Standard - X

MATHEMATICS

ആമുഖം

കൊല്ലം ജില്ലാ പഞ്ചായത്തും പൊതുവിദ്യാഭ്യാസ വകുപ്പും കൊല്ലം ഡയറ്റിന്റെ അക്കാദമിക പിന്തുണയോടെ പത്താം ക്ലാസ്സിലെ വിദ്യാർത്ഥികൾക്ക് വേണ്ടി തയ്യാറാക്കിയിട്ടുള്ള പഠനസാമഗ്രിയാണ് **'ഉജ്ജ്വലം'**. എസ്.എസ്.എൽ.സി. പരീക്ഷ എഴുതുന്ന കൊല്ലം ജില്ലയിലെ മുഴുവൻ വിദ്യാർത്ഥികൾക്കും പഠനനേട്ടങ്ങൾ ഉറപ്പാക്കി പരീക്ഷാഫലം മെച്ചപ്പെടുത്തുക എന്നതാണ് പദ്ധതിയുടെ ലക്ഷ്യം. കോവിഡ് കാലഘട്ടത്തിൽ സ്വാഭാവിക ക്ലാസ്സന്തരീക്ഷം സാധ്യമല്ലാതിരുന്ന സാഹചര്യത്തിലുണ്ടായ പഠനവിടവ് പരിഹരിക്കുന്നതിനും വിദ്യാഭ്യാസ പ്രവർത്തനങ്ങൾക്ക് ശക്തിപകരുന്നതിനും വേണ്ടി നൂതന സാങ്കേതികവിദ്യയുടെ കൂടി സഹായത്തോടെ തയ്യാറാക്കിയ പഠനസാമഗ്രിയാണിത്. കഴിഞ്ഞ വിദ്യാലയ വർഷം എസ്.എസ്.എൽ.സി പരീക്ഷയിൽ 'A+' കളുടെ എണ്ണം വർദ്ധിപ്പിക്കുന്നതിന് ഉജ്ജ്വലത്തിലൂടെ സാധിച്ചിട്ടുണ്ട്. ഈ പഠനസഹായി ഉപയോഗിച്ചുകൊണ്ട് 60% പരീക്ഷാചോദ്യങ്ങൾക്ക് ഉത്തരം കണ്ടെത്താൻ കുട്ടികൾക്ക് കഴിഞ്ഞു. എസ്.എസ്.എൽ.സി. പരീക്ഷയെ സ്വാഭാവിക സന്ദർഭത്തിൽ ആത്മവിശ്വാസത്തോടെ ഏറ്റെടുക്കുന്നതിന് വിദ്യാർത്ഥികളെ സഹായിക്കുന്ന വിധത്തിലാണ് ഇതിലെ ഓരോ യൂണിറ്റും ക്രമീകരിച്ചിരിക്കുന്നത്. ഒപ്പം മാതൃകാചോദ്യങ്ങളും ഉൾപ്പെടുത്തിയിട്ടുണ്ട്. കുട്ടികൾക്ക് സ്വന്തമായും അദ്ധ്യാപകരുടെ ഇടപെടലോടെയും പഠനം ഉറപ്പാക്കുന്ന രീതിയിലാണ് ഇത് രൂപകല്പന ചെയ്തിരിക്കുന്നത്. ക്ലാസ്റൂം പഠന പ്രവർത്തനങ്ങളോടൊപ്പം സ്വാഭാവിക സന്ദർഭം കൂടി ഒരുക്കി കുട്ടികളെ ആത്മവിശ്വാസത്തോടെ പരീക്ഷ എഴുതാൻ പ്രാപ്തരാക്കുന്നതിന് അദ്ധ്യാപകർ ശ്രദ്ധിക്കേണ്ടതാണ്. **'ഉജ്ജ്വലം'** പഠനസാമഗ്രി പരമാവധി പ്രയോജനപ്പെടുത്തി പ്രതിജ്ഞാബദ്ധതയോടുകൂടി അക്കാദമിക പ്രവർത്തനങ്ങളിലേർപ്പെട്ട് ജില്ലയിലെ എസ്.എസ്.എൽ.സി വിജയം 100% ഉറപ്പാക്കുന്നതിന് എല്ലാവിധ ആശംസകളും നേർന്നുകൊള്ളുന്നു.

വിജയാശംസകളോടെ...

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CHAPTER - 1
ARITHMETIC SEQUENCES

1. A set of numbers written as the first , second , third and so on , according to a particular rule is called a number sequence .
2. A sequence got by starting with any number and adding a fixed number repeatedly is called an arithmetic sequence. The fixed number is called common difference (d).
3. The difference between any two terms of an arithmetic sequence is the product of the difference of positions and the common difference.

   If x_m and x_n are any two terms of an arithmetic sequence

   then $x_m - x_n = (m-n)d$.

   $$\text{Common difference, } d = \frac{x_m - x_n}{m - n} = \frac{\text{term difference}}{\text{position difference}}$$

4. All terms of an arithmetic sequence leave same remainder on division by common difference.
5. The algebraic expression of an arithmetic sequence is $x_n = an + b$ and the algebraic expression of sum of terms of an arithmetic sequence is $Sn = an^2 + bn$.

   If $x_n = an + b$ then $x_1 = a+b$ and $d = a$.

   If $s_n = an^2 + bn$ then $x_1 = a+b$ and $d = 2a$.

6. Number of terms of an arithmetic sequence, $= \dfrac{\text{last term} - \text{first term}}{\text{common difference}} + 1$
7. If a , b , c are three consecutive terms of an arithmetic sequence then

   $a + c = 2b$.

   If n is an odd number sum n terms = n × Middle term .

8. If n is an even number sum n terms

   = ½ n × sum of equidistant pair.

   $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 3\ (x_1 + x_6)$

   OR

   $= 3\ (x_2 + x_5)$

   OR

   $= 3\ (x_3 + x_4)$

9. The difference of sum of first n terms and the sum of next n terms of an arithmetic sequence with common difference 'd' is 'n^2d' .

10. The difference of corresponding terms of two arithmetic sequence with same common difference is equal.

    sum of first ‘n’ natural numbers,

    $$1 + 2 + 3 + \ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}$$

    sum of first ‘n’ odd numbers = n^2

    Sum of first ‘n’ even number = n (n+1)

PART 1 (2 Marks)

1. a) Write an arithmetic sequence with first term 5 and common difference 3.

   b) Find the 13^{th} term of the sequence.

2. a) Write an arithmetic sequence with common difference 5.

   b) Find 11^{th} term of the sequence.

3. a) What is the common difference of the sequence 7, 11, 15,....... ?

   b) Is 2022 a term of this sequence ? Justify your answer.

4. a) Find the common difference of the sequence 11, 21, 31, ...

   b) What is the first 3 digit term of this sequence ?

5. The algebraic expression of an arithmetic sequence is $3n+2$.

   a) What is its first term ?

   b) Find common difference.

6. Consider the sequence 20, 15 , 10.......

   a) Find the 10 th term of the sequence ?

   b) Write its algebraic expression ?

7. How much more the sum of first 30 terms of an arithmetic sequence 5,11,17 .... if the sum of first 30 terms of the arithmetic sequence 7,13,19....... ?

8. How much more the sum of first 40 terms of the arithmetic sequence 4,8,12 .... if the sum of first 40 terms of the arithmetic sequence 9,17,25....... ?

9. The sum of first 5 terms of an arithmetic sequence is 25 , sum of first 12 terms is 144 then what is the sum of first 17 terms ?

10. a) Write the sequence of two digit natural numbers leaves remainder 1 on division by 4.

    b) How many terms does it have ?

ANSWERS

1. a) 5 , 8 , 11,.........

   b) To get 13 th term add 12 times common difference to 5.

   13^{th} term , $X_{13} = 5 + 12 \times 3$

   $= 5 + 36$

   $= 41$

2. Take any number as the first term.

3. 7, 11, 15,.......

   a) common difference = 11– 7 = 4

   b) All terms of this sequence are odd numbers .

   But 2022 is an even number. So 2022 is not a term of this sequence.

4. 11, 21, 31, ...

   a) common difference = 10

   b) 101

5. First term = 3 + 2 = 5

   common difference = 3

6. 20, 15 , 10.......

   10^{th} term $= 20 + 9 \times -5$

   $= 20 - 45 = -25$

7. 5,11,17 .... 7,13,19.......

   The difference of terms at same positions is 2.

   So the difference of 30 terms = 30×2

   $= 60$

8. 4,8,12 .... 9,17,25.......

   The difference of terms : 5 , 9 , 13 , ... is an arithmetic sequence .

   Sum of first 40 terms

$= 20\ (10 + 39 \times 4)$

$= 1720$

9. $S_5 = 25 = 5^2$

$S_{12} = 144 = 12^2$

$S_{17} = 17^2 = 289$

10. a) 13,17, 21 97

b) Number of terms $= \dfrac{97-13}{4} + 1$

$= \dfrac{84}{4} + 1$

$= 21 + 1$

$= 22$

PART 2 (3 marks)

1. Complete the square such that numbers in the rows and numbers in the columns are in an arithmetic sequence .

2		18
10		34

2. a) Write the 10th term of the sequence a + 2 , a + 4 , a + 6 ....

b) Find its common difference .

c) Write the algebraic expression of the sequence.

