DATE : 12/03/2020

SET-1

Code No. 430/1/1

Aakash
Medical | IIT-JEE | Foundations
(Divisions of Aakash Educational Services Limited)

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Max. Marks : 80

Class X

Time : 3 Hrs.

Mathematics (Basic)

(CBSE 2020)

GENERAL INSTRUCTIONS :

(i) This question paper comprises four sections - A, B, C and D. This question paper carries 40 questions. All questions are compulsory.

(ii) Section A : Q. No. 1 to 20 comprises of 20 questions of one mark each.

(iii) Section B : Q. No. 21 to 26 comprises of 6 questions of two marks each.

(iv) Section C : Q. No. 27 to 34 comprises of 8 questions of three marks each.

(v) Section D : Q. No. 35 to 40 comprises of 6 questions of four marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to **attempt only one of the choices** in such questions.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

Section-A

Q 1 – 10 are multiple choice questions. Select the most appropriate answer from the given options.

1. HCF of 144 and 198 is **[1]**

(a) 9 (b) 18

(c) 6 (d) 12

Answer (b)

Sol. $144 = 2^4 \times 3^2$

$198 = 2 \times 3^2 \times 11$

HCF $= 2 \times 3^2$

$= 18$

Hence, option (b) is correct. **[1]**

2. The median and mode respectively of a frequency distribution are 26 and 29. Then its mean is **[1]**

(a) 27.5 (b) 24.5

(c) 28.4 (d) 25.8

Answer (b)

Sol. 3 Median – 2 Mean = Mode

$\Rightarrow$ 3 × 26 – 2 Mean = 29

$\Rightarrow$ Mean = 24.5

Hence, option (b) is correct. **[1]**

3. In Fig. 1, on a circle of radius 7 cm, tangent PT is drawn from a point P such that PT = 24 cm. If O is the centre of the circle, then the length of PR is **[1]**

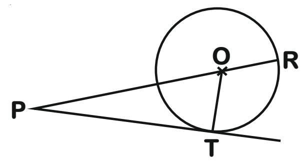


Fig. 1

(a) 30 cm (b) 28 cm

(c) 32 cm (d) 25 cm

Answer (c)

Sol. In $\triangle$POT,

$(OP)^2 = (OT)^2 + (PT)^2$

$\Rightarrow OP^2 = (7)^2 + (24)^2$

$\Rightarrow OP^2 = (25)^2$

$\Rightarrow$ OP = 25 cm

$\therefore$ PR = OP + OR = 25 + 7

= 32 cm

Hence, option (c) is correct. **[1]**

4. 225 can be expressed as **[1]**

(a) 5×3^2 (b) $5^2 \times 3$

(c) $5^2 \times 3^2$ (d) $5^3 \times 3$

Answer (c)

Sol. Prime factorisation of 225 is given below,

3	225
3	75
5	25
5	5
	1

$\therefore$ $225 = 3^2 \times 5^2$

Option (c) is correct. **[1]**

5. The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4 is **[1]**

(a) $\frac{4}{15}$ (b) $\frac{2}{15}$

(c) $\frac{1}{15}$ (d) $\frac{1}{5}$

Answer (d)

Sol. Favourable outcomes are 4, 8, 12, i.e., 3 outcomes and total number of outcomes = 15

$\therefore$ Required probability $= \frac{3}{15} = \frac{1}{5}$

Option (d) is correct. **[1]**

6. If one zero of a quadratic polynomial $(kx^2 + 3x + k)$ is 2, then the value of k is **[1]**

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$

(c) $\frac{6}{5}$ (d) $-\frac{6}{5}$

Answer (d)

Sol. 2 is a zero of polynomial $p(x) = kx^2 + 3x + k$.

$\Rightarrow$ $p(2) = 0$

$\Rightarrow$ $k(2^2) + 3(2) + k = 0$

$\Rightarrow$ $4k + 6 + k = 0$

$\Rightarrow$ $5k = -6$

$\therefore$ $k = \frac{-6}{5}$

Option (d) is correct. **[1]**

7. $2.\overline{35}$ is **[1]**

(a) an integer (b) a rational number

(c) an irrational number (d) a natural number

Answer (b)

Sol. $2.\overline{35}$ is a non-terminating recurring decimal. **[1]**

8. The graph of a polynomial is shown in Fig. 2, then the number of its zeroes is **[1]**

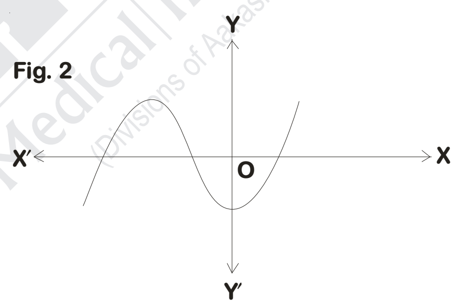


Fig. 2

(a) 3 (b) 1

(c) 2 (d) 4

Answer (a)

Sol. Graph of given polynomial cuts the x-axis at 3 distinct points. **[1]**

$\therefore$ No. of zeroes is 3.

