SET 3

SECTION A.

1) 2 (b)
2) (b) 22.
3) (b) −2.
4) (d) 36 : 49.
5) (c) 45.
6) (b) 4.
7) (a) 3
8) (d) 4.
9) (b) 1
10) (d) 2.

OR.

option not present. answer is 44.

11) 5/2.
12) 25

OR.

120

13) 8 cm.
14) 2 units
15) 90°.
16) $\frac{n(n+1)}{2}$
17) 0.15
18) 4.51
19) 3 : 1
20) a^2b^2.

21) CSA of cylinder $= 2\pi r h$

$$= 2 \times \frac{22}{\cancel{7}} \times \cancel{21}^{3} \times \frac{\cancel{35}^{7}}{\cancel{10}_{2}}$$

$= 462 \text{ cm}^2$.

(i) Cost of silver coating

$$= 462 \times \frac{5}{100}$$

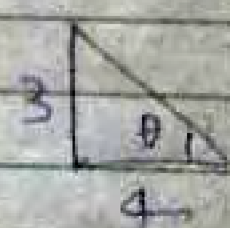
$= ₹\ 23.10$

(ii) CSA of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{\cancel{7}} \times \cancel{21}^{3} \times 21$$

$= 2772 \text{ cm}^2$

22) $\tan\theta = \frac{3}{4} = \frac{\text{opp side}}{\text{adj. side}}$.



hyp $= \sqrt{3^2+4^2}$
$= 5$

hyp $= 5$

$\cos\theta = \frac{4}{5}$.

$$\frac{1-\cos^2\theta}{1+\cos^2\theta} = \frac{1-\frac{16}{25}}{1+\frac{16}{25}} = \frac{\frac{25-16}{25}}{\frac{25+16}{25}} = \frac{9}{41}$$

OR

$\tan\theta = \frac{\sqrt{3}}{1} = \frac{\text{opp}}{\text{adj}}$

hyp $= 2$.

$$\frac{2\sec\theta}{1+\tan^2\theta} = \frac{2\sec\theta}{\sec^2\theta} = \frac{2}{\sec\theta} = 2\cos\theta = \cancel{2 \times \frac{\sqrt{3}}{2}}$$

$$= 2 \times \frac{1}{2} = 1$$

23) $12, 8, 4, \ldots, -84$

$-84, -80, \ldots, 4, 8, 12$

Let $a = -84$.

$d = 4$.

$a_{11} = a + (n-1)d$.

$= -84 + 10 \times 4$

$= -84 + 40$

$= \underline{\underline{-44}}$

OR.

$1 + 5 + 9 + 13 + \ldots x = 1326$

$S_x = \frac{x}{2}(a_1 + a_x) = 1326$.

$\frac{x}{2}(1 + x) = 1326$

$x^2 + x = 2652$

$x^2 + x - 2652 = 0$.

$x^2 + 52x - 51x - 2652 = 0$

$x(x + 52) - 51(x + 52) = 0$

$(x - 51)(x + 52) = 0$

$x = 51, -52$.

Here $x = 51$.

24)

Class	Freq	x_i
2-4	6	3
4-6	8	5
6-8	15	7
8-10	p	9
10-12	8	11
12-14	4	13

$\frac{\sum f_i x_i}{\sum f_i} = 7.5$

$\frac{6 \times 3 + 8 \times 5 + 15 \times 7 + 9 \times p + 8 \times 11 + 4 \times 13}{41 + p} = 7.5$

