SHRI KRISHNA ACADEMY

NEET,JEE AND BOARD EXAM COACHING CENTRE

SBM SCHOOL CAMPUS,TRICHY MAIN ROAD,NAMAKKAL

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QUATERLY EXAMINATION -2019

XII - MATHEMATICS **TENTATIVE ANSWER KEY**

ANSWER KEY		
Q.No	PART – I	MARKS
1.	(b) 1	1
2.	(b) $\frac{\pi}{4}$	1
3.	(a) 2	1
4.	(a) 3	1
5.	(c) $\frac{-q}{r}$	1
6.	(b) $[0,\pi]\backslash\left\{\frac{\pi}{2}\right\}$	1
7.	(d) 10	1
8.	(c) $2\sqrt{3}$	1
9.	(a) $k \neq 0$	1
10.	(c) ± 1	1
11.	(a) $\text{Re}\left(z^2\right)=0$	1
12.	(a) form a square	1
13.	(c) $\frac{-24}{25}$	1
14.	(a) rational	1
15.	(c) $\frac{\sqrt{3}}{2}$	1
16.	(a) [0,1]	1
17.	(c) (-5,0)	1
18.	(a) $\left(\frac{1}{2},\frac{1}{2}\right)$	1
19.	(a) $\left[\vec{a}\vec{b}\vec{c}\right]^4$	1
20.	(a) (3,-4,-2)	1
	PART – II	
21.	$\lvert adjA\rvert = 9$ $A^{-1} = \frac{1}{\sqrt{\lvert adjA\rvert}} adjA \Rightarrow \pm\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$	1 1

22.	$\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}=(i)^{18}\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^{18}$ $=-(\cos 3\pi - i\sin 3\pi)$ $=1$	1 1
23.	$x=\left(\frac{2}{3}\right)^{\frac{1}{4}}\Rightarrow x^4=\frac{2}{3}$ $\Rightarrow 3x^4-2=0$	1 1
24.	$\tan^{-1}\left(\tan\frac{3\pi}{5}\right)=\tan^{-1}\left(\tan\left(\frac{-2\pi}{5}\right)\right)$ $=\frac{-2\pi}{5}\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$	1 1
25.	Let (x_1,y_1) be tangent point $y_1=mx_1+c$ ----(1) $yy_1=-xx_1+a^2$ ----(2) From(1),(2) $\frac{y_1}{1}=\frac{-x_1}{m}=\frac{a^2}{c}\Rightarrow c=\pm a\sqrt{1+m^2}$	 1 1
26.	$\begin{vmatrix}2 & -1 & 3\\ 3 & 2 & 1\\ 1 & m & 4\end{vmatrix}=0$ $m=-3$	1 1
27.	Let $z=\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ $\bar{z}=-\left[\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}\right]$ $\bar{z}=-z$	 1 1
28.	Let $P(x)=x^9-5x^2-14x^7$ P(x) has only one sign change $P(-x)=-x^9-5x^8+14x^7$ P(-x) has only one sign change $\therefore$ There is atmost one +ve and one –ve root	 1 1
29.	Let $\sin^{-1}\frac{4}{5}=y\Rightarrow\sin y=\frac{4}{5}$ and $\cos y=\frac{3}{5}\Rightarrow y=\cos^{-1}\frac{3}{5}$ $\cot\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{3}{5}\right)=\cot\left(\frac{\pi}{2}\right)$ $=0$	 1 1
30.	$\begin{vmatrix}1 & 2a & a\\ 1 & 3b & b\\ 1 & 4c & c\end{vmatrix}=0$ $-bc+2ac-ab=0\Rightarrow\frac{2}{b}=\frac{1}{c}+\frac{1}{a}$ $\therefore a,b,c$ are in H.P	 1 1