3. Find the sum of terms of the sequences given below.

a) 1 + 2 + 3 + ... + 50

b) 2 + 4 + 6 + ... + 100

c) 3 + 6 + 9 + ... +150

4. The algebraic expression of an arithmetic sequence is 3n - 2 .

   a) What is its common difference?

   b) Find the remainder on dividing the terms this sequence by 3.

   c) Is 103 a term of this sequence ?

5. 8^{th} term of an arithmetic sequence is 24 and its 24^{th} term is 8.

   a) What is its common difference?

   b) Find the first term ?

   c) Find 32^{nd} term ?

6. a) Which is the first integer term of the sequence $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \ldots$.

   b) Find the sum of first 5 terms of this sequence.

   c) Find the sum of first 50 terms of this sequence.

7. 11^{th} term of an arithmetic sequence is 30.

   a) Find the sum of 4^{th} term and 18^{th} term of this sequence.

   b) Find the sum of first 22 terms of this sequence.

8. The sum of first 15 terms of an arithmetic sequence is 300.

   a) Find its 8^{th} term.

   b) If the first term is 6 , what is its common difference ?

   c) Write the algebraic expression of the sequence.

9. a) Which are the possible remainders got when a perfect square is dividing by 3 ?

   b) Prove that there is no perfect squares in the sequence 8,11,14 ....

10. The sum of 11^{th} term and 18^{th} term of an arithmetic sequence is 80. Its 23rd term is 60.

    a) Find its 6^{th} term.

    b) Find the sum of first 11 terms.

11. Write the sequence of natural numbers which leaves remainder 5 on division by 6.

    b) Write the algebraic expression of this sequence .

    c) Is 2000 a term of this sequence.

12. The sum of first and 15^{th} terms of an arithmetic sequence is 60.

    a) Find the sum of 2^{nd} and 14^{th} terms of this sequence?

    b) Find its 8^{th} term ?

    c) Find the sum of first 15 terms of this sequence ?

13. If 8^{th} term of an arithmetic sequence is 33 and the sum of first 20 terms is 860.

    a) Find the sum 10^{th} and 11^{th} terms.

    b) Find its 13^{th} term.

ANSWER

PART 2

1.

2	10	18
6	16	26
10	22	34

2. a) a + 2 , a+4 , a+6 ....

   10^{th} term = a + 20

   b) Common difference = 2

   c) Algebraic expression, $x_n = a + 2n$

3. a) 1+2+3+ ....+50 = = 1275

   b) 2+4+6+ ....+ 100 = 2 × 1275 = 2550

   c) 3+6+9+ ....+150 = 3 × 1275 = 3825

4. Algebraic expression, $x_n = 3n - 2$

   a) Common difference = 3

   b) Remainder on dividing the terms of this sequence by 3 = 3 - 2 = 1

   c) 103 is a term of this sequence.

5. 8th term = 24

24^{th} term = 8

a) Common difference = $\frac{8-24}{24-8}$

$= -1$

b) $x_1 + 7d = 24$

$x_1 + 7 \times -1 = 24$

$x_1 - 7 = 24$

$x_1 = 24 + 7$

First term = 31

c) 32^{nd} term = $x^1 + 31d$

$= 31 + 31x - 1$

$= 31 - 31$

$= 0$

6 . a) First integer term $= \frac{5}{5} = 1$

b) Sum of first 5 terms $= \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5} = \frac{15}{5} = 3$

Sum of next 5 terms $= \frac{6}{5} + \frac{7}{5} + \frac{8}{5} + \frac{9}{5} + \frac{10}{5}$

$$= \left(1+\frac{1}{5}\right) + \left(1+\frac{2}{5}\right) + \left(1+\frac{3}{5}\right) + \left(1+\frac{4}{5}\right) + \left(1+\frac{5}{5}\right) = 8$$

c) Sum of first 50 terms = 3 + 8 + 13 + ... 10 terms

= 255

7. 11^{th} term = 30

   a) Sum of 4th and 18th terms = 30×2

   $= 60$

   b) Sum of first 22 terms = 11 x 60 = 660

8. a) 8^{th} term = 20

   b) Common difference = 2

   c) Algebraic Expression, $x_n = 2n + 4$

9. a) Possible remainders got when a perfect square is dividing by 3 are 0 , 1

   b) Algebraic expression of an arithmetic sequence is 8,11,14 .... is 3n + 5.

   That is 3(n+1)+2

   So the remainder when any term of this sequence is dividing by 3 is 2.

   Perfect squares cannot be a term of this sequence.

10. Sum of 11^{th} term and 18^{th} term is 80.

$$x_{11} + x_{18} = 80 \text{ m}, \qquad x_{23} = 60$$

$$x_1 + 10d + x_1 + 17d = 80$$

$$2x_1 + 27d = 80 \qquad \dots\dots\dots\dots\dots\dots (1)$$

$$x_{23} = 60$$

$$x_1 + 22d = 60 \qquad \dots\dots\dots\dots\dots\dots (2)$$

$$(1) — (2) \Rightarrow \qquad x_1 + 5d = 20$$

   a) 6^{th} term = 20

   b) Sum of 11 terms = $11 \times 20 = 220$

11. Sequence of natural numbers which leaves remainder 5 on division by 6 is 11, 17 , 23 ....

   b) Algebraic expression of the sequence, $x_n = 6n - 1$

   c) 2000 is not a term of this sequence.

12. $x_1 + x_{15} = 60$

a) $x_2 + x_{14} = 60$

b) $x_8 = 30$

c) Sum of first 15 terms $= 15 \times 30 = 450$

13. $S_{20} = 860$, $X_8 = 33$

a) $x_{10} + x_{11} = 86$

b) $x_8 + x_{13} = 86$ $\quad x_{13} = 86 - 33 = 53$

PART 3 (4 marks)

1. If 2x + 1, 4x-1, 5x.....is an arithmetic sequence, then

a) Find x.

b) Find its first term.

c) Find the common difference .

d) Write the algebraic expression of the sequence .

2. Among three digit numbers which leaves remainder 1 on division by 9.

a) Which is the First number ?

b) Which is the Last number ?

c) Find the number of terms of the sequence ?

d) Find their sum ?

3. Find the sum of the series given below.

a) 1+2+3+ .... +40

b) 2+4+6+................ +80

c) 3+6+9+.............. +120

d) 4+7+10+...............+ 121

4. 3 times 7^{th} term of an arithmetic sequence is equal to 4 times its 10^{th} term . Find its 19^{th} term?

5. Sum of first 11 terms of an arithmetic sequence is 77.

a) Find its 6^{th} term.

b) If the common difference is 3, write the first two terms ?

c) Write another sequence with sum of first 11 terms is 77.

6. a) Write the sequence of odd numbers greater than 1.

   b) Write its algebraic expression .

   c) What is the algebraic expression of the sequence .

   d) Prove that no integer belongs to this sequence ?

7. The sum of 5^{th} and 21^{st} terms of an arithmetic sequence is 130 . Also the sum of 8^{th} and 28^{st} terms is 100

   a) Find the common difference ?

   b) Find its first term ?

   c) Can the difference of any two terms of this sequence be 2020 ? Why ?

8. The sum of first n terms of an arithmetic sequence is $2n^2 + 3n$

   a) Find the first term.

   b) Find the common difference.

   c) Write the sequence .

   d) Write the algebraic expression of the sequence .

PART 3 (Answers)

1. a) $2(4x-1) = 2x + 1 + 5x$

   $8x-2 = 7x + 1$

   $x = 1+2 = 3$

   b) First term = $2 \times 3 + 1 = 7$

   Second term = $4 \times 3 - 1 = 11$

   c) Common difference = $11-7 = 4$

   d) Algebraic expression of the sequence is $4n + 3$

2. Among three digit numbers leaves remainder 1 on division by 9.

a) Smallest number = 100

b) Largest number = 991

c) Number of three digit numbers leaves remainder 1 on division by 9 = $\frac{991-100}{9}+1$=100

d) Sum = 50 (100 + 991) =54550

3. a) 1+2+3+ .... + 40 = 820

b) 2+4+6+................ + 80 = 820 × 2 = 1640

c) 3+6+9+............... + 120 = 820 × 3 = 2460

d) 4+7+10+...............+ 121 = 2460 + 40 = 2500

4. $3x_7 = 4x_{10}$

$3x_7 = 4(x_7 + 3d)$

$3x_7 = 4x_7 + 12d$

$x_7 = -12d$

19th term , $x_{19} = x_7 + 12d$

$= -12d + 12d = 0$

5 Sum of 11 terms is 77

a) 6^{th} term = 7

b) $x_1+5d = 7$

$x_1= 7-5d$

$= 7 - 5x3$

$= 7 - 15 = -8$

First two terms = -8 , -5

c) -13 , -9 , -5 ......(when take common difference as 4)

6. (a) 3, 5, 7, 9 ......