$\frac{18 + 40 + 105 + 9p + 88 + 52}{41 + p} = 7.5$

$10(303 + 9p) = 75(41 + p)$

$3030 + 90p = 3075 + 75p$

$15p = 45$

$p = \underline{\underline{3}}$

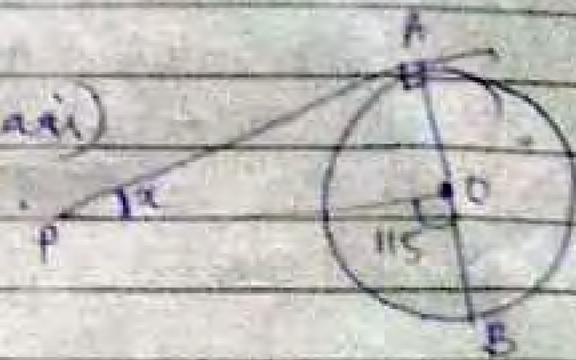
25) Let G = girl & B = boy

(GGG) (GGB) (GBG) (BGG) (GBB) (BGB)

(BBG) (BBB)

$P(\text{atleast one boy}) = \frac{7}{8}$

26) To find $\angle APO$.



$\angle POA = 180 - 115$ (linear pair)

$= 65$

In $\triangle APO$

$x + 90 + 65 = 180$

$x + 155 = 180$

$x = 180 - 155$

$= 25$

SECTION C

27) $l = 80m \quad b = 50m$

Vol. displaced $= 0.04 m^3$

$lbh = 0.04 \times 500$

$80 \times 50 \times h = 0.04 \times 500$

$$h = \frac{4 \times 500}{100 \times 80 \times 50} = \frac{5}{1000} = 0.005m$$

28) $\sin\theta + \cos\theta = p$

$\sec\theta + \operatorname{cosec}\theta = q$

$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$

$\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = q$

$2p = 2(\sin\theta + \cos\theta)$

$$q(p^2-1) = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\left[(\sin\theta + \cos\theta)^2 - 1\right]$$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\left[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\right]$$

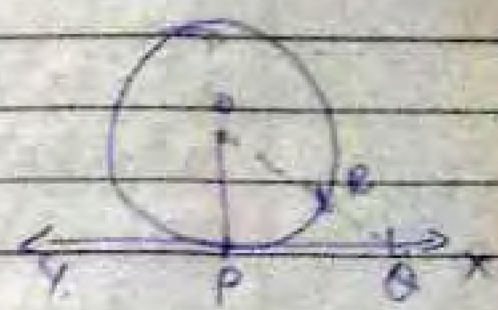
$\frac{(\sin\theta + \cos\theta)(1 + 2\sin\theta\cos\theta - 1)}{\sin\theta\cos\theta}$

$= \frac{(\sin\theta + \cos\theta) \times 2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= 2(\sin\theta + \cos\theta)$

$\therefore$ LHS = RHS.

29) Given - A circle with centre O, and tangent XY with P being the point of contact.



To prove $OP \perp XY$

Proof. Let Q be the point on XY. Join OQ. It touches the circle on R.

$OQ > OR$.

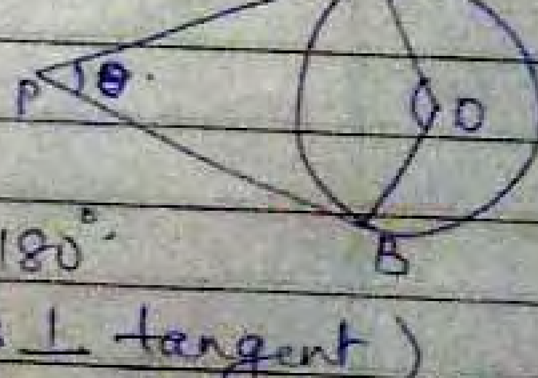
$OQ > OP$. $(OP = OR = \text{Radius})$

i.e. OP is the smallest line that connects XY similar is the case of all pts on the circle.

$\therefore OP \perp XY$

OR.

Let PA & PB be tangents to a circle with centre O from an external point P.



To prove $\angle APB + \angle AOB = 180°$.

$\angle OAP = \angle OBP = 90$ (Radius $\perp$ tangent)

In quadrilateral OAPB.

$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360$

$90 + \angle APB + 90 + \angle AOB = 360$

$\angle APB + \angle AOB + 180 = 360$

$\angle APB + \angle AOB = 360 - 180 = 180 \Rightarrow$ Hence proved.

