

$$= \left(x^2/2\right)_1^2 = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

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(38). $\Delta^4 y_0 = 0 \therefore (E-1)^4 y_0 = 0$

$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1) y_0 = 0$

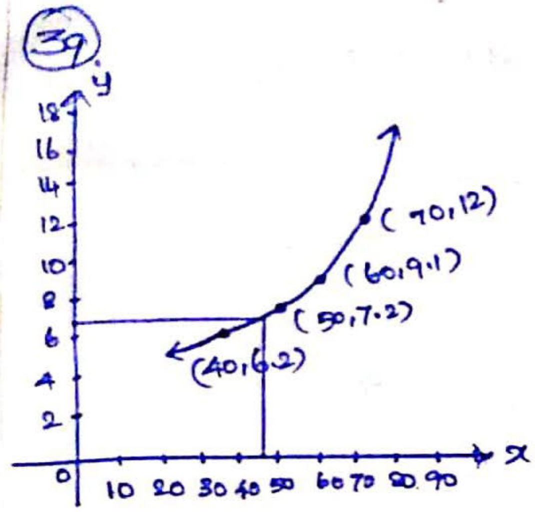
$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$256 - 4y_3 + 96 - 16 + 1 = 0$

$-4y_3 = -337$

$y_3 = 84.25$

(39)



From the Graph
when $x=48$, $y=6.8$.

(40) $\Delta = -33 \neq 0$

$\Delta x = -33$

$\Delta y = -66$

$\Delta z = -99$

By Cramer's Rule,

$x = \frac{\Delta x}{\Delta} = \frac{-33}{-33} = 1$

$y = \frac{\Delta y}{\Delta} = \frac{-66}{-33} = 2$

$z = \frac{\Delta z}{\Delta} = \frac{-99}{-33} = 3$

PART-IV

Five Marks Question Key:

(41) (a)

$AX = B$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

$$(A|B) = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 6 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 8 & 3 \end{pmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix} R_3 \to R_3 - 3R_2$$

From the last equivalent matrix is in echelon form.

$\therefore \rho(A) = 3$; $\rho(A|B) = 3$ and $n = 3$

$\therefore \rho(A|B) = \rho(A) = n = 3$

$\therefore$ The system is consistent and has unique solution.

$$\therefore \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \left.\begin{matrix} x+y+z=3 \\ y+2z=1 \\ 2z=0 \end{matrix}\right\} \Rightarrow \begin{matrix} x=2 \\ y=1 \\ z=0. \end{matrix}$$

(41) (b).

$$\int \frac{3x+2}{(x-2)(x-3)}dx. = \int\left[\frac{-8}{x-2} + \frac{11}{x-3}\right]dx.$$

$= 11\log(x-3) - 8\log(x-2)$

$= \log\left[(x-3)^{11}/(x-2)^{8}\right] + C.$

(42) (a) ([illegible])

At market equilibrium, $P_d = P_s$

$25 - 3x = 5 + 2x \Rightarrow x = 4$

$\therefore x_0 = 4$

when $x_0 = 4$, $P_0 = 25 - 3(4) = 13$

$\therefore P_0 x_0 = 52$

$CS = \int_0^{x_0} f(x)dx - P_0x_0 = \int_0^4 (25-3x)dx - 52$

$CS = \left[25x - \frac{3x^2}{2}\right]_0^4 - 52 = 24$ units.

$PS = P_0x_0 - \int_0^{x_0} g(x)dx = 52 - \int_0^4 (5+2x)dx$

$PS = 52 - \left[5x + x^2\right]_0^4 = 16$

$\therefore CS = 24$ Units

$PS = 16$ Units.