(b) 2n + 1

(c) $\frac{2n+1}{8}$

(d) $\frac{\text{odd number}}{\text{even number}} \neq$ natural number

There is no natural numbers in this sequence.

7. a) $x_5 + x_{21} = 130$

$x_1+4d+x_1+20d=130$

$2x_1+24d=130$

$x_8+x_{28}=100$

$x_1+7d+x_1+27d=100$

$2x_1+34d=100$

$2x_1 + 24d + 10d = 100$

$130 + 10d = 100$

$10d = -30$

$d = -3$

b) $2x_1 + 24d = 130$

$2x_1 + 24x-3 = 130$

$2x_1 - 72 = 130$

$2x_1 = 130 + 72$

$2x_1 = 202$

$x_1 = 101$

8. $Sn = 2n^2 + 3n$

a) First term = $2\times1^2 + 3\times1 = 5$

b) d = 2x2= 4

c) Sequence : 5 , 9 , 13 ....

d) Algebraic expression : 4n + 1

PART 4 (5 marks)

1.

		1		
	2		3	
	4	5	6	
7	8	8	9	10

....

a) Write next two lines of this sequence ?

b) How many numbers are there in the 10th line of this pattern ?

c) What is the first term of 11th line ?

d) What is the last term of 11th line ?

e) Find the sum all numbers in 11th line ?

2.

4					
7	10				
13	16	19			
22	25	28	31		
...	...	...	...	...	
...	...	...	...	...	...

a) Write next two lines of this sequence ?

b) How many numbers are there in the 14th line of this pattern ?

c) What is the first term of 15th line ?

d) What is the last term of 15th line ?

e) Find the sum all numbers in 15th line ?

3. Consider the numbers between 100 and 500 which leaves remainder 1 on division by 3.

a) Which is the first number ?

b) Which is the last number ?

c) How many such numbers are there in this sequence ?

d) Find their sum.

4. If the sum of first 8 terms of an arithmetic sequence with common difference 4 is equal to the sum next 7 terms,

   a) what is the difference between first term and 9th term?

   b) what is the difference between 9th term and 15th term ?

   c) Find 8th term ?

   d) Find first term ?

5. If the terms of the sequence $\frac{2}{9}, \frac{3}{9}, \frac{4}{9}, ...$ are written as x_1, x_2, x_3 then

   a) $x_1+x_2+x_3 =$ ....

   b) $x_4 + x_5 + x_6 =$ ....

   c) $x_7 + x_8 + x_9 =$ ....

   d) Find the sum first 9 terms of the sequence ?

   e) Find the sum of first 300 terms of the sequence ?

Part 4 (Answers)

1.

1

2 3

4 5 6

7 8 9 10

....

a) 11, 12, 13, 14, 15

b) Total numbers in first 10 lines = 1 + 2 + 3 + ... + 10 = 55

c) 56

d) Sum = $\frac{11}{2}$ (56 + 56 + 10)= 671

2. a) 34 37 40 43 46

   49 52 55 58 61 64

   b) Number in first 14 lines = 1 + 2 + 3 + ... + 14 = 105

c) First number in 15th line = $4 + 105\ d = 319$

d) Last number = $319 + 14d = 319 + 14 \times 3 = 361$

e) Sum = $\frac{15}{2}(319 + 361) = \frac{15}{2} \times 680 = 5100$

3. Among numbers leaves remainder 1 on division by 3 lies between 200 and 500.

a) First number = 202

b) Last number = 499

c) $n = \frac{499 - 202}{3} + 1 = 100$

d) Sum = $50\ (202 + 499) = 35050$

4. $d = 4$

a) $x_9 - x_1 = 8d = 8 \times 4 = 32$

b) $x_{15} - x_7 = 8d = 8 \times 4 = 32$

c) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15}$

$$x_8 = (x_9 - x_1) + (x_{10} - x_2) + ... + (x_{15} - x_7)$$

$$x_8 = (32 + 32 + ... + 32)$$

$$= 32 \times 7 = 224$$

d) $x_1 = 224 - 7 \times 4$

$= 196$

5. a) $x_1 + x_2 + x_3 = 1$

b) $x_4 + x_5 + x_6 = 2$

c) $x_7 + x_8 + x_9 = 3$

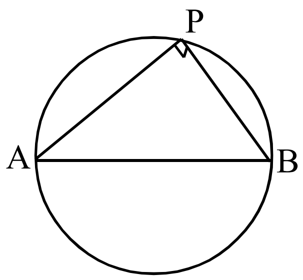
d) $1 + 2 + 3 = 6$

e) $1 + 2 + 3 + ... + 100 = 5050$

CHAPTER - 2
CIRCLE

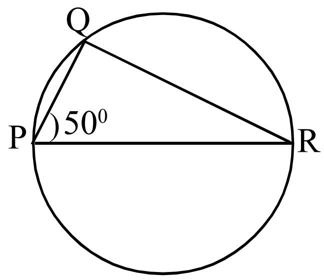
Previous knowledge

- In any circle, if the ends of diameter are joined to another point on a circle, then the angle at that point is right angle.

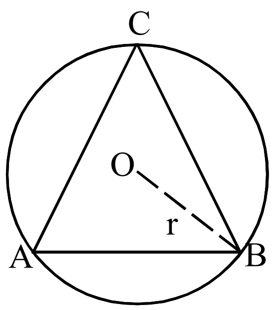


Activity - 1

In the figure, PQR is a triangle in which $\angle P = 50^0$ then $\angle Q =$ ...., $\angle R =$ ......



- Circle passes through all the vertices of the triangle is called its circumcircle.
- To construct the circumcircle of a triangle, take the point of intersection of any two sides of the triangle as centre and distance of centre from any vertices of the triangle as radius.



- Angle around a point is 360^0

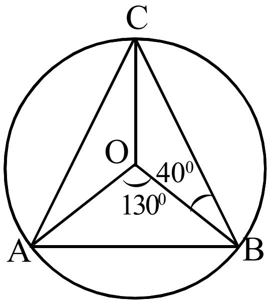
Activity - 2

In the figure, $\angle AOB$ is obtained by joining the ends of the chord AB with the centre of the circle.

(a) $\angle OAB$ = ...

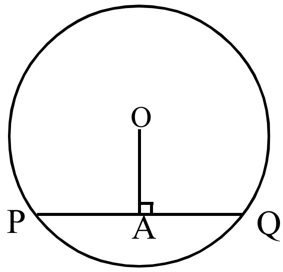
(b) $\angle OBC = 40^0$ then

$\angle BOC$ = ... and $\angle AOC$ = ...

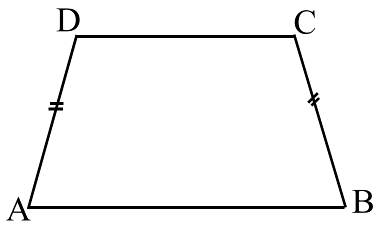


- Perpendicular from the centre to a chord bisect it. OA is perpendicular to PQ

$\therefore$ PA = AQ



- Base angles of an isosceles trapezium are equal.

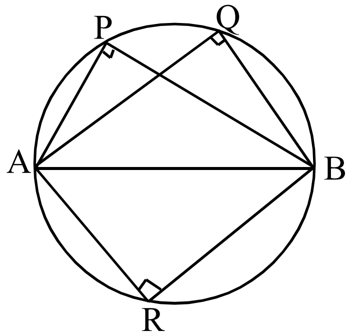


In the isosceles trapezium ABCD, AD = BC then $\Rightarrow$ $\angle A = \angle B$,

$\angle D = \angle C$

Important Concepts

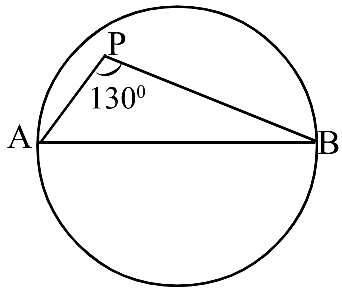
- Angle in a semicircle is right angle. That is, if we join the ends of a diameter of a circle to a point on the circle, we get a right angle $\angle APB = \angle AQB = \angle ARB = 90^0$



- If we join the ends of a diameter of a circle to a point inside the circle, the angle at the point is more than right angle.

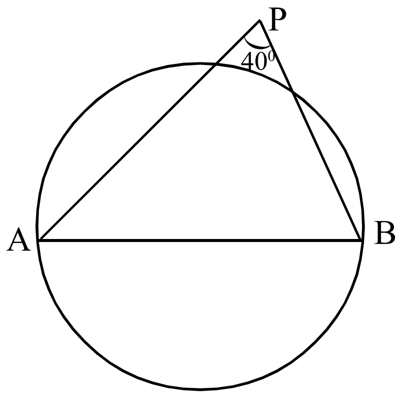
  $\angle APB > 90^0$

  $\angle APB = 130^0 > 90^0$

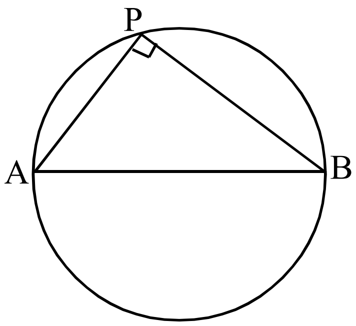


- If we join the ends of a diameter of a circle to a point outside the circle, the angle at that point is less than a right angle.