9. Distance of point P(3, 4) from x-axis is **[1]**

(a) 3 units (b) 4 units

(c) 5 units (d) 1 unit

Answer (b) **[1]**

Sol. Distance of point (3, 4) from x-axis is its y-coordinate.

10. If the distance between the points A(4, p) and B(1, 0) is 5 units, then the value(s) of p is (are) **[1]**

(a) 4 only (b) –4 only

(c) ±4 (d) 0

Answer (c) **[1]**

Sol. A(4, p)

B(1, 0)

AB = 5

$\therefore \quad \sqrt{(4-1)^2+(p-0)^2}=5$

$\Rightarrow \quad 9 + p^2 = 25$

$\Rightarrow \quad p^2 = 16$

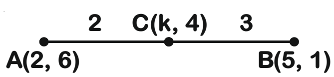
$\Rightarrow \quad p = \pm 4$

(Q11 – 15) Fill in the blanks:

11. If the point C(k, 4) divides the line segment joining two points A(2, 6) and B(5, 1) in ratio 2 : 3, the value of k is ________. **[1]**

Answer $\frac{16}{5}$ **[1]**

Sol.



C(k, 4) divides AB in the ratio 2 : 3

$\Rightarrow \quad C(k, 4)=\left(\frac{2\times 3+5\times 2}{2+3}, \frac{6\times 3+1\times 2}{2+3}\right)$

$\Rightarrow \quad (k, 4)=\left(\frac{16}{5}, \frac{20}{5}\right)$

$\Rightarrow \quad k=\frac{16}{5}$

OR

If points A(–3, 12), B(7, 6) and C(x, 9) are collinear, then the value of x is ________. **[1]**

Answer 2

Points A(–3, 12), B(7, 6) and C(x, 9) are collinear.

$\Rightarrow \quad ar(\Delta ABC) = 0$

$\Rightarrow \quad \frac{1}{2}\left|-3(6-9)+7(9-12)+x(12-6)\right|=0$

$\Rightarrow \quad \left|9-21+6x\right|=0$

$\Rightarrow \quad \left|6x-12\right|=0$

$\Rightarrow \quad x=\frac{12}{6}=2$ **[1]**

12. If the equations kx – 2y = 3 and 3x + y = 5 represent two intersecting lines at unique point, then the value of k is __________. [1]

Answer For any real number except k = –6 [1]

Sol. kx – 2y = 3 and 3x + y = 5 represent lines intersecting at a unique point.

$\Rightarrow \quad \frac{k}{3} \neq \frac{-2}{1}$

$\Rightarrow \quad k \neq -6$

For any real number except k = –6

The given equation represent two intersecting lines at unique point.

OR

If quadratic equation $3x^2 - 4x + k = 0$ has equal roots, then the value of k is __________. [1]

Answer $\frac{4}{3}$ [1]

Quadratic equation $3x^2 - 4x + k = 0$ has equal roots

$\Rightarrow \quad D = b^2 - 4ac = 0$, where a = 3, b = –4 and c = k

$\Rightarrow \quad (-4)^2 - 4 \times 3 \times k = 0$

$\Rightarrow \quad 16 - 12k = 0$

$\Rightarrow \quad k = \frac{16}{12} = \frac{4}{3}$

13. The value of (sin20° cos70° + sin70° cos20°) is __________. [1]

Answer 1 [1]

Sol. sin20° cos70° + sin70° cos20°

= cos(90° – 20°)cos70° + sin70° sin(90° – 20°)

= cos 70° cos70° + sin70° sin70°

$= \cos^2 70° + \sin^2 70°$

= 1

14. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $A > B$, then the value of A is __________. [1]

Answer 45° [1]

Sol. $\tan(A+B) = \sqrt{3}$

$\Rightarrow \quad$ A + B = 60° ...(i)

Also, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad$ A – B = 30° ...(ii) $[\because A > B]$

On adding (i) and (ii), we get

2A = 90°

$\Rightarrow \quad$ A = 45°

15. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is ________. **[1]**

Answer 5.4 cm **[1]**

Sol. Let perimeters of two similar triangles be P_1 and P_2 and their corresponding sides be a_1 and a_2

$$\therefore \frac{P_1}{P_2} = \frac{a_1}{a_2}$$

$$\Rightarrow \frac{25}{15} = \frac{9}{a_2}$$

$$\Rightarrow a_2 = 5.4 \text{ cm}$$

(Q 16 – 20) Answer the following:

