

காலாண்டு தேர்வு = 2019

வகுப்பு-12 கணிதம்
வினாக்களுக்கீழ்

- I
- 1) b) 1 (BB)
 - 2) b) $\frac{\pi}{4}$ (BB) (சு.காந்தத்திசையை நிகுவாண்டு)
 - 3) a) 2 (BB)
 - 4) a) 3 (சகா. 2.11)
 - 5) c) $\frac{-9}{8}$ (BB)
 - 6) c) $(0, \pi) \setminus \{\frac{\pi}{2}\}$
 - 7) d) 10 (BB)
 - 8) c) $2\sqrt{3}$ (BB)
 - 9) a) $k \neq 0$
 - 10) c) ± 1

11) $\text{Re}(z) = \text{Im}(z)$
 $z = a + ia$
 $z^2 = (a+ia)(a+ia)$
 $= a^2 - a^2 + i(a^2 + a^2)$
 $z^2 = 0 + i(2a^2)$

a) $\text{Re}(z^2) = 0$

12) a) ஆடு சதுரத்தை அமைக்கும்

13) $\sin 2\alpha = 2 \sin \alpha \cos \alpha$
 $= 2 \alpha \beta$
 $= 2 \left(\frac{k}{a} \right)$
 $= 2 \left(\frac{-12}{25} \right)$
 c) $= \frac{-24}{25}$

14) $\Delta = B^2 - 4AC$
 $= (2a+b)^2 - 4(2a)(b)$

$$= 4a^2 + b^2 + 4ab - 8ab$$

$$= 4a^2 + b^2 - 4ab$$

$$\Delta = (2a-b)^2 > 0$$

சூடுவாக்கம்

a) விகிதமுறு எண்கள்

15) $4 \cos^2 x + \sin^2 x = \pi$

$$3 \cos^2 x + \cos^2 x + \sin^2 x = \pi$$

$$3 \cos^2 x + 1 = \pi$$

$$3 \cos^2 x = \pi - 1$$

$$\cos^2 x = \frac{\pi - 1}{3}$$

$$x = \cos^{-1} \sqrt{\frac{\pi - 1}{3}}$$

c) $\frac{\sqrt{3}}{2}$

16) $-1 \leq 2x - 1 \leq 1$

$$0 \leq 2x \leq 2$$

$$0 \leq x \leq 1$$

a) $[0, 1]$

17) $x = -\frac{9}{5}$ $a^2 = 9$ $a = 3$
 $b^2 = 16$

c) $(-5, 0)$

$$e = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

$$c = ae = 5$$

$$\text{கூலியம் } (\pm ae, 0) = (\pm 5, 0)$$

18) $a = b$ $(-g, -f) = (\frac{1}{2}, \frac{1}{2})$
 $2a - 4 = 0$
 $a = 2 = b$

a) $\frac{1}{2}, \frac{1}{2}$

19) $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

$$= [\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]^2$$

$$([\vec{a} \vec{b} \vec{c}])^2 = [\vec{a} \vec{b} \vec{c}]^4$$

a) $[\vec{a} \vec{b} \vec{c}]^4$

20) $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ $(1, 0, 0)$

எசும்கூறு எவகூறு $(2\lambda) \hat{i} + (-3\lambda - 1) \hat{j} + (8\lambda - 10) \hat{k}$

$$\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$$

$$\vec{b} \cdot \vec{d} = 0$$

$$4\lambda + 9\lambda + 3 + 64\lambda - 80 = 0$$

$$77\lambda - 77 = 0$$

$$\lambda = 1$$

அடிப்பள்ளி $(2+1, -3-1, (8-10))$

$$= (3, -4, -2)$$

a) $3, -4, -2$

II) 21) $|adj A| = 9 \rightarrow ①$
 $A^{-1} = \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \rightarrow ①$

22) $i \left(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6} \right)^{18} \rightarrow ①$
 $= -1 (\cos 3\pi - i \sin 3\pi)$
 $= -1(-1 - i0) = 1 \rightarrow ①$

23) $x = \left(\frac{2}{3}\right)^{1/4} \rightarrow ①$
 $x^4 = \frac{2}{3} \quad 3x^4 - 2 = 0 \rightarrow ①$