  $\angle APB = 40^0 \angle 90^0$

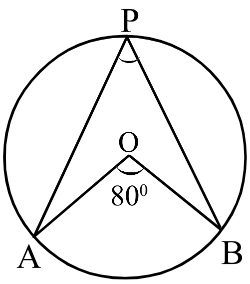


- All pairs of mutually perpendicular lines, drawn from the ends of a fixed line, meet on the circle with that line as diameter.



- The angle made by any are of a circle on the alternate are is half the angle made at the centre.

$\angle APB = \frac{1}{2} \angle AOB$



(i) In the figure,

$\angle AOB = 80^0$

then $<APB = \frac{1}{2} \times 80^0$

$= 40^0$

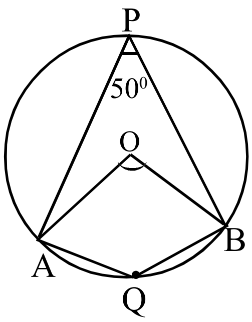
(ii) In the figure, $<APB = 50^0$

then $<AOB = ..., <AQB = ...$

$<AOB = 2\times 50 = 100^0$

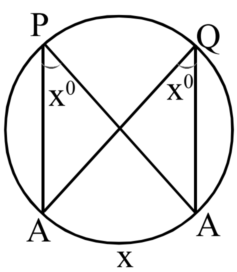
$<AQB = \frac{1}{2} (360^0 - <AOB)$

$= \frac{1}{2} \times 260^0 = 130^0$



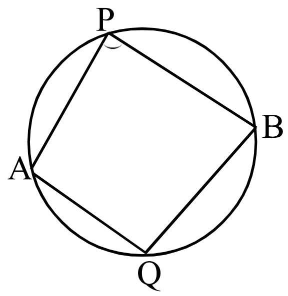
- All angles made by an are on the alternate are arc equal

Angle APB and AQB
are made by the are
AXB at the alternate are.
$\angle APB = AQB = x^0$



- A pair of angles on an are and its alternate are arc are supplementary.

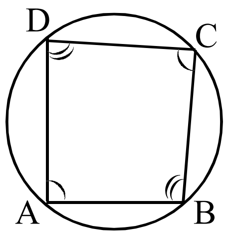
$\angle APB + \angle AQB = 180^0$



- If all four vertices of a quadrilateral are on a circle, then its opposite angles are supplementary.

$\angle A + \angle C = 180^0$

$\angle B + \angle D = 180^0$



In the figure, $\angle A = 80^0$, $\angle B=120^0$. Find the measures of $\angle C$ and $\angle D$?

$\angle C = 180^0 - \angle A$

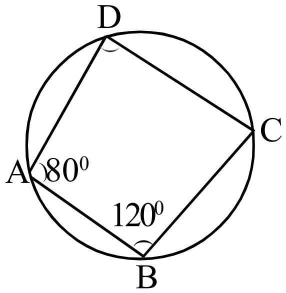
$= 180^0 - 80^0$

$= 100^0$

$\angle D = 180^0 - \angle B$

$= 180^0 - 120^0$

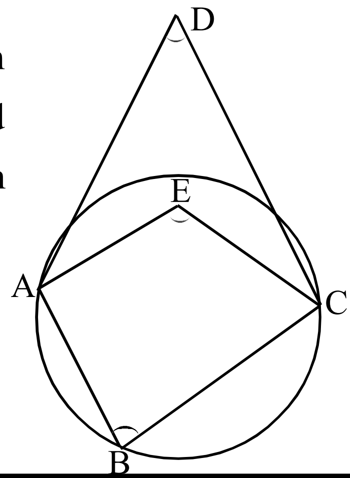
$= 60^0$



- If the opposite angles of a quadrilateral are supplementary, we can draw a circle passing through all four of its vertices.
- If one vertex of a quadrilateral is outside the circle drawn through the other three vertices, then the sum of the angles at this vertex and the opposite vertex is less than 180^0, if it is inside the circle the sum is more than 180^0.

$\angle B + \angle D < 180^0$

$\angle B + \angle E > 180^0$

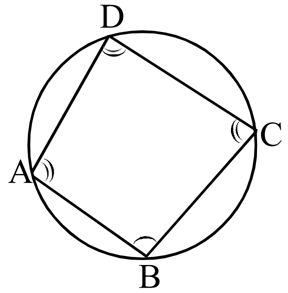


- If all the four vertices of a quadrilateral are on a circle, it is called a cyclic quadrilateral.

  Opposite angles of a cyclic quadrilateral is supplementary.

  $\angle A + \angle C = 180^0$

  $\angle B + \angle D = 180^0$



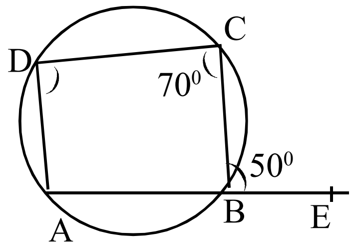
- The outerangle at vertex of a cyclicquadrilateral is equal to the inner angle at its opposite vertex.

  ie, $\angle CBE = \angle D$

  In the figure,

  $\angle CBE = 50^0$,

  $\angle C = 70^0$ then



  find the measures of all angles of quadrilateral ABCD?

  Here, $\angle D = \angle CBE = 50^0$

  $\angle C + \angle A = 180^0$

  $\therefore \angle A = 180^0 - \angle C$

  $= 180^0 - 70^0 = 110^0$

  $\angle ABC = 180^0 - 50^0 = 130^0$

- If two chords of a circle intersect within the circle, then the products of the parts of the two chords are equal.

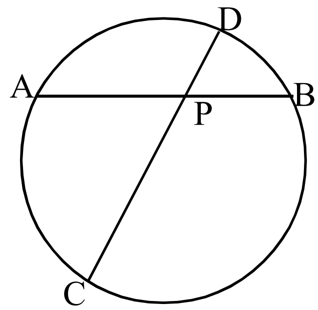
  $PA \times PB = PC \times PD$

  In the figure,

  AB = 18 cm, PB = 6 cm, PD = 8 cm

  Find PC?

  PB = 6 cm



$PA = 18 - 6 = 12$ cm

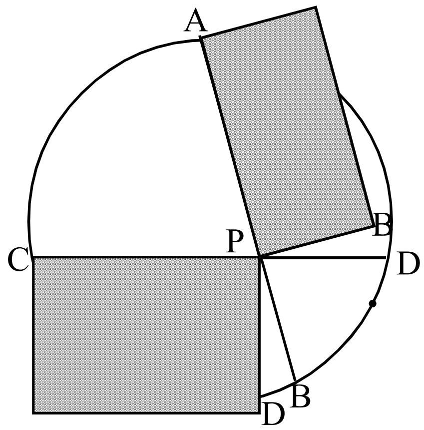
$PD = 8$ cm

$PA \times PB = PC \times PD$

$12 \times 6 = PC \times 8$

$PC = \frac{12 \times 6}{8} = 9$ cm

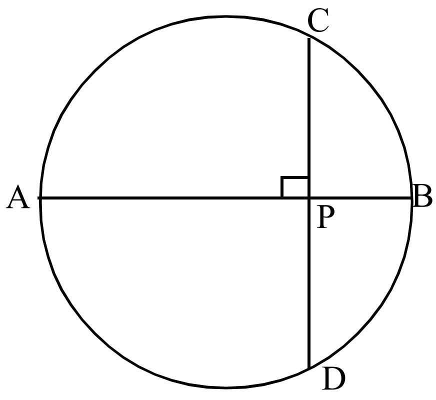
- If two chords of circle intersect within a circle, then the rectangles formed by the parts of the same chord have equal area.



- If AB is the diameter and CD is perpendicular to AB, then

$PA \times PB = PC^2$ ($\because$ PC = PD)

In the figure, PC = 6 cm, PB = 4 cm. Find the radius of the circle.



$PA \times PB = PC^2$

$PA \times 4 = 6^2$

$PA = 9$ cm

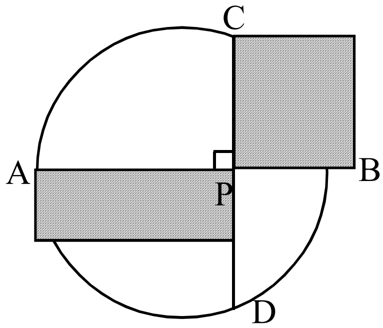
$AB = 9 + 4 = 13$ cm

Radius $= \frac{13}{2} = 6.5$ cm

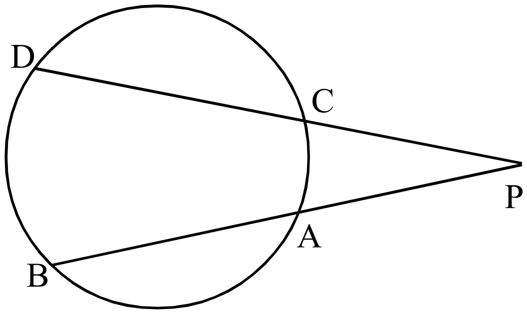
- The product of the parts into which a diameter of a circle is cut by a perpendicular chord, is equal to the square of half the chord.