16. If $5\tan\theta = 3$, then what is the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$? **[1]**

Sol. $\tan\theta = \frac{3}{5}$

Now, $\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta} = \frac{5\tan\theta - 3}{4\tan\theta + 3}$ [Dividing numerator and denominator by $\cos\theta$] **[½]**

$$= \frac{3-3}{\frac{12+15}{5}}$$

$= 0$ **[½]**

17. The areas of two circles are in the ratio 9 : 4, then what is the ratio of their circumferences? **[1]**

Sol. Let radius of two circles be r_1 and r_2

$$\therefore \frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{4}$$

$\therefore \frac{r_1}{r_2} = \frac{3}{2}$ **[½]**

Now ratio of circumferences is $\frac{2\pi r_1}{2\pi r_2}$

$= \frac{r_1}{r_2} = \frac{3}{2}$ **[½]**

18. If a pair of dice is thrown once, then what is the probability of getting a sum of 8? **[1]**

Sol. Total outcomes = 36

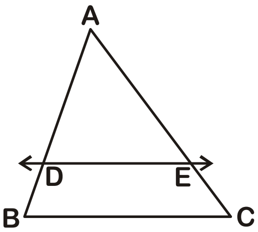
Favourable outcomes {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)} **[½]**

Number of favourable outcomes = 5

$P(\text{sum } 8) = \frac{5}{36}$ **[½]**

19. In Fig. 3, in $\triangle ABC$, DE || BC such that AD = 2.4 cm, AB = 3.2 cm and AC = 8 cm, then what is the legnth of AE? **[1]**

Fig. 3



Sol. $\because$ DE || BC

$\therefore \dfrac{AD}{AB} = \dfrac{AE}{AC}$ **[½]**

$\Rightarrow \dfrac{2.4}{3.2} = \dfrac{AE}{8}$

$\therefore$ $AE = \dfrac{24}{32} \times 8 = 6\,\text{cm}$ **[½]**

20. The nth term of an AP is (7 – 4n), then what is its common difference? **[1]**

Sol. $T_n = 7 - 4n$

$T_1 = 7 - 4(1) = 3$

$T_2 = 7 - 4(2) = 7 - 8 = -1$ **[½]**

$\therefore$ Common difference $= T_2 - T_1$

$= -1 - 3 = -4$ **[½]**

Section-B

Q. Nos. 21 – 26 carry two marks each.

21. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag. **[2]**

Sol. Let the number of blue balls be x.

So, total number of balls in the bag = (x + 5) **[½]**

According to the question,

$\dfrac{x}{x+5} = 3 \times \dfrac{5}{x+5}$ **[1]**

$\Rightarrow$ x = 15

$\therefore$ Number of blue balls = 15 **[½]**

22. Prove that $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta.$ **[2]**

Sol. L.H.S.

$= \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\dfrac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$ [On rationalisation] **[½]**

$= \dfrac{1-\sin\theta}{\cos\theta}$ $[\because 1-\sin^2\theta = \cos^2\theta]$ **[½]**

$= \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$ **[½]**

$= (\sec\theta - \tan\theta)$ **[½]**

L.H.S. = R.H.S.

OR

Prove that $\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}=1$ **[2]**

Sol. L.H.S.

$$=\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}$$

$$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta}+\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\csc^2\theta} \quad [\because \sec^2\theta=1+\tan^2\theta, \text{cosec}^2\theta=1+\cot^2\theta]$$ [½]

$$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}+\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}} \quad \left[\because \sec^2\theta=\frac{1}{\cos^2\theta}, \text{cosec}^2\theta=\frac{1}{\sin^2\theta}\right]$$ [½]

= $\sin^2\theta + \cos^2\theta$ [½]

= 1 [½]

L.H.S. = R.H.S.

23. Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5. **[2]**

Sol. Total number of outcomes = 6 × 6 = 36 [½]

Favourable outcomes = {(1, 1)(1, 2)(1, 3)(2, 1)(2, 2)(3, 1)} [½]

Number of favourable outcomes = 6 [½]

∴ P(less than 5) = $\frac{6}{36}=\frac{1}{6}$ [½]

OR

Find the probability that 5 Sundays occur in the month of November of a randomly selected year. **[2]**

Sol. In month of November 4 sundays are fixed.