	PART – III	
31.	$(A\|I) = \left(\begin{array}{cc\|cc} 0 & 5 & 1 & 0 \\ -1 & 6 & 0 & 1 \end{array}\right)$	1
	$= \left(\begin{array}{cc\|cc} 1 & 0 & 6/5 & -1 \\ 0 & 1 & 1/5 & 0 \end{array}\right)$	1
	$A^{-1} = \frac{1}{5}\begin{bmatrix} 6 & -5 \\ 1 & 0 \end{bmatrix}$	1
32.	$x + y = 100$	
	$x - \frac{1}{4}y = 80$	1
	$\Delta = -5, \Delta_1 = -420, \Delta_2 = -80$	1
	$x = 84; y = 16$	1
33.	Let z_1, z_2 be two complex number, then $\|z_1 + z_2\| \le \|z_1\| + \|z_2\|$	1
	$\|z_1 + z_2\|^2 = (z_1 + z_2)\left(\overline{z_1 + z_2}\right)$	
	$= \|z_1\|^2 + \|z_2\|^2 + 2\,\mathrm{Re}\left(z_1 \overline{z_1}\right)$	1
	$\|z_1 + z_2\|^2 \le \left(\|z_1\| + \|z_2\|\right)^2$	
	$\|z_1 + z_2\| \le \|z_1\| + \|z_2\|$	1
34.	Let the roots $\frac{\alpha}{\lambda}, \alpha, \alpha\lambda$	
	$\sum\alpha = \alpha\left(\frac{1}{\lambda} + 1 + \lambda\right) = \frac{-b}{a}; \sum\alpha\beta = \alpha^2\left(\frac{1}{\lambda} + 1 + \lambda\right) = \frac{c}{a}; \sum\alpha\beta\gamma = \alpha^2\left(\frac{1}{\lambda} + 1 + \lambda\right) = \frac{c}{a}$	2
	$\therefore \alpha = \frac{-c}{b} \Rightarrow ac^3 = db^3$	1
35.	$-1 \le 2 - 3x^2 \le 1$ and $x^2 \ge \frac{1}{3} \Rightarrow \frac{1}{\sqrt{3}} \le \|x\| \le 1$	1
	$x \in \left[-1, \frac{-1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, 1\right]$	2
36.	$(x+1)^2 = 4a(y+2)$	1
	It passes (3,6) then $a = \frac{1}{\sqrt{2}}$	1
	$\therefore (x+1)^2 = 2(y+2)$ (or) $x^2 + 2x - 2y - 3 = 0$	1
37.	The equation of tangent at t_1 is $y + xt_1 = 2at_1 + at_1^3$	1
	It passes $\left(at_2^2, 2at_2\right)$	1
	$\frac{-2}{t_1} = t_2 + t_1 \Rightarrow t_2 = -\left(t_1 + \frac{2}{t_1}\right)$	1
38.	$\hat{a} \times \left(\hat{b} \times \hat{c}\right) = \frac{1}{2}\hat{b}$	
	$\left(\hat{a}.\hat{c}\right)\hat{b} - \left(\hat{a}.\hat{b}\right)\hat{c} = \frac{1}{2}\hat{b}$	1

	$\hat{a}.\hat{c}=\frac{1}{2}\Rightarrow\cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$	2
39.	$\vec{a}=2\hat{i}+6\hat{j}+3\hat{k};\vec{b}=2\hat{i}+3\hat{j}+4\hat{k};\vec{c}=2\hat{j}-3\hat{k};\vec{d}=\hat{i}+2\hat{j}+3\hat{k}$ $\vec{b}\times\vec{d}=\hat{i}-2\hat{j}+\hat{k}$ $(\vec{c}-\vec{a}).(\vec{b}\times\vec{d})=0$ $d=\frac{\left\|(\vec{c}-\vec{a}).(\vec{b}\times\vec{d})\right\|}{\left\|\vec{b}\times\vec{d}\right\|}=0$	1 1 1
40.	Let $\frac{1}{2}\cos^{-1}\frac{a}{b}=\theta\Rightarrow\frac{a}{b}=\cos 2\theta\Rightarrow\sec 2\theta=\frac{b}{a}$ $LHS=\tan^{-1}\left(\frac{\pi}{4}+\theta\right)+\tan^{-1}\left(\frac{\pi}{4}-\theta\right)$ $=\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}$ $=\frac{2\sec^2\theta}{\left(\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\right)}=\frac{2}{\cos 2\theta}=\frac{2b}{a}$	1 2
	PART – IV	
41	(a) $x_1C_5H_8+x_2O_5=x_3CO_2+x_4H_2O$ $5x_1-x_3=0;4x_1-x_4=0;2x_2-2x_3-x_4=0$ $[A\backslash B]=\left[\begin{array}{cccc\|c}5&0&-1&0&0\\4&0&0&-1&0\\0&2&-2&-1&0\end{array}\right]$ $\sim\begin{bmatrix}4&0&0&-1&0\\0&2&-2&-1&0\\0&0&-4&5&0\end{bmatrix}$ $\rho(A)=\rho([A\|B])=3<4$ $\therefore 4x_1-x_4=0$ and $2x_2-2x_3-x_4=0$ and $-4x_3+5x_4=0$ $\Rightarrow x_1=1;x_2=7;x_3=5;x_4=4$ Balanced equation is $C_5H_8+7O_2\rightarrow 5CO_2+4H_2O$	1 1 2 1
	(b) $6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$-----(1) Put $x+\frac{1}{x}=y$ and $x^2+\frac{1}{x^2}=y^2-2$ (1)*becomes* $6y^2-35y+50=0$ $y=\frac{10}{3},\frac{5}{2}$ when $y=\frac{10}{3}$, $x=3,\frac{1}{3}$ and when $y=\frac{5}{2}$, $x=2,\frac{1}{2}$ *The* roots are $2,\frac{1}{2},3,\frac{1}{3}$	1 2 2