3a) $p(x) = g(x) \times q(x) + r(x)$

$p(x) = x^3 - 3x^2 + x + 2$

$q(x) = x - 2$

$r(x) = -2x + 4$

$\frac{p(x) - r(x)}{q(x)} = g(x)$

$x^3 - 3x^2 + x + 2 + 2x - 4 = p(x) - r(x)$

$x^3 - 3x^2 + 3x - 2$

$\frac{p(x) - r(x)}{q(x)}$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
\hline
 & x - 2
\end{array}
$$

$\therefore g(x) = x^2 - x + 1$

OR

$f(x) = x^2 - 8x + k$

Let α & β be the zeros.

$\alpha^2 + \beta^2 = 40$

$\alpha + \beta = \frac{-b}{a} = 8$ $\qquad \alpha\beta = \frac{c}{a} = k$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$

$8^2 - 2k = 40$

$64 - 2k = 40$

$-2k = 40 - 64$

$-2k = -24$

$k = 12$

31) $a, 7, b, 23, c$

Let $d = 7 - a$.

$a_4 = a + 3d$

$23 = a + 3(7 - a)$

$23 = a + 21 - 3a$.

$2 = -2a$

$a = -1$

$d = 7 - \cancel{a}$

$= 7 + 1$

$= 8$.

$b = 7 + 8 = 15$

$c = 23 + 8 = 31$

OR.

$a_m \times m = a_n \times n$.

$m[a + (m-1)d] = n[a + (n-1)d]$

$am + m^2 d - md = an + n^2 d - nd$

$am - an + m^2 d - n^2 d - md + nd = 0$.

$a(m-n) + d[m^2 - n^2 - (m-n)] = 0$.

$(m-n)[a + d[m+n-1]] = 0$.

$a + d(m+n-1) = 0$.

ie $a_{(m+n)} = 0 \implies$ Hence proved.

32) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$\frac{x-7-x-4}{x^2-3x-28} = \frac{11}{30}$

$-\cancel{11} \times 30 = \cancel{11}(x^2 - 3x - 28)$

$x^2 - 3x - 28 + 30 = 0$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 1, 2$

33) $A(-1,1)$ $B(5,7)$ $C(8,10)$

For the points to be collinear ar $\Delta ABC = 0$

$$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]=0$$

$$-1(7-10)+5(10-1)+8(1-7)$$

$$-1\times-3+5\times9+8\times-6=0$$

$$3+45-48$$

$= 0$ $\therefore$ the pts are collinear.

34) For two similar Δs

corresponding $\angle$s are equal

are ratio of corresponding ~~sig~~ sides are equal

Let ΔABC & ΔPQR be similar.

$\angle A = \angle P$ $\angle B = \angle Q$ $\angle C = \angle R$.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}.$$

Given ratio of areas are equal.

i.e $\frac{ar\,\Delta ABC}{ar\,\Delta PQR} = 1$

$$\frac{ar\,\Delta ABC}{ar\,\Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = 1$$