24) $\tan^{-1} \tan(108^\circ) = \tan^{-1} \tan(180 - 72^\circ)$
 $= \tan^{-1} \tan(-72^\circ)$
 $= -72^\circ = -\frac{2\pi}{5} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

25) $y_1 = mx + c$
 $yy_1 = -xx_1 + a^2$
 $\frac{y_1}{1} = \frac{-x_1}{m} = \frac{a^2}{c} \rightarrow ①$
 $c = \pm a \sqrt{1+m^2} \rightarrow ①$

26) $\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & m & 4 \end{vmatrix} = 0 \rightarrow ① \quad m = -3 \rightarrow ①$

27) $\bar{z} = (2 - i\sqrt{3})^{10} - (2 + i\sqrt{3})^{10} \rightarrow ①$
 $\bar{z} = -z$
 இருவதும் கற்பனையானவை $\rightarrow ①$

28) $p(x)$ இதுபொருள் = 1
 மிகைகூடுதல் = 1 $\rightarrow ①$
 $p(-x)$ இதுபொருள் = 1
 குறைகூடுதல் = 1 $\rightarrow ①$
 சமச்சாரம் $\rightarrow ①$

29) $\cot\left(\sin^{-1} \frac{3}{5} + \cos^{-1} \frac{3}{5}\right) \rightarrow ①$
 $= \cot \pi/2 = 0 \rightarrow ①$

30) $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \rightarrow ①$
 $-bc + 2ac - ab = 0$
 $2ac = ab + bc$
 $\frac{2}{b} = \frac{1}{c} + \frac{1}{a}$
 a, b, c HP ன் இடத்தில்.

III) 31) $[A|I] = \left[\begin{array}{cc|cc} 0 & 5 & 1 & 0 \\ -1 & 6 & 0 & 1 \end{array} \right] \rightarrow ①$
 $= \frac{1}{5} \begin{bmatrix} 6 & -5 \\ 1 & 0 \end{bmatrix} \rightarrow ②$

32) $\begin{cases} x+y=100 \\ 4x-y=320 \end{cases} \rightarrow ①$
 $x=84, y=16 \rightarrow ②$

33) மிகை $\rightarrow ①$
 நிழையதல் $\rightarrow ②$

34) சமச்சாரம் $\frac{a}{\lambda}, \alpha, \alpha\lambda$ சமச்சாரம் $\rightarrow ①$
 $\frac{a}{\alpha} = \frac{\alpha}{\alpha\lambda} \Rightarrow \alpha^2 = a\lambda$
 $\alpha = \frac{a}{\lambda} \rightarrow ①$
 $ac^3 = db^3 \rightarrow ①$

35) $-1 \leq 2-3x^2 \leq 1 \rightarrow ①$
 $x^2 \geq \frac{1}{3} \rightarrow ①$
 $x \in \left[-1, -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, 1\right] \rightarrow ①$

36) $(x+1)^2 = 4a(y+2) \rightarrow ①$
 $a = \frac{1}{4} \rightarrow ①$
 (or) $x^2 + 2x - 2y - 3 = 0 \quad (x+1)^2 = 2(y+2) \rightarrow ①$

37) t_1 இல் $at_1^2, 2at_1$ சமச்சாரம் $\rightarrow ①$
 $y + xt_1 = 2at_1 + at_1^3 \rightarrow ①$
 $(at_2^2, 2at_2)$ சமச்சாரம் $\rightarrow ①$
 $t_2 = -\left(t_1 + \frac{2}{t_1}\right) \rightarrow ①$

38) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{2}\vec{b} \rightarrow ①$
 $\vec{a} \cdot \vec{c} = \frac{1}{2} \rightarrow ①$
 $\theta = \pi/3 \rightarrow ①$

39) $\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$
 $(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d}) = 0 \rightarrow ①$
 இவை நேர செங்குதானவை, தூரம் = 0 $\rightarrow ①$