But there are two extra days. They may be {(Sun, Mon), (Mon, Tues), (Tues, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat), (Sat, Sun)} **[1]**

Number of favourable outcomes = 2 [½]

∴ Required probability (5 sundays) = $\frac{2}{7}$ [½]

24. In Fig. 4, a circle touches all the four sides of a quadrilateral ABCD. If AB = 6 cm, BC = 9 cm and CD = 8 cm, then find the length of AD. **[2]**

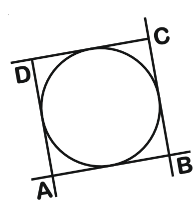
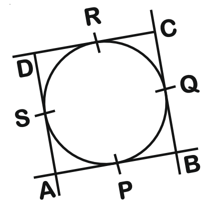


Fig. 4

Sol. $\because$ Tangents from external point are equal in length.



$\therefore$ AP = AS ...(1)

BP = BQ ...(2)

CR = CQ ...(3)

DR = DS ...(4)

Adding equations (1 + 2 + 3 + 4)

AP + BP + CR + DR = AS + BQ + CQ + DS **[1]**

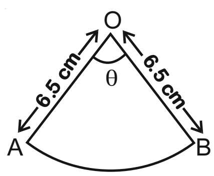
AB + CD = AD + BC

6 + 8 = AD + 9

AD = 14 – 9 = 5 cm **[1]**

25. The perimeter of a sector of a circle with radius 6.5 cm is 31 cm, then find the area of the sector. **[2]**

Sol.



Perimeter of sector OAB = OA + OB + length of arc AB = $\left(6.5+6.5+\frac{2\pi r\theta}{360^\circ}\right)$cm

$31 = 13 + 2 \times \pi \times r \times \frac{\theta}{360^\circ}$ **[½]**

$\frac{\pi r\theta}{360^\circ} = 9$ cm **[½]**

Area of sector = $\frac{\pi r^2\theta}{360^\circ}$

$= \frac{\pi r\theta}{360^\circ} \times r = 9 \times 6.5$ **[½]**

= 58.5 cm^2 **[½]**

26. Divide the polynomial $(4x^2 + 4x + 5)$ by $(2x + 1)$ and write the quotient and the remainder. **[2]**

Sol.

$$\begin{array}{r|l} & 2x+1 \\ 2x+1 & 4x^2+4x+5 \\ & -4x^2 \pm 2x \\ \hline & \quad 2x+5 \\ & \quad -2x \pm 1 \\ \hline & \qquad 4 \end{array}$$

[1½]

$\therefore$ Quotient on dividing $(4x^2 + 4x + 5)$ by $(2x + 1)$ is $2x + 1$ and remiander = 4 **[½]**

Section-C

Q. Nos. 27 to 34 carry 3 marks each.

27. If α and β are the zeroes of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$. **[3]**

Sol. α and β are zeroes of the polynomial $f(x) = x^2 - 4x - 5$

$\Rightarrow \quad \alpha + \beta = -\frac{b}{a} = 4$ and $\alpha\beta = \frac{c}{a} = -5$, where a = 1, b = –4, c = –5 **[1]**

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ **[½]**

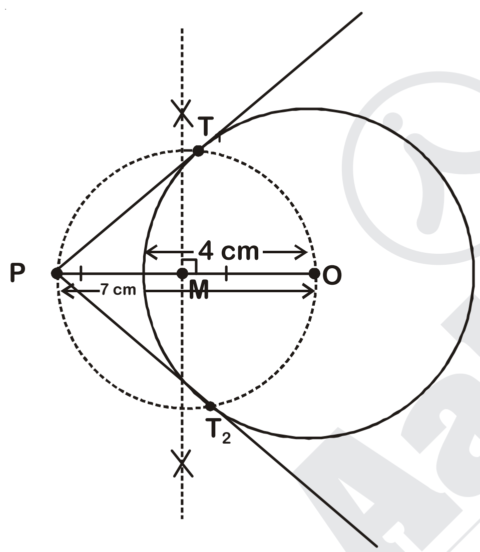
$= (4)^2 - 2(-5)$ **[½]**

$= 16 + 10$ **[½]**

$= 26$ **[½]**

28. Draw a circle of radius 4 cm. From a point 7 cm away from the centre of circle. Construct a pair of tangents to the circle. **[3]**

Sol. **[3]**

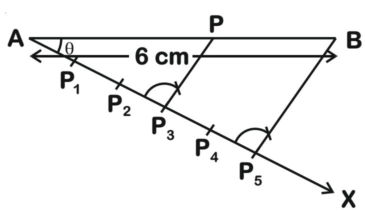


PT_1 and PT_2 are required tangents.

OR

Draw a line segment of 6 cm and divide it in the ratio 3 : 2.