42) (b)

$y^2dx + (xy + x^2)dy = 0$

$\frac{dy}{dx} = \frac{-y^2}{xy+x^2}$ —— (1)

put $y = vx$ and

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (1),

$v + x\frac{dv}{dx} = \frac{-v^2x^2}{xvx + x^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{-(v+2v^2)}{1+v}$

$\int \frac{1+v}{v(1+2v)}dv = \int -\frac{dx}{x}$

$\int\left(\frac{1}{v} - \frac{1}{1+2v}\right)dv = -\int\frac{dx}{x}$

$\Rightarrow \log\left(\frac{v}{\sqrt{1+2v}}\right) = \log\left(\frac{c}{x}\right)$

$\Rightarrow \frac{v}{\sqrt{1+2v}} = \frac{c}{x}$

$\Rightarrow \frac{y^2x}{x+2y} = k$ where $k = c^2$

43) (a) Given: $Q_d = Q_s$

$13 - 6p + 2\frac{dp}{dt} + \frac{d^2p}{dt^2} = -3 + 2p$

$\frac{d^2p}{dt^2} + 2\frac{dp}{dt} - 8p = -16$

The A.E is $m^2 + 2m - 8 = 0$

$(m+4)(m-2) = 0$

$m = -4, 2$

$\therefore CF = Ae^{-4t} + Be^{2t}$

$P.I = \frac{1}{D^2+2D-8}(-16) = \frac{-16}{-8} = 2$

$\therefore p = Ae^{-4t} + Be^{2t} + 2.$

43) (b) Formulae

$y = \frac{(8)(5)(-2)}{(-3)(-6)(-13)}(30) + \frac{(11)(5)(-2)}{(3)(-3)(-10)}(33)$

$+ \frac{(11)(8)(-2)}{(6)(3)(-7)}(37) + \frac{(11)(8)(5)}{(13)(10)(7)}(40)$

$= 10.26 - 40.33 + 51.68 + 19.34$

$y = 40.95$

44) (a) Given:

$3e^x \tan y\, dx + (1+e^x)\sec^2 y\, dy = 0$

$\int \frac{3e^x}{1+e^x}dx + \int \frac{\sec^2 y}{\tan y}dy = \int 0$

$\log(1+e^x)^3 + \log\tan y = \log C$

$\log[(1+e^x)^3 \tan y] = \log C$

$(1+e^x)^3 \tan y = C$ —— (1)

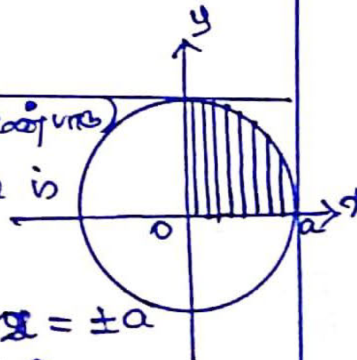
$y(0) = \pi/4 \Rightarrow (1+e^0)^3 \tan\frac{\pi}{4} = C$

$2^3 = C$

$C = 8$

$\therefore (1+e^x)^3 \tan y = 8$

44) (b) (வட்டத்தின் பரப்பு)



Equation of a circle is

$x^2 + y^2 = a^2$

In x-axis $y = 0$ $\therefore x = \pm a$

Area of a circle $= 4\int_0^a y\,dx$

$A = \int_0^a \sqrt{a^2 - x^2}\,dx$

$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$

$= 4\frac{a^2}{2}\cdot\frac{\pi}{2}$

$A = \pi a^2$ Sq. units.

45 (a) (மாறுதல் நிகழ்தகவு அணி)

Transition probability matrix is

$$T = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} \end{array}$$

(ஓராண்டுக்கு பின் சந்தை பங்கீடு)
Percentage after one year is

$$(0.15 \quad 0.85)\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix}$$

$$= (0.48 \quad 0.52)$$

$\therefore A = 48\% \quad B = 52\%$

At equilibrium, (சமநிலையில்)

$(A \quad B)T = (A \quad B)$

$$(A \quad B)\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} = (A \quad B)$$

$0.65A + 0.45B = A$

$0.65A + 0.45(1-A) = A$

$\therefore 0.45 = 0.8A$

$A = \dfrac{0.45}{0.8}$

$A = 56.25$

$\therefore B = 1 - A = 1 - 56.25$

$B = 43.75\%$

45 (b)

$$\int_1^2 (2x+5)\,dx = \int_a^b f(x)\,dx$$

$\Rightarrow a = 1 \ ; \ b = 2 \ ; \ h = \dfrac{1}{n}$

$f(a+rh) = 7 + \dfrac{2r}{n}$

$$\int_1^2 (2x+5)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left[7 + \frac{2r}{n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{7}{n}\Sigma_1(1) + \frac{2}{n^2}\Sigma_1(r)\right]$$

$$= \lim_{n\to\infty}\left[\frac{7}{n}(n) + \frac{2}{n^2}\frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty}\left[8 + \frac{1}{n}\right] = 8$$