40) $\frac{1}{2} \cos^{-1} \frac{a}{b} = \theta \rightarrow ①$
 $\cos^{-1} \frac{a}{b} = 2\theta$
 $\frac{a}{b} = \cos 2\theta$
 $\sec 2\theta = \frac{b}{a}$
 $\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta} = \frac{2(1+\tan^2\theta)}{1-\tan^2\theta} = \frac{2 \sec^2\theta}{\cos^2\theta - \sin^2\theta} = \frac{2}{\cos 2\theta} = 2 \sec 2\theta$
 $LHS = \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = \frac{2}{\cos 2\theta} = 2 \sec 2\theta$

41) a) $x_1 C_5H_8 + x_2 O_2 \rightarrow x_3 CO_2 + x_4 H_2O$ (3)

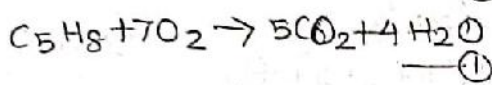
$$\left. \begin{aligned} 5x_1 - x_3 &= 0 \\ 4x_1 - x_4 &= 0 \\ 2x_2 - 2x_3 - x_4 &= 0 \end{aligned} \right\} \text{--- (2)}$$

$$[A|B] = \left[\begin{array}{cccc|c} 5 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 4 & 0 & 0 & -1 & 0 \end{array} \right] \text{--- (1)}$$

$$= \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 5 & 0 \end{array} \right]$$

$x_1 = t$ எனில்

$$(x_1, x_2, x_3, x_4) = \left(\frac{t}{4}, \frac{7t}{4}, \frac{5t}{4}, t \right) \text{--- (1)}$$



b) $x + \frac{1}{x} = y$ --- (1)

$$6(y^2 - 2) - 35y + 62 = 0$$

$$6y^2 - 35y + 50 = 0 \text{--- (1)}$$

$$y = \frac{10}{3} \quad y = \frac{5}{2} \text{--- (1)}$$

$$x = 3, \frac{1}{3} \quad x = \frac{1}{2}, 2 \text{--- (1)}$$

42) a) $[A|B] = \left[\begin{array}{ccc|c} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{array} \right] \text{--- (1)}$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 0 & 2(k-1) & 1-k & -3 \\ 0 & 0 & (k+2) & -(k+2) \end{array} \right]$$

(i) $k=1$ எனில் தீர்வு இல்லை

(ii) $k \neq 1, k \neq -2$ ஆக தீர்வு

(iii) $k=-2$ எனில் எண்ணற்ற தீர்வு

b) $z = z_1 e^{i2\pi/3} = (1+i\sqrt{3}) \left(\frac{-1+i\sqrt{3}}{2} \right)$

$$z_2 = -2 \text{--- (2)}$$

$$z_2 e^{i2\pi/3} = -2 \left(\frac{-1+i\sqrt{3}}{2} \right) = 1 - i\sqrt{3} \text{--- (2)}$$

43) $\arg(z-i) - \arg(z+2) = \pi/4$ --- (1)

$$\tan^{-1} \left(\frac{y-1}{x} \right) - \tan^{-1} \left(\frac{y}{x+2} \right) = \frac{\pi}{4} \text{--- (2)}$$

$$x^2 + y^2 + 3x - 3y + 2 = 0 \text{ தீர்வு --- (2)}$$

b) $\alpha_1 = 2+i, \alpha_2 = 2-i \} \alpha_5, \alpha_6 = ?$
 $\alpha_3 = 3-i\sqrt{2}, \alpha_4 = 3+i\sqrt{2} \} \text{--- (1)}$

$$\alpha_5 + \alpha_6 = 3 \text{--- (1)}$$

$$\alpha_5 \alpha_6 = -4 \text{--- (1)}$$

காரணி சமன்பாடு $x^2 - 3x - 4 = 0$ --- (1)

மற்ற காரணி $-1, 4$ --- (1)

44) $\tan^{-1}(-1) = -\frac{\pi}{4}$ --- (1)

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \text{--- (1)} \left(= \frac{\pi}{12} \text{--- (2)} \right)$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \text{--- (1)}$$

b) $\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$ --- (1)

கூலியம்மைகள் $(-3, 1)$ --- (1)

கூலியம்மைகள் $(-3, 2\sqrt{3}+1)$ $(-3, -2\sqrt{3}+1)$ --- (1)