$PA \times PB = PC^2$

- The area of the rectangle formed of parts into which a diameter of a circle is cut by a perpendicular chord is equal to the area of the square formed by half the chord.



- If two chords intersect at a point outside the circle, then PA × PB = PC × PD.

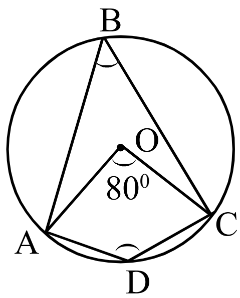


Part A 2 mark Questions

1. In the figure, 'O' is the centre of the circle. $\angle AOC = 80^0$.

   (a) What is the measure of $\angle ABC$?

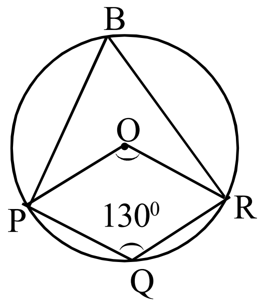
   (b) What is the measure of $\angle ADC$?



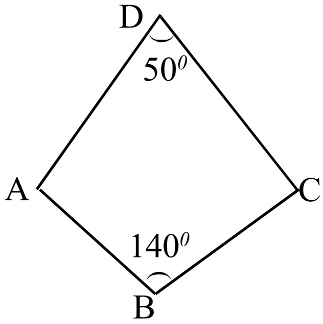
2. P, Q, R, S are points on the circle with centre 'O'. $\angle Q = 130^0$.

   (a) What is the measure of $\angle S$?

   (b) What is the measure of $\angle POR$?



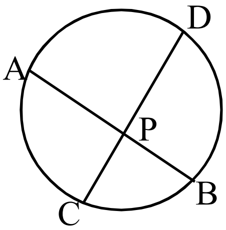
3. In the figure, If we draw a circle with AC as diameter, where is the position of the points B and D why?



4. Chord AB and CD meet at the point P. AB = 22 cm, PB = 6 cm, PD = 12 cm

(a) What is the length of PA?

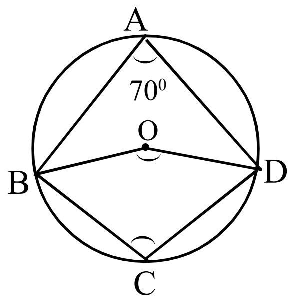
(b) What is the length of PC?



5. In the figure, 'O' is the centre of the circle. $\angle A = 70^0$.

(a) What is the measure of $\angle BOD$?

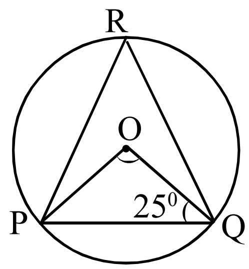
(b) What isa the measure of $\angle BCD$?



6. In the figure, 'O' is the centre of the circle. P,Q,R are points on the circle. $\angle OQP = 25^0$.

(a) $\angle POQ$ = ...

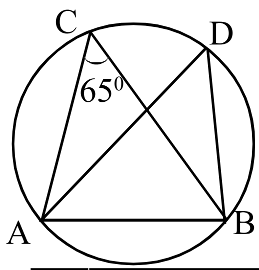
(b) $\angle PRQ$ = ...



7. In triangle ABC, $\angle C = 65^0$. AD is the diameter of the circle.

(a) What is the measure of $\angle ADB$?

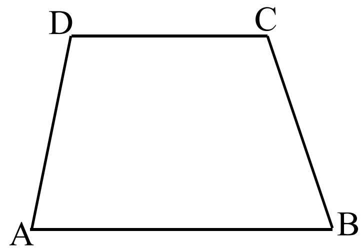
(b) What is the measure of $\angle ABD$?



8. ABCD is a quadrilateral in which AB and CD are parallel. $\angle A + \angle C = 195^0$

(a) If we draw a circle passes through A, B and D, then where will be the position of the vertex ‘C’?

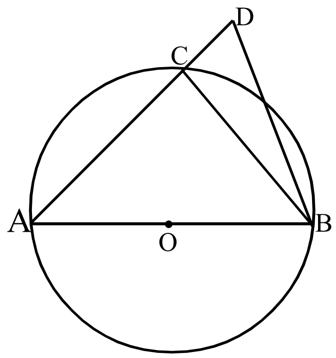
(Inside the circle, Outside the circle, on the circle)



(b) If AD = BC, can we draw a circle passes through A, B, C and D? Justify?

9. In the figure, AB is the diameter of the circle with centre. ‘O’. ‘C’ is a point on the circle. Among $\angle$ACB and $\angle$ADB, measure of one is double that of the other.

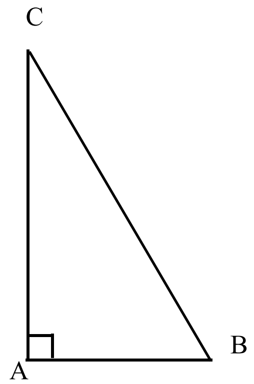
Find the measure of $\angle$ACB and $\angle$ADB?



10. In the figure, ABC is a right triangle.

(a) If we draw a circle with BC as diameter, where will be the position of the vertex ‘A’?

(b) If we draw a circle with AB as diameter, where will be the position of the vertex ‘C’?



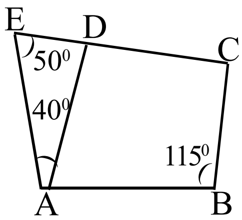
Part B

3 marks questions

1. In the figure, $\angle EAD = 40^0$, $\angle E=50^0$, $\angle B = 115^0$

   (a) Find the measure of $\angle ADC$?

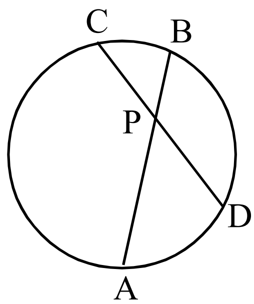
   (b) If we draw a circle through A, B and C where will be the position of E and D



2. Chords AB and CD of a circle are intersect at P. AB = 10cm, PB = 4 cm, PC = 3 cm.

   (a) Find the length of PA?

   (b) Find the length of PD?

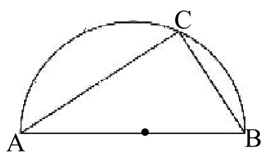


3. In the figure, ‘C’ is a point on the circle with diameter AB. then,

   (a) $\angle ACB$ = ...

   (b) If $\angle B = 2 \times \angle A$,

   then find the measures of $\angle A$ and $\angle B$?

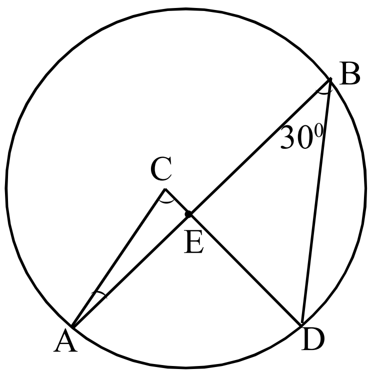


4. In the figure ‘C’ is the centre of the circle. $\angle ABD = 30^0$.

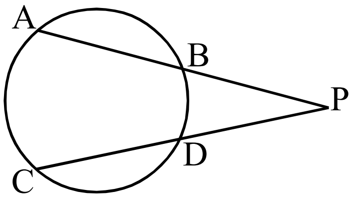
   (a) What is the measure of $\angle ACD$?

   (b) If $\angle ABD = \angle CAB$ and AB = 6 cm.

   Find the radius of the circle?



5. Draw an isosceles right triangle with hypotenuse 5 cm.
6. Draw a circle with radius 3 cm. Draw a triangle in which two angles are 40^0 and 60^0 and all vertices are points on the circle.
7. In the figure, chords AB and CD meet at a point P outside the Circle. PB = 13cm, AB = 8 cm, PD = 10 cm.

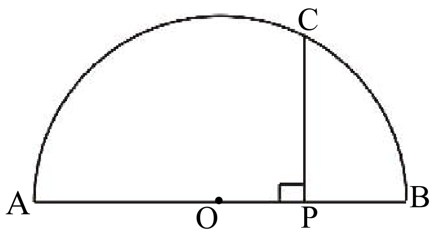


(a) What is PA?

(b) What is PC?

(c) What is CD?

8. Draw a square of area $12cm^2$.
9. In the figure, AB is the diameter of the semicircle. PC is perpendicular to AB. Length of PA is 40 cm more than that of PB.

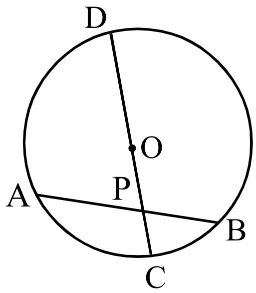


(a) Consider PB and x, find the length of PA?

(b) What is the relation between PA, PB and PC?

(c) If PC = 15 cm, find the diameter of the semicircle?