i.e $AB^2 = PQ^2$ or $AB = PQ$,

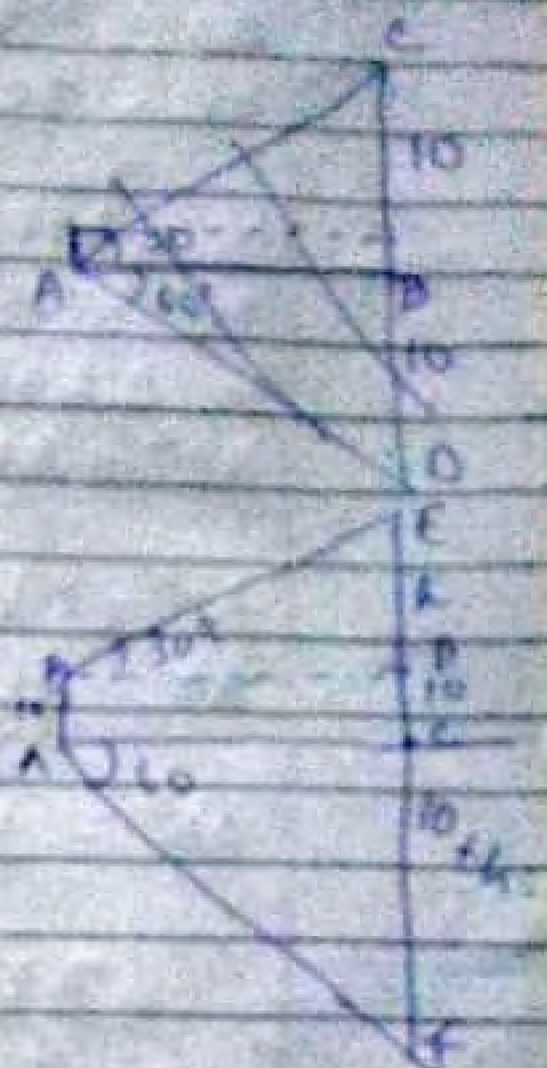
$BC^2 = QR^2$ " $BC = QR$

$AC^2 = PR^2$ or $AC = PR$.

when corresponding sides and corresponding angles are equal, the triangles are congruent.

i.e $\Delta ABC \cong \Delta PQR$

SECTION D



25) Height of the cloud from the lake = AC

$\tan 30 = \frac{BC}{}$

Height of the cloud from the surface of the lake = ~~AE~~ ~~BD~~ ~~EB+ED~~ CE

$\tan 30 = \frac{DE}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{BD}$

$BD = h\sqrt{3}$

$\tan 60 = \frac{10+h}{AC}$

$\sqrt{3} = \frac{10+h}{h\sqrt{3}}$

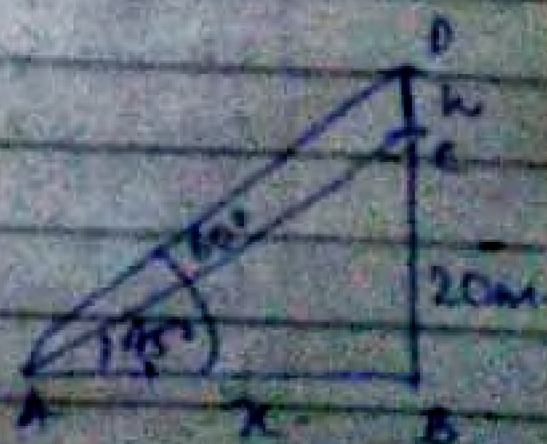
$3h = 10+h$

$2h = 10$

$h = 5$.

$EC = 10+5 = \underline{\underline{15m}}$

OR



$\tan 45 = \frac{BC}{x}$

$1 = \frac{20}{x}$

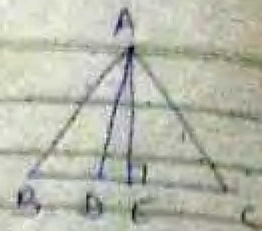
$x = 20$

$\tan 60 = \frac{h+20}{}$

$\sqrt{3} = \frac{h+20}{20}$

$20\sqrt{3} = h+20$

$h = \underline{\underline{20(\sqrt{3}-1)}}$



36) $AB = BC = AC$

$BD = \frac{1}{3} BC$

$BE = EC = \frac{1}{2} BC$

In ΔABE

$AE^2 + BE^2 = AB^2$ $\quad AE^2 = AB^2 - BE^2$

In ΔADE

$AE^2 + DE^2 = AD^2$ $\quad AE^2 = AD^2 - DE^2$

$AB^2 - BE^2 = AD^2 - DE^2$

$AB^2 - \left(\frac{1}{2} BC\right)^2 = AD^2 - (BE - BD)^2$

$AB^2 - \frac{AB^2}{4} = AD^2 - \left(\frac{AB}{2} - \frac{AB}{3}\right)^2$

$\frac{3AB^2}{4} = AD^2 - \frac{AB^2}{36}$

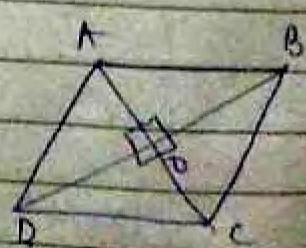
$\frac{3AB^2}{4} + \frac{AB^2}{36} = AD^2$