46 (a) formulae

$x_0 = 1891, \ h = 10, \ n = 1.4$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	98752				
		35333			
1901	132285		2258		
		35791		-10435	
1911	168076		-8177		41376
		27614		30941	
1921	195680		22764		
		50360			
1931	246050				

$y = 98725 + 46946.2 + 639.8 + 584.36 + 1390.23$

$y = 148312.59$

$y = 148313.$

46 (b).

$$(3D^2 + D - 14)y = 4 - 13e^{-\frac{7}{3}x}$$

The A.E is

$3m^2 + m - 14 = 0$

$(3m+7)(m-2) = 0$

$m = \dfrac{-7}{3}, 2$

$\therefore CF = Ae^{-7/3x} + Be^{2x}$

$$P.I_1 = \frac{1}{3D^2+D-14}\cdot 4 = \frac{4}{-14} = \frac{-2}{7}$$

$$P.I_2 = \frac{1}{3D^2+D-14}(-13)\cdot e^{-7/3x}$$

$$= \frac{1}{(3D+7)(D-2)}(-13)e^{-7/3x}$$

$$= \frac{1}{3(D+\frac{7}{3})(D-2)}(-13)e^{-7/3x}$$

$$= \frac{x\cdot 1}{3(\frac{-7}{3}-2)}(-13)e^{-7/3x}$$

$P.I_2 = xe^{-7/3x}$

$\therefore y = Ae^{-7/3x} + Be^{2x} - \dfrac{2}{7} + xe^{-7/3x}$

47 (a) Given: $C'(x) = 50 + \frac{x}{50}$

$\therefore C(x) = \int\left(50 + \frac{x}{50}\right)dx$

$C(x) = 50x + \frac{x^2}{100} + K_1$

When $x = 0$, $C = 200$ $\therefore K_1 = 200$

$\therefore C(x) = 50x + \frac{x^2}{100} + 200 \rightarrow (1)$

Given: $R'(x) = 60$

$\therefore R(x) = \int 60\,dx = 60x + K_2$

Since, $x = 0$, $R = 0$ $\therefore K_2 = 0$

$\therefore R(x) = 60x \rightarrow (2)$

Profit $= R(x) - C(x)$

$= 60x - 50x - \frac{x^2}{100} - 200$

$P = 10x - \frac{x^2}{100} - 200$

$\frac{dP}{dx} = 10 - \frac{x}{50}$; $\frac{d^2P}{dx^2} = -\frac{1}{50} < 0$

$\frac{dP}{dx} = 0 \Rightarrow 10 - \frac{x}{50} = 0 \Rightarrow x = 500$

$\therefore$ Maximum Profit $= P(500)$

$= 10(500) - \frac{(500)^2}{100} - 200$

$=$ Rs. 2300

47 (b) Given

$\frac{dy}{dx} - 3y\cot x = \sin 2x$.

The above is

$\frac{dy}{dx} + Py = Q$

$\therefore P = -3\cot x$; $Q = \sin 2x$

$\therefore \int P\,dx = \int -3\cot x\,dx$

$\int P\,dx = \log \frac{1}{\sin^3 x}$

$\therefore I.F = \frac{1}{\sin^3 x}$

The solution is

$y(I.F) = \int Q.(I.F)\,dx + C$

$y\left(\frac{1}{\sin^3 x}\right) = \int \sin 2x \cdot \frac{1}{\sin^3 x}\,dx$

$= 2\int \operatorname{cosec} x \cdot \cot x\,dx$

$\frac{y}{\sin^3 x} = -2\operatorname{cosec} x + C \rightarrow (1)$

Now, $y = 2$ when $x = \pi/2$

$\therefore (1) \Rightarrow 2 = -2 + C$

$C = 4$

$\therefore (1) \Rightarrow y\left(\frac{1}{\sin^3 x}\right) = -2\operatorname{cosec} x + 4$.

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