10. In the figure, Radius of the circle with centre 'O' is 5 cm. PA = 5 cm, PB = 4 cm. Take the length of OP as 'x', then

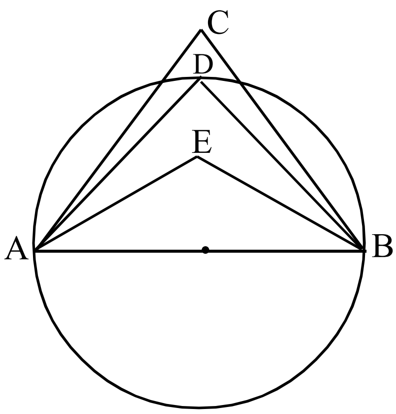


(a) PC = ...

(b) PD = ...

(c) Find the length of OP?

11. In the figure, AB is the diameter of the circle and D is a point on the circle.



$\angle ACB + \angle ADB + \angle AEB = 260^0$

Measure of one among these angles is 110^0. Find the measures of angles $\angle AEB$, $\angle ADB$ and $\angle ACB$?

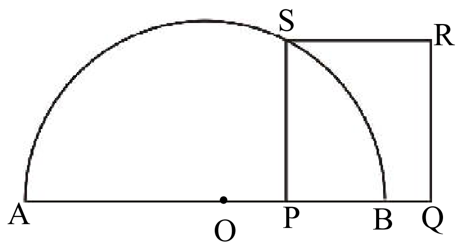
12. In the figure, radius of the circle with centre ‘O’ is 5 cm.

PB = 2cm

(a) What is the length of OP?

(b) What is the length of PA?

(c) What is the area of the square PQRS?



Part - C

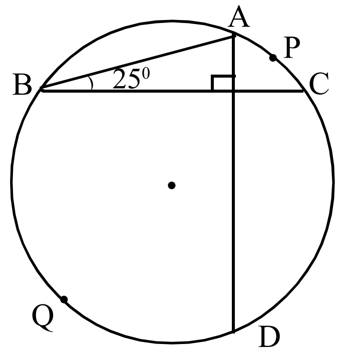
4 marks questions

1. AB and CD are two mutually perpendicular chords in a circle with centre ‘O’. $\angle B = 25^0$

(a) What is the measure of $\angle A$?

(b) What is the central angle of arc BQD?

(c) What is the central angle of arc ADC?



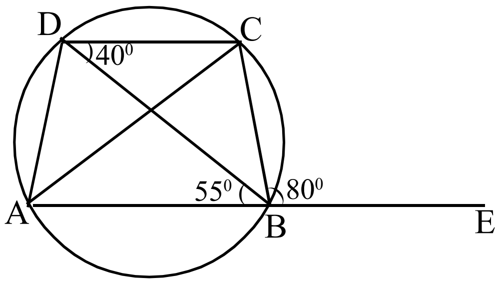
2. In the figure, ABCD is a cyclic quadrilateral.

$\angle CBE = 80^0$, $\angle BDC = 40^0$

$\angle ABD = 55^0$

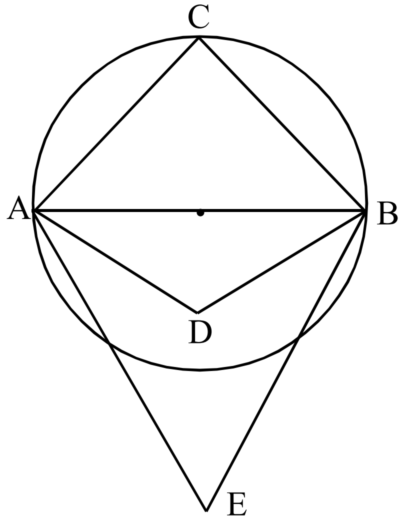
(a) Find the measure of $\angle DBC$?

(b) Find the measure of all angles of triangle ABC?



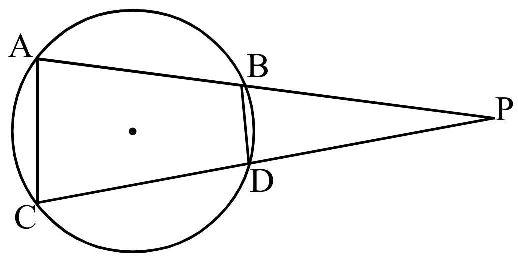
3. In the figure, AB is the diameter of the circle. For the angles given in column I choose suitable measures from column-II

Column - I	Column - II
$\angle ACB$	140^0
$\angle ADB$	90^0
$\angle AEB$	180^0
	70^0



4. In the figure, chords AB and CD proceeded to meet at 'P'.

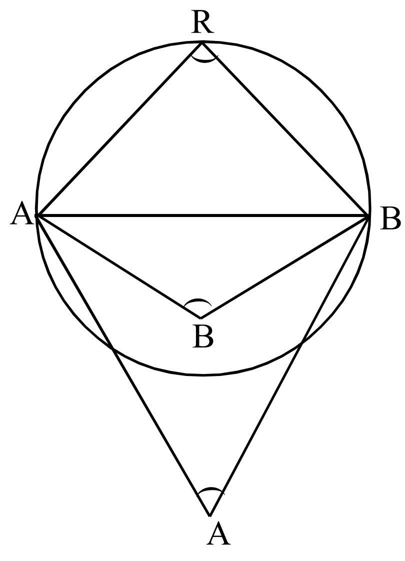
(a) If $\angle A = 80^0$, $\angle C = 60^0$ then find the measures of $\angle P$ and $\angle PBD$?



(b) If $\angle P = 40^0$, PB = PD then find the measures of $\angle A$ and $\angle C$?

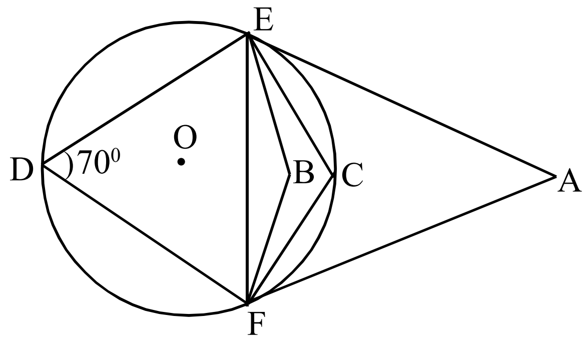
5. In the figures PQ is the diameter of the circle. R is a point on the circle. Measures of $\angle PQR$, $\angle A$, $\angle R$ and $\angle B$ are in an arithmetic sequence.

The measure of one among there angles is 70^0, then find the measures of $\angle PQR$, $\angle A$, $\angle R$ and $\angle B$.



6. In the figure, C, D, E and F are points on the circle. $\angle D = 70^0$. For the angles given in column I, choose suitable measures from column II.

Column-I	Column II
$\angle$ECF	120^0
$\angle$EBF	60^0
$\angle$EAF	110^0
	180^0

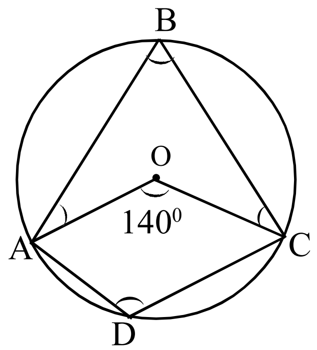


7. In the figure, 'O' is the centre of the circle and $\angle AOC = 140^0$.

(a) Find the measure of $\angle$ABC?

(b) Find the measure of $\angle$ADC?

(c) Calculate $\angle OAB + \angle OCB$.

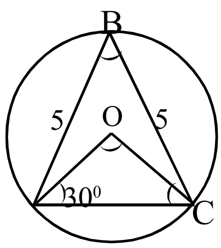


8. A, B, C are points on the circle with centre 'O'. $\angle OAC = 30^0$, BC = AB = 5 cm

(a) Find the measure of $\angle$OCA?

(b) Find the measures of $\angle$AOC and $\angle$A BC?

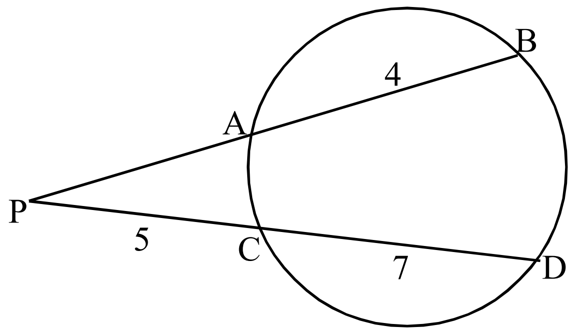
(c) Find the length of AC?



Part D

5 marks questions

1. In the figure, chords AB and CD meet at a point 'P' out side the circle. AB = 4 cm, PC = 5cm, CD = 7cm.



(a) What is the length of PD?

(b) Consider PA as x, ind the length of PB?

(c) Write the relation between PA, PB, PC and PD.

(d) Find the length of PA?

2. (a) Draw a rectangle with sides 6 cm and 4 cm.