<table>
<tr><td>42.</td><td>(a) $[A|B] \sim \begin{bmatrix} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{bmatrix}$

$[A|B] \sim \begin{bmatrix} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & (k+2)(1-k) & -k-2 \end{bmatrix}$

case (i): when k=1 $\Rightarrow \rho(A) \neq \rho([A|B])$
No Solution.
case (ii): when $k \neq 1$, $k \neq -2 \Rightarrow \rho(A) = \rho([A|B]) = 3$
Unique Solution.
case (iii): when k=−2 $\Rightarrow \rho(A) = \rho([A|B]) = 2 < 3$
Infinitely many Solution.</td><td>1

1

1

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1</td></tr>
<tr><td></td><td>(b) $z_1 = 1 + i\sqrt{3}$

$z_2 = z_1 e^{\frac{i2\pi}{3}} = (1+i\sqrt{3})\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \Rightarrow z_2 = -2$

$z_3 = z_2 e^{\frac{i2\pi}{3}} = (-2)e^{\frac{i2\pi}{3}} = -2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \Rightarrow z_3 = 1 - i\sqrt{3}$</td><td>1

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2</td></tr>
<tr><td>43.</td><td>(a) $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{\frac{(x+2)(y-1)-xy}{x(x+2)}}{\frac{x(x+2)+y(y-1)}{x(x+2)}}\right) = 1$

$\frac{(x+2)(y-1)-xy}{x(x+2)+y(y-1)} = 1$

$x^2 + y^2 + 3x - 3y + 2 = 0$</td><td>1

2

2</td></tr>
<tr><td></td><td>(b) The factor of the given polynomial is
$((x-(2+i))(x-(2-i))(x-(3-\sqrt{2}))(x-(3+\sqrt{2}))$
$(x^2-4x+5)(x^2-6x+7)$ *is a factor*
Dividing the given polynomial by this factor ,
we get other factor as (x^2-3x-4)
$\therefore$ 4 and -1 are other two roots.
$\therefore$ The roots of the given polynomial are $2+i, 2-i, 3+\sqrt{2}, 3-\sqrt{2}, -1$ and 4.</td><td>2

1

1
1</td></tr>
<tr><td>44.</td><td>(a)(i) $\tan^{-1}(-1) = -\frac{\pi}{4}; \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}; \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{4} + \frac{\pi}{3} - \frac{\pi}{6} = -\frac{\pi}{12}$</td><td>2