Sol. **[3]**



Required AP : PB = 3 : 2

29. A solid metallic cuboid of dimension 24 cm × 11 cm × 7 cm is melted and recast into solid cones of base radius 3.5 cm and height 6 cm. Find the number of cones so formed. **[3]**

Sol. Volume of cuboid = $24 \times 11 \times 7 \text{ cm}^3$

Volume of 1 cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6 \text{ cm}^3$ **[1]**

Let no. of cones formed = n

$\therefore$ Volume of n cones = $n \times \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6 \text{ cm}^3$

Now, according of question

Volume of n cones = volume of cuboid

$\Rightarrow \quad n \times \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 6 = 24 \times 11 \times 7$ **[1]**

$n = \frac{24 \times 11 \times 7 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 24$ **[1]**

$\therefore$ Number of cones formed are 24.

30. Prove that (1 + tan A – sec A) × (1 + tan A + sec A) = 2 tan A **[3]**

Sol. LHS = (1 + tan A – sec A) × (1 + tan A + sec A)

$\because \quad (x - y)(x + y) = x^2 - y^2$

here x = 1 + tan A

y = sec A

LHS = $(1 + \tan A)^2 - (\sec A)^2$ **[1]**

$= 1 + \tan^2 A + 2 \tan A - \sec^2 A$ **[1]**

$= \sec^2 A + 2 \tan A - \sec^2 A \quad (1 + \tan^2 A = \sec^2 A)$

$= 2 \tan A$ = RHS **[1]**

Hence, proved.

OR

Prove that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

Sol. LHS = $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1}$

$= \operatorname{cosec}\theta\left(\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1}\right)$

$= \operatorname{cosec}\theta\left(\frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}\right)$ **[1]**

$= \operatorname{cosec}\theta\left(\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}\right)$

$= \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta} \quad \left[\begin{array}{l}\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta \\ \Rightarrow \operatorname{cosec}^2\theta - 1 = \cot^2\theta\end{array}\right]$ **[1]**

$= \frac{2 \times \frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} \quad \left[\begin{array}{l}\because \operatorname{cosec}\theta = \frac{1}{\sin\theta} \\ \cot\theta = \frac{\cos\theta}{\sin\theta}\end{array}\right]$

$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$ = RHS **[1]**

Hence, proved.

31. Given that $\sqrt{3}$ is an irrational number, show that $(5+2\sqrt{3})$ is an irrational number. **[3]**

Sol. Let $5+2\sqrt{3}$ be a rational number.

$5+2\sqrt{3}=\frac{p}{q}$, where p and q are co-prime integers. [½]

$\Rightarrow \quad 2\sqrt{3}=\frac{p}{q}-5$

$=\frac{p-5q}{q}$ [½]

$\Rightarrow \quad \sqrt{3}=\frac{p-5q}{2q}$ [½]

Here, $\frac{p-5q}{2q}$ is rational as p and q are integers. [½]

But it is given that $\sqrt{3}$ is irrational.

$\Rightarrow$ LHS is irrational and RHS is rational. [½]

Which contradicts our assumption that $5+2\sqrt{3}$ is a rational number.

$\therefore$ $5+2\sqrt{3}$ is an irrational number. [½]

Or

An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. For maximum number of columns, we need to find highest common factor i.e., HCF of 612 and 48. [½]

Now,

612 = 48 × 12 + 36 [½]

48 = 36 × 1 + 12 [½]

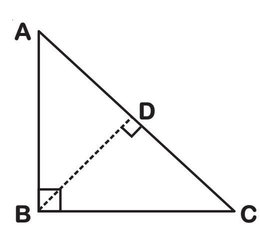
36 = 12 × 3 + 0 [½]

$\therefore$ HCF of 612 and 48 is 12. [½]

$\therefore$ Maximum number of columns in which they can march is 12. [½]

32. Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. **[3]**

Sol. Given : ΔABC is a right triangle right angled at B.



To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw BD $\perp$ AC.

Proof : In ΔABC and ΔADB, [½]

$\angle ABC = \angle ADB$ [Each 90°]

and $\angle BAC = \angle DAB$ [common]

$\therefore \Delta ABC \sim \Delta ADB$ [By AA] [½]

$\Rightarrow \quad \frac{AB}{AD}=\frac{AC}{AB}$ [Corresponding sides of similar triangles are proportional]

$\therefore \quad AB^2 = AC \times AD$ ...(i) [½]

Now,

In ΔABC and ΔBDC,

$\angle ABC = \angle BDC$ [Each 90°]

and $\angle ACB = \angle BCD$ [common]

$\therefore$ $\Delta ABC \sim \Delta BDC$ [By AA] [½]

$\Rightarrow \frac{AC}{BC} = \frac{BC}{CD}$ [Corresponding sides of similar triangles are proportional]

$\therefore$ $BC^2 = AC \times CD$ ...(ii) [½]

Adding equation (i) and (ii), we get

$AB^2 + BC^2 = AC \times AD + AC \times CD$

$= AC(AD + CD)$

$= AC \times AC = AC^2$

$\therefore$ $AC^2 = AB^2 + BC^2$

Hence, proved. [½]

Read the following passage carefully and then answer the questions given at the end.

33. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Fig. 5. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag.

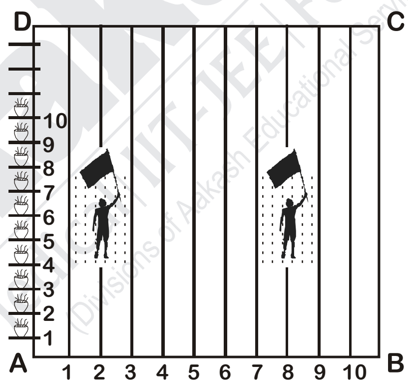


Fig. 5

(i) What is the distance between the two flags?

(ii) If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post the blue flag? [3]

Sol. AD = 100 × 1 m

= 100 m

Niharika runs $\frac{1}{4}$th of $AD = \frac{100}{4} = 25\text{ m}$ on 2nd line.

$\therefore$ Coordinates of green flag posted by Niharika are (2, 25)

Preet runs $\frac{1}{5}$th of $AD = \frac{100}{5} = 20\text{ m}$ on 8th line.

$\therefore$ Coordinates of red flag posted by Preet are (8, 20) [1]

(i) Distance between two flags $= \sqrt{(8-2)^2 + (20-25)^2}$

$$= \sqrt{6^2 + (-5)^2}$$

$$= \sqrt{36+25}$$

$$= \sqrt{61}\text{ m}$$ [1]

(ii) Mid-point of line segment joining the two flags = $\left(\frac{8+2}{2}, \frac{25+20}{2}\right)$

$$= \left(5, \frac{45}{2}\right) = (5, 22.5)$$

∴ Rashmi will post a blue flag on fifth line at the distance of 22.5 m. [1]

34. Solve graphically : 2x + 3y = 2, x – 2y = 8 [3]

Sol. Given lines are 2x + 3y = 2 and x – 2y = 8

2x + 3y = 2

$\Rightarrow \quad y = \frac{2-2x}{3}$

x	1	–2	4
y	0	2	–2

[½]

and x – 2y = 8

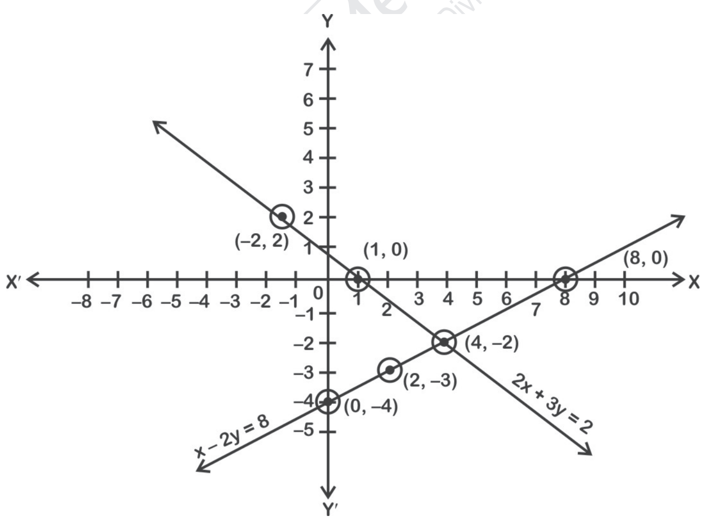
$\Rightarrow \quad y = \frac{x-8}{2}$

x	0	8	2
y	–4	0	–3

[½]

∴ We will plot the points (1, 0), (–2, 2) and (4, –2) and join them to get the graph of 2x + 3y = 2 and we will plot the points (0, –4), (8, 0) and (2, –3) and join them to get the graph of x – 2y = 8

[1½]



The graph of two given equations intersect at (4, –2)

∴ Solution of 2x + 3y = 2 and x – 2y = 8 is x = 4 and y = –2 [½]

Section-D

Q. Nos. 35 to 40 carry 4 marks each.

35. A two digit number is such that the product of its digits is 14. If 45 is added to the number; the digits interchange their places. Find the number. **[4]**

Sol. Let the units digit of the two digit number be x.