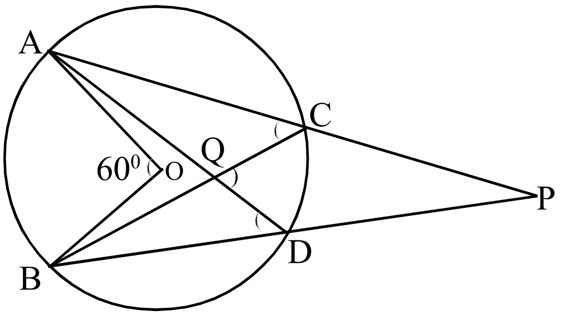
(b) Draw a square having the same area as that of the rectangle.

3. In the figure, 0 is the centre of the circle. A, B, C and D are points on the circle. $\angle AOB = 60^0$.

(a) Write the measures of $\angle ACB$ and $\angle ADB$.

(b) Write the measures of $\angle BCP$ and $\angle ADP$.

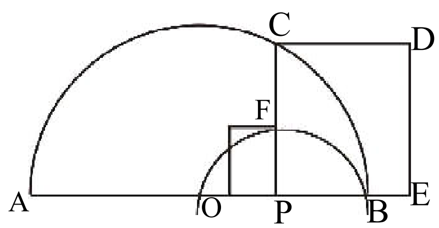
(c) Find $\angle CQD + \angle P$.



4. In the figure, radius of the circle with centre 'O' is 5 cm. PB = 2 cm.

(a) What is the length of OP?

(b) What is the length of PA?



(c) What is the area of the square PCDE?

(d) What is the area of the square with side PF.

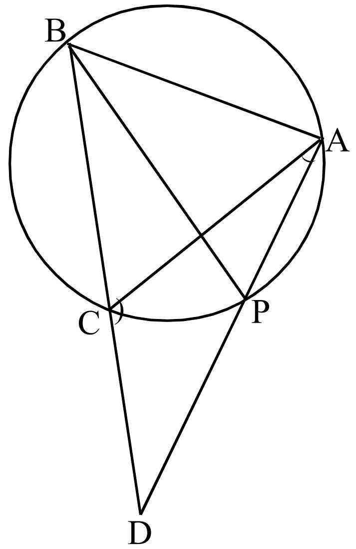
5. In the figure, AB = AC. Bisector of $\angle$B meets the circle at the point P.

(a) Write the angle which is equal to $\angle$CAP.

(b) Consider $\angle$CAP as x, what is the measure of $\angle$ACD?

(c) Find the measure of $\angle$D?

(d) Show that CA = CD.



6. In the figure, O is the centre of the circle, A, B, C and D are points on the circle. Compute the angles given below.

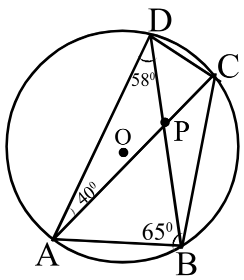
(a) $\angle$ACB = ...

(b) $\angle$CBD = ...

(c) $\angle$BCD =..............

(d) $\angle$BAC = ...

(e) $\angle$DPC = ...

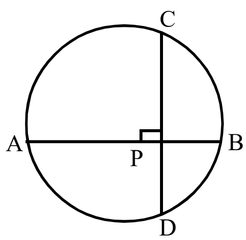


7. In the figure, AB is the diameter of the circle. CD is perpendicular to AB. They meet at the point P. PA = 6 cm, PB = 2 cm.

(a) Write the relation between PA, PB and PD.

(b) What is the length of PD?

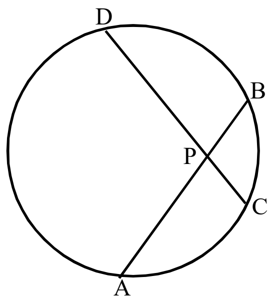
(c) Draw an equilateral triangle of side $4\sqrt{3}$ cm.



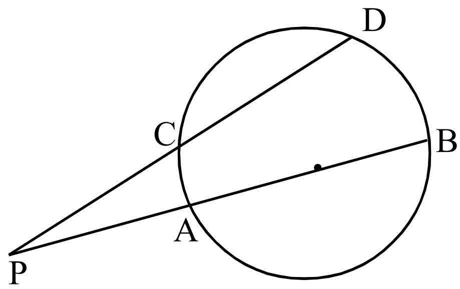
8. In the figure chords AB and CD interest at ‘P’. AB = 19 cm, PA = 15 cm

(a) Find the length of PB?

(b) If PC : PD = 5:3, then find the length of CD?



9. AB is the diameter of the circle. PC = CD = 6 cm, Length of PA is 3 cm less than the radius.



(a) What is the length of PD?

(b) Take the radius as ‘x’ and then write PA?

(c) Write the length of PB?

(d) Find the radius of the circle?

Answers

Part A (2 marks)

1. (a) $\angle ABC = \frac{1}{2} \times 80^0 = 40^0$

(b) $\angle ADC = 180^0 - 40^0 = 140^0$

2. (a) $\angle S = 180^0 - 130^0 = 50^0$

(b) $\angle POR = 2 \times 50^0 = 100^0$

3. Since the measure of $\angle B$ is 140^0 (more than 90^0), position of B is inside the circle.

Since the measure of $\angle D$ is 50^0 (less than 90^0), the position of D is outside the circle.

4. (a) PA = AB – PB

$= 22 - 6 = 16$ cm

(b) PA × PB = PC × PD

$16 \times 6 = PC \times 12$

$PC = \frac{16 \times 6}{12} = 8$ cm.

5. (a) $\angle BOD = 2 \times 70^0 = 140^0$

(b) $\angle BCD = 180^0 - 70^0 = 110^0$

6. (a) OP = OQ

$\therefore \angle OPQ = \angle OQP = 25^0$

$\therefore \angle POQ = 180^0 - (25^0 + 25^0)$

$= 180^0 - 50^0$

$= 130^0$

(b) $\angle PRQ = 65^0$

7. (a) $\angle ADB = 65^0$

(b) $\angle ABD = 90^0$ ($\because$ AD is the diameter of the circle)

8. (a) inside the circle.

(b) yes, If AD = BC; ABCD is an isosceles trapezium.

Since enery isosceles trapezium is cyclic; ABCD is also cyclic.

9. Since 'C' is a point on the circle and AB is a diameter,

$\angle ACB = 90^0$

$\therefore \angle ADB = 45^0$ (One is the double of other)

10.(a) Position of A is on the circle.

$(\because \angle A = 90^0)$

(b) Position of C is outside the circle.

$(\because \angle C$ is less than $90^0)$

Part B (3 marks)

1. (a) $= \angle ADC = 40^0 + 50^0$ (outer angle of a triangle is sum of its oppisite inner angles)

(b) Since $\angle E + \angle B = 165^0$, position of E will be outside the circle.

Since $\angle D + \angle B = 205^0$, position of D will be inside the circle.

2. (a) PA = 10 – 4 = 6 cm

(b) $PA \times PB = PC \times PD$

$6 \times 4 = 3 \times PD$

PD = 8 cm

3. (a) $\angle C = 90^0$

(b) $\angle A + \angle B + \angle C = 180^0$

$\angle A + 2\angle A + 90^0 = 180^0$

$3\angle A = 90$

$\angle A = 30^0$

$\angle B = 2 \times 30^0 = 60^0$

4. (a) $\angle ACD = 2 \times 30^0 = 60^0$

(b) $\angle ABD = \angle CAB = 30^0$

$\angle AEC = 90^0$

$AB = 6$ cm.

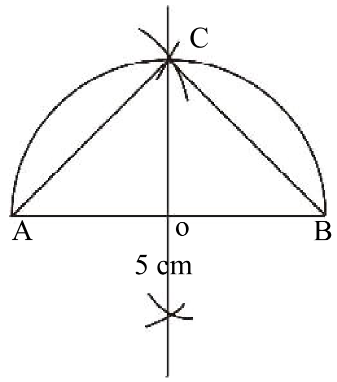
Since the perpendicular from the centre to a chord bisect it,

$AE = \frac{AB}{2} = 3$ cm

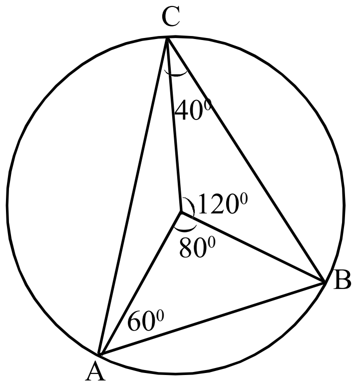
Radius $= AC = \frac{3}{\sqrt{3}} \times 2 = \frac{6}{\sqrt{3}}$

$= 2\sqrt{3}$ cm

5. * Draw AB = 5 cm
   * Draw the perpendicular bisector of AB
   * Draw a semicircle with radius OA
   * Join AC and BC.



6. * Draw a circle of radius 3 cm
   * Draw radius and mark 80^0 at the centre.
   * Again mark 120^0 at the centre.
   * Join AB, BC and AC.