கூலியம்மைகள் $(1, 5), (1, -3)$ --- (1)

கூலியம்மைகள் 2 --- (1)

45) $\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}3x - \tan^{-1}x$

$$\tan^{-1}\left(\frac{2x}{1-x^2+1}\right) = \tan^{-1}\left(\frac{2x}{1+3x^2}\right) \text{--- (2)}$$

$$6x^3 + 2x = 4x - 2x^3 \text{--- (1)}$$

$$8x^3 - 2x = 0 \text{--- (1)}$$

$$\text{கூலியம்மைகள்} = 3 \text{--- (1)}$$

45(b) U.L.M --- (1)

$(x, 50)$ யம்மை --- (1) $x_1 = 45.4$ $\text{உ.L.M} = 90.82$ --- (1)

$(x_2, -100)$ யம்மை --- (1) $\text{உ.L.M} = 148.98$ --- (1)

46) $\bar{a} = 2i + 2j + k$
 $\bar{b} = i - 2j + 3k$
 $\bar{c} = -3i - 4j - 5k$ } --- (1)

$$\bar{r} = \bar{a} + s(\bar{b} - \bar{a}) + t\bar{c}$$

$$\bar{r} = (2i + 2j + k) + s(-i - 4j + 2k) + t(-3i - 4j - 5k)$$

$$(\bar{b} - \bar{a}) \times \bar{c} = 12i - 11j - 16k \text{--- (1)}$$

காரணி சமன்பாடு $12x - 11y - 16z + 14 = 0$ --- (1)

b) U.L.M --- (1)

$$\bar{a} \cdot \bar{c} - \bar{a} \cdot \bar{b} = 0 \text{--- (1)}$$

$$\bar{a} \cdot \bar{b} - \bar{b} \cdot \bar{c} = 0 \text{--- (1)}$$

$$\overline{OC} \perp \overline{BA} \text{--- (2)}$$

தீர்வு

47) $a = \cos \theta + i \sin \theta$
 $b = \cos \phi + i \sin \phi$ எனில்

$$a+b = (\cos \theta + \cos \phi) + i(\sin \theta + \sin \phi)$$

$$a+b=0 \quad \text{--- (1)}$$

$$\Rightarrow a^2 + b^2 = -2ab$$

$$(\cos \theta + i \sin \theta)^2 + (\cos \phi + i \sin \phi)^2 = -2(\cos \theta + i \sin \theta)(\cos \phi + i \sin \phi)$$

$$(\cos 2\theta + \cos 2\phi) + i(\sin 2\theta + \sin 2\phi) = -2 \cos(\theta + \phi) - 2i \sin(\theta + \phi) \quad \text{--- (2)}$$

மேல் பகுதி

$$\cos 2\theta + \cos 2\phi = -2 \cos(\theta + \phi)$$

$$\cos 2\theta + \cos 2\phi = 2 \cos(\pi + \theta + \phi) \quad \text{--- (1)}$$

$$\boxed{\cos(\pi + x) = -\cos x}$$

கீழ்ப்பகுதி

$$\sin 2\theta + \sin 2\phi = -2 \sin(\theta + \phi)$$

$$\sin(\pi + x) = -\sin x$$

$$\sin 2\theta + \sin 2\phi = 2 \sin(\pi + \theta + \phi) \quad \text{--- (1)}$$

b). மூலக்கோட்டின் மையம்

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad \text{--- (1)}$$

மையம்

$$h = \frac{a^2}{x_1} \quad k = \frac{b^2}{y_1} \quad \text{--- (1)}$$

$$x_1 = \frac{a^2}{h} \quad y_1 = \frac{b^2}{k} \quad \text{--- (1)}$$

$$(x_1, y_1) = \left(\frac{a^2}{h}, \frac{b^2}{k} \right) \text{ மையம் } \quad \text{--- (1)}$$

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$$

$$\frac{a^2 a^2}{h^2 a^2} + \frac{b^2 b^2}{k^2 b^2} = 1$$

$$\frac{a^2}{h^2} + \frac{b^2}{k^2} = 1 \quad \text{--- (1)}$$

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