1</td></tr>
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	(ii) $\cot^{-1}\left(\frac{1}{7}\right)=\theta,\ \theta\in(0,\pi)$ $\cot\theta=\frac{1}{7}$ $\tan\theta=7$ and θ is acute $\cos\theta=\frac{1}{5\sqrt{2}}$	1 1
	(b) $4x^2+24x+y^2-2y+21=0$ $\frac{(x+3)^2}{4}+\frac{(y-1)^2}{16}=1$ Major axis is parallel to y-axis $c^2=12\Rightarrow c=\pm2\sqrt{3}$ Foci are $\left(-3,2\sqrt{3}+1\right)$ and $\left(-3,-2\sqrt{3}+1\right)$ Vertices are $(1,5)$ and $(1,-3)$; L.L.R= 2 unit	1 1 1 2
45.	(a) $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$ $\tan^{-1}\left(\frac{4x-x^3}{2-3x^2}\right)=\tan^{-1}3x$ $\frac{4x-x^3}{2-3x^2}=3x\Rightarrow 8x^3-2x=0$ There are 3 solutions.	 2 2 1
	(b) $\frac{x^2}{30^2}-\frac{y^2}{44^2}=1$ $(x_1,50)$ is a point of hyperbola $\frac{x_1^2}{30^2}-\frac{y_1^2}{44^2}=1\Rightarrow x_1=45.40$ and $2x_1=90.8$ $(x_2,-100)$ is a point of hyperbola $\frac{x_2^2}{30^2}-\frac{(-100)^2}{44^2}=1\Rightarrow x_2=74.48$ and $2x_2=148.96$	1 1 1 1 1
46.	(a) Let $\vec{a}=2\vec{i}+2\vec{j}+\vec{k}$, $\vec{b}=\vec{i}-2\vec{j}+3\vec{k}$, $\vec{c}=-3\vec{i}+4\vec{j}-5\vec{k}$ Parametric form of Vector Equation is , $\vec{r}=(2\vec{i}+2\vec{j}+\vec{k})+s(-3\vec{i}-4\vec{j}+2\vec{k})+t(-3\vec{i}+4\vec{j}-5\vec{k}),\ s,t\in R$ Cartesian form : $\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ c_1 & c_2 & c_3\end{vmatrix}=0\Rightarrow\begin{vmatrix}x-2 & y-2 & z-1\\ -1 & -4 & 2\\ -3 & 4 & -5\end{vmatrix}=0$ $12x-11y-16z+14=0$	1 2 2

	(b) Diagram Let $\overrightarrow{OA}=\vec{a},\overrightarrow{OB}=\vec{b},\overrightarrow{OC}=\vec{c}$ $\overrightarrow{AD}\perp\overrightarrow{BC}\Rightarrow\vec{a}.\vec{c}-\vec{a}.\vec{b}=0$-----(1) $\overrightarrow{BE}\perp\overrightarrow{CA}\Rightarrow\vec{b}.\vec{a}-\vec{b}.\vec{c}=0$-----(2) (1)+ (2) $\vec{a}.\vec{b}-\vec{b}.\vec{c}=0\Rightarrow\vec{c}.(\vec{a}-\vec{b})=0$ $\overrightarrow{CF}\perp\overrightarrow{AB}$ Hence opposite sides of triangle are concurrent.	1 1 1 1 1
47.	(a) Given $\cos\theta+\cos\phi=\sin\theta+\sin\phi=0$ Let $a=\cos\theta+i\sin\theta, b=\cos\phi+\sin\phi$ $a+b=(\cos\theta+\cos\phi)+i(\sin\theta+\sin\phi)=0$ Since $a+b=0$ then $(a+b)^2=0$ $a^2+b^2+2ab=0\Rightarrow a^2+b^2=2(-1)ab$ $\therefore(\cos\theta+i\sin\theta)^2+(\cos\phi+i\sin\phi)^2=2(\cos\pi+i\sin\pi)(\cos\theta+i\sin\theta)(\cos\phi+i\sin\phi)$ $(\cos 2\theta+\cos 2\phi)+i(\sin 2\theta+\sin 2\phi)=2[\cos(\pi+\theta+\phi)+i\sin(\pi+\theta+\phi)]$ Equating real and imaginary parts $\cos 2\theta+\cos 2\phi=2\cos(\pi+\theta+\phi)$ $\sin 2\theta+\sin 2\phi=2\sin(\pi+\theta+\phi)$	1 2 2
	(b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Tangent at (x_1,y_1) is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1\Rightarrow\frac{x}{\left(\frac{a^2}{x_1}\right)}+\frac{y}{\left(\frac{b^2}{y_1}\right)}=1$ $\therefore h=\frac{a^2}{x_1}\Rightarrow x_1=\frac{a^2}{h}$ and $k=\frac{b^2}{y_1}\Rightarrow y_1=\frac{b^2}{k}$ $(x_1,y_1)=\left(\frac{a^2}{h},\frac{b^2}{k}\right)$ is a point on the ellipse $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1\Rightarrow\frac{(a^2)^2}{h^2(a^2)}+\frac{(b^2)^2}{k^2(b^2)}=1$ $\therefore\frac{a^2}{h^2}+\frac{b^2}{k^2}=1$	1 2 1 1