$\therefore$ Ten's digit will be $\frac{14}{x}$. **[½]**

According to question,

$10\times\frac{14}{x}+x+45=10x+\frac{14}{x}$ **[1]**

$\Rightarrow \frac{140}{x}+x+45=\frac{10x^2+14}{x}$

$\Rightarrow \frac{140+x^2+45x}{x}=\frac{10x^2+14}{x}$ **[½]**

$\Rightarrow 9x^2 - 45x - 126 = 0$

$\Rightarrow 9x^2 - 63x + 18x - 126 = 0$

$\Rightarrow 9x(x - 7) + 18(x - 7) = 0$ **[½]**

$\Rightarrow (x - 7)(9x + 18) = 0$

$\Rightarrow$ Either $x = 7$ or $x = -2$ **[½]**

$\therefore x = 7 \quad [\because x \neq -2]$

$\therefore$ Ten's digit $=\frac{14}{7}=2$ **[½]**

So, the number is 27. **[½]**

36. If 4 times the 4th term of an AP is equal to 18 times the 18th term, then find the 22nd term. **[4]**

Sol. Let the first term and common difference be a and d.

According to the question,

$4(a + 3d) = 18 \times (a + 17d)$ **[1]**

$\Rightarrow 4a + 12d = 18a + 306d$

$\Rightarrow 14a + 294d = 0$ **[1]**

$\Rightarrow a + 21d = 0$ **[1]**

$\therefore a_{22} = a + 21d$

$= 0$ **[1]**

OR

How many terms of the AP : 24, 21, 18, ... must be taken so that their sum is 78? **[4]**

Sol. Given A.P.

24, 21, 18,.....

$\therefore$ First term = 24 = a

and common difference = – 3 = d ...(i) **[1]**

Let no. of terms is n.

$\therefore$ Sum of n terms $=\frac{n}{2}[2a+(n-1)d]$ **[1]**

According to question

$\Rightarrow$ $78 = \frac{n}{2}[2 \times 24 - 3x(n-1)]$ [from (i) and given]

$\Rightarrow$ $78 = \frac{n}{2}[51 - 3n]$

$\Rightarrow$ $n^2 - 17n + 52 = 0$ **[1]**

$\Rightarrow$ $n^2 - 13n - 4n + 52 = 0$

$\Rightarrow$ $n(n - 13) - 4(n - 13) = 0$

$(n - 13)(n - 4) = 0$

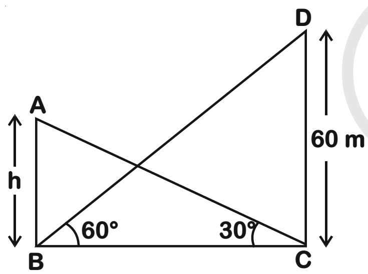
$n = 13, 4$

For first 4 terms and first 13 terms in both case we get sum 78. **[1]**

37. The angle of elevation of the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building. **[4]**

Sol. Let AB = h m be the height of building and CD be height of tower.

$\therefore$ CD = 60 m



[1]

In ΔBDC, $\tan 60° = \frac{CD}{BC}$

$\Rightarrow$ $BC = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ m ... (i) **[1]**

In ΔABC,

$\tan 30° = \frac{AB}{BC}$ **[1]**

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{AB}{20\sqrt{3}}$ [From (i)]

$\Rightarrow$ AB = 20 m

$\therefore$ Height of building = 20 m. **[1]**

38. In Fig. 6, DEFG is a squre in a triantgle ABC right angled at A.

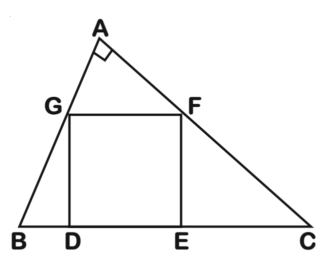
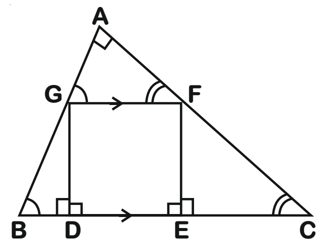


Fig. 6

Prove that

(i) $\Delta AGF \sim \Delta DBG$, (ii) $\Delta AGF \sim \Delta EFC$ **[4]**

Sol. Given : DEFG is a square and ΔABC is a right triangle right angled at A.



To prove : (i) $\Delta AGF \sim \Delta DBG$

(ii) $\Delta AGF \sim \Delta EFC$

Proof :

(i) In ΔAGF and ΔDBG

$\angle A = \angle D = 90°$

and $\angle AGF = \angle GBD = 90°$ ($\because$ GF || BC $\Rightarrow$ Corresponding angles) **[1]**

By AA similarity

$\Delta AGF \sim \Delta DBG$ **[1]**

(ii) In ΔAGF and ΔEFC

$\angle A = \angle E = 90°$

$\angle AFG = \angle ECF = 90°$ ($\because$ GF || BC $\Rightarrow$ Corresponding angles) **[1]**

By AA similarity

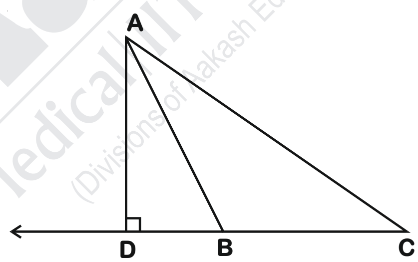
$\Delta AGF \sim \Delta EFC$ **[1]**

Hence proved.