$\frac{28AB^2}{36} = AD^2$

$\frac{7AB^2}{9} = AD^2$

$7AB^2 = 9AD^2$

OR



To prove. $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

$OA = OC = \frac{1}{2} AC$

$OB = OD = \frac{1}{2} BD$

In ΔAOB $\quad AO^2 + OB^2 = AB^2$ — (1)

In ΔBOC $\quad BO^2 + OC^2 = BC^2$ — (2)

In ΔCOD $\quad OC^2 + OD^2 = CD^2$ — (3)

In ΔAOD $\quad AO^2 + OD^2 = AD^2$ — (4)

(1) + (2) + (3) + (4)

$AO^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OD^2 + OA^2 = AB^2 + BC^2 + CD^2 + AD^2$

$2(AO^2 + OB^2 + OC^2 + OD^2) = AB^2 + BC^2 + CD^2 + AD^2$

$2(2AO^2 + 2OB^2) = AB^2 + BC^2 + CD^2 + AD^2$

$2\left(\frac{AC^2}{2} + \frac{BD^2}{2}\right) = AB^2 + BC^2 + CD^2 + AD^2$

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

Fraction: $\frac{x}{y}$

$\frac{x-1}{y} = \frac{1}{3}$ $\qquad \frac{x}{y}$

$3(x-1) = y$

$3x - 3 = y$

$3x - y = 3 \rightarrow (1)$

$\frac{x}{y+8} = \frac{1}{4}$

$4x = y+8$

$4x - y = 8 \rightarrow (2)$

~~$3x - y = 3$~~ Using elimination method.

$4x - y = 8$

$3x - y = 3$

$(-) \quad (+) \quad (-)$

$x = 5$

$3x - y = 3$

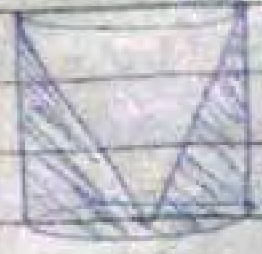
$3 \times 5 - y = 3$

$15 - y = 3$

$y = 12$

Fraction: $\frac{x}{y} = \frac{5}{12}$

40. Height = 2.4 cm Length = 2.5 cm
Radius = 0.7 cm



area of the remaining solid
$= $ CSA of hemisphere + CSA of cone + area of circle
$= 2\pi rh + \pi rl + \pi r^2$
$= 2 \times \frac{22}{7} \times \frac{7}{10} \times \frac{24}{10} + \frac{22}{7} \times \frac{7}{10} \times \frac{25}{10} + \frac{22}{7} \times \frac{7}{10} \times \frac{7}{10}$

$= \frac{44 \times 24}{100} + \frac{22 \times 24}{100} + \frac{154}{100}$

$= \frac{1056 + 550 + 154}{100} = \cancel{\frac{1732}{100}} = \cancel{17.32} = 17.6 \text{ cm}^2$

37) $(12)^n = (2^2 \times 3)^n$
For a number to end with 0, its factors must be 2 and 5.
For a number to end with 5, its factor must be 5.
Since $(12)^n$ has 2 and 3 as factors, it cannot end with 0 or 5.

OR.

$(\sqrt{2} + \sqrt{5})$
Assume $\sqrt{2}$ is rational
$\sqrt{2} = a/b$ where $b \neq 0$ and a & b are co prime.
$b\sqrt{2} = a$
$2b^2 = a^2$
Since a^2 is a factor of $2b^2$, a is a factor of $2b$.
Let $a = 2c$
$a^2 = 4c^2$
$2b^2 = 4c^2$
$b^2 = 2c^2$.