OR

In an obtuse ΔABC($\angle B$ is obtuse), AD is perpendicular to CB produced. Then prove that $AC^2 = AB^2 + BC^2 + 2BC \times BD$.

Sol. Given : In ΔABC, $\angle B$ is obtuse angle.



AD $\perp$ CB produced.

To prove : $AC^2 = AB^2 + BC^2 + 2BC \times BD$ **[1]**

Proof : In ΔADC, $\angle D = 90°$

$AC^2 = AD^2 + DC^2$ ... (1) **[½]**

In ΔABD, $\angle D = 90°$

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2$ ... (2) **[½]**

From (1) and (2)

$AC^2 = AB^2 - BD^2 + DC^2$ **[½]**

$= AB^2 - BD^2 + (BD + BC)^2$ **[½]**

$= AB^2 - BD^2 + BD^2 + BC^2 + 2BC \times BD$

$AC^2 = AB^2 + BC^2 + 2BC \times BD$ **[1]**

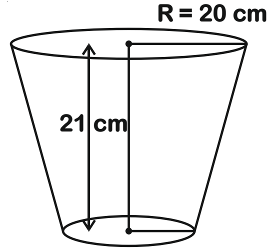
39. An open metal bucket is in the shape of a frustum of cone of height 21 cm with radii of its lower and upper ends are 10 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹ 40 per litre. **[4]**

Sol. For given frustum

h = 21 cm

r = 10 cm

R = 20 cm **[1]**



Volume of frustum = $\frac{1}{3}\pi(r^2+R^2+rR)h$ **[½]**

$=\frac{1}{3}\times\frac{22}{7}(100+400+200)\times 21$

$=\frac{1}{3}\times\frac{22}{7}\times 700\times 21$ **[½]**

= 15400 cm^3 | 1 litre = 1000 cm^3

= 15.4 litre | 1 cm^3 = $\frac{1}{1000}$ litre

∴ Total quantity of milk = 15.4 litre **[1]**

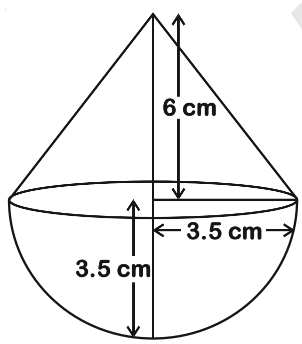
Cost of 1 litre milk = Rs. 40

∴ Cost of 15.4 litre milk = 15.4 × 40 = Rs. 616 **[1]**

OR

A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Sol. According to the question, we get following figure.



[1]

∴ Volume of solid = Volume of cone + volume of hemisphere

⇒ Volume $=\frac{1}{3}\pi r^2 h+\frac{2}{3}\pi r^3$ **[1]**

⇒ Volume $=\frac{1}{3}\pi(3.5)^2\times 6+\frac{2}{3}\pi(3.5)^3$

⇒ Volume $=\frac{1}{3}\pi(3.5)^2[6+3.5\times 2]$

⇒ Volume $=\frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}[6+7]$ **[1]**

⇒ Volume $=\frac{1}{3}\times\frac{22}{7}\times\frac{49}{4}\times 13$

$\Rightarrow$ Volume $= \frac{1}{3} \times \frac{2002}{4} = \frac{1001}{6}$ [½]

$\Rightarrow$ Volume $= 166\frac{5}{6}$ cm^3

$\therefore$ Volume of solid = $= 166\frac{5}{6}$ cm^3 [½]

40. Find the mean of the following data: [4]

Classes	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	20	35	52	44	38	31

Sol. [2]

Classes	x_i	f_i	A = 50 $d_i = x_i - A$	$u_i = \frac{x_i - A}{h}$ h = 20	f_iu_i
0-20	10	20	10 – 50 = –40	–2	–40
20-40	30	35	30 – 50 = –20	–1	–35
40-60	50	52	50 – 50 = 0	0	0
60-80	70	44	70 – 50 = 20	1	44
80-100	90	38	90 – 50 = 40	2	76
100-120	110	31	110 – 50 = 60	3	93
		$\Sigma f_i = 220$			$\Sigma f_iu_i = 138$

$\bar{x} = A + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$ [1]

$= 50 + \frac{138}{220} \times 20$

$= 50 + 12.55$

$= 62.55$ [1]

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