7. PB = 12 cm, AB = 8 cm

(a) PA = PB + AB = 12 + 8 = 20 cm

PD = 10 cm

(b) PA × PB = PC × PD

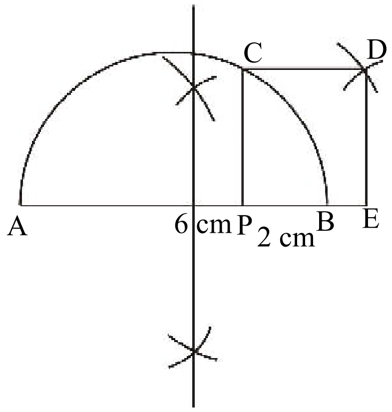
20 × 12 = PC × 10

PC = 24 cm

(e) CD = PC – PD

= 24-10 = 14 cm

8. $12 = 6 \times 2$



* Draw AB = 8 cm (6 + 2)
* Draw the perpendicular bisector of AB.
* Draw a semicircle with AB as diameter.
* Draw a perpendicular at 'P'.
* Complete the square PEDC with PC as side.

9. (a) $PB = x$

$PA = x + 40$

(b) $PA \times PB = PC^2$

(c) $(x + 40)\, x = (15)^2$

$x^2 + 40x = 225$

$x^2 + 40x + 400 = 625$

$(x + 20)^2 = (25)^2$

$x + 20 = 25$

$x = 25 - 20 = 5$

$\therefore$ PB = 5 cm

PA = 5 + 40 = 45 cm

Diameter AB = PA + PB

= 45 + 5

= 50 cm

10. PA = 5 cm

PB = 4 cm

OC = OD = 5 cm

(a) PC = 5 - x

(b) PD = 5 + x

(c) $PA \times PB = PC \times PD$

$5 \times 4 = (5-x)(5+x)$

$20 = 25 - x^2$

$x^2 = 5$

$x = \sqrt{5}$ cm.

11. $\angle ACD + \angle ADB + \angle AEB = 260^0$

But $\angle ADB = 90^0$

$\angle AEB = 110^0$

$\therefore \angle ACB + 90^0 + 110^0 = 260^0$

$\angle ACB + 200^0 = 260^0$

$\therefore \angle ACB = 260^0 - 200^0 = 60^0$

12. OA = OB = 5 cm

PB = 2 cm

(a) OP = OB – PB = 5 – 2 = 3 cm

(b) PA = OA + OP = 5 + 3 = 8 cm

(c) Area = $PS^2 = PA \times PB$

$= 8 \times 2 = 16\ cm^2$

Part C (4 mark)

1. (a) $\angle A = 65^0$

(b) Central angle of arc BQD = $2 \times 65^0 = 130^0$

(c) Central angle of arc APC = $2 \times 25^0 = 50^0$

2. (a) $\angle DBC = 180^0 - (55^0 + 80^0)$

$= 180^0 - 135^0 = 45^0$

(b) $\angle ABC = 55^0 + 45^0 = 100^0$

$\angle BAC = \angle BDC = 40^0$

$\angle ACB = 180 - (100^0 + 40^0)$

$= 40^0$

3. $\angle ACB = 90^0$

$\angle ADB = 140^0$

$\angle AEB = 70^0$

4. (a) $\angle P = 180^0 - (80^0 + 60^0)$

$= 180^0 - 140^0$

$= 40^0$

$\angle PBD = \angle C = 60^0$

(b) $\angle P = 40^0$, PB = PD

$\angle A = \angle C = 70^0$

5. $\angle A = 70^0$ ($\because$ A is outside the circle)

$\angle PRQ = 90^0$

Since $\angle PQR$, $\angle A$, $\angle R$ and $\angle B$ are in arithmetic sequence, common difference is $\angle R - \angle A = 90 - 70 = 20^0$

$\therefore \angle PQR = 70^0 - 20^0 = 50^0$

$\angle B = 90^0 + 20^0 = 110^0$

6. $\angle ECF = 110^0$

$\angle EBF = 120^0$

$\angle EAF = 60^0$

7. (a) $\angle ABC = \dfrac{140^0}{2} = 70^0$

(b) $\angle ADC = 180^0 - 70^0 = 110^0$

(c) $\angle OAB + \angle OCB = \angle ABC = 70^0$

8. (a) $\angle OCA = 30^0$

(b) $\angle AOC = 120^0$

(c) $\angle ABC = 60^0$

(d) AC = 5 cm ($\because$ ABC is an equilateral triangle)

Part D (5 marks)

1. (a) PD = 5 + 7 = 12 cm

(b) PB = x + 4

(c) PA × PB = PC × PD

(d) x (x + 4) = 5 × 12

$x^2 + 4x = 60$

$x^2 + 4x + 4 = 64$

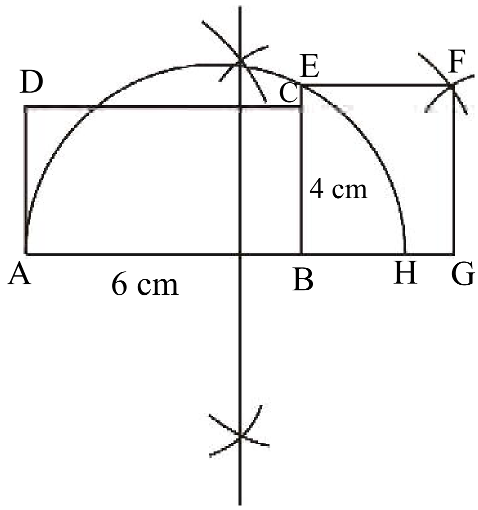
$(x+2)^2 = (8)^2$

$x + 2 = 8$

$x = 8 - 2 = 6$

PA = 6 cm

2.



* Draw the rectange ABCD.

* Exlend AB such that BH = 4 cm

* Draw a semicircle with diameter AH.

* Produce BC to BE.

* Complete the square with side BE.

3. (a) $\angle ACB = \angle ADB = \frac{1}{2} \times 60^0 = 30^0$

(b) $\angle BCP = \angle ADP = 180^0 - 30^0 = 150^0$

(c) QDPC is a quadrilateral.

$\therefore \angle CQD + \angle P = 360^0 - (150^0 + 150^0)$

$= 360^0 - 300^0$

$= 60^0$

4. OA = OB = 5 cm

PB = 2 cm

(a) OP = 5 –2 = 3 cm

(b) PA = 5 + 3 = 8 cm

(c) Area of the square PCDE = PC^2

$= PA \times PB$

$= 8 \times 2 = 16\ cm^2$

(d) Area of the square with side PF

$= PF^2$

$= PO \times PB$

$= 3 \times 2 = 6\ cm^2$

5. (a) $\angle CAP = \angle CBP$

(b) $\angle CAP = x^0 \Rightarrow \angle CBP = x^0$.

$\therefore \angle PBA = x^0$ ($\because$ BP bisects $\angle B$)

$\because \angle CBA = x^0 + x^0 = 2x^0$

Since AB = AC

$\angle ACB = \angle CBA = 2x^0$

$\because \angle ACD = 180^0 - 2x^0$

(c) $\angle D = 180^0 - (180^0 - 2x^0 + x^0)$

$= x^0.$

(d) Since $\angle D = \angle CAD = x^0$,

CA = CD

6. (a) $\angle ACB = 58^0$

(b) $\angle CBD = 40^0$

(c) $\angle BCD = \angle BCA + \angle ACD$

$= 58^0 + 65^0$

$= 123^0$

(d) $\angle BAC = 180^0 - (\angle ACB + \angle ABC)$

$= 180^0 - (58^0 + 105^0)$

$= 180^0 - 163^0$

$= 17^0$

(e) $\angle DPC = 180^0 - (\angle BDC + \angle ACD)$

$= 180^0 - (17^0 + 65^0)$

$= 180^0 - 82^0$

$= 98^0$

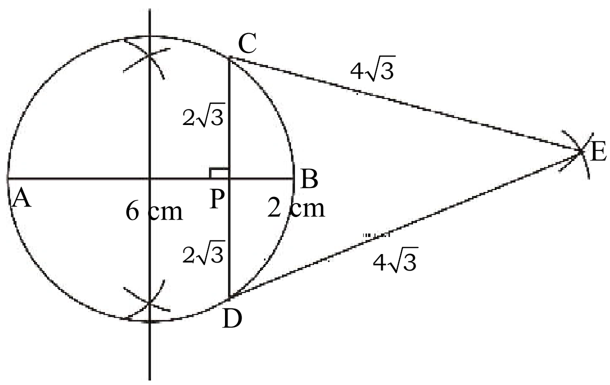
7. (a) $PA \times PB = PD^2$

(b) $PD^2 = 6 \times 2$

$= 12$

$PD = \sqrt{12} = 2\sqrt{3}$ cm

(c)



* Draw AB = 8 cm
* Draw a circle with AB as diameter
* Mark P on AB at a distance of 6 cm from A.
* Draw CD through P.

$PC = PD = 2\sqrt{3}$ $(PA \times PB = PC^2)$

* Draw an equilateral triangle with side CD.

8. $AB = 19$ cm, $PA = 15$ cm

(a) $PB = AB - PA = 19 - 15 = 4$ cm

(b) $PC : PD = 5 : 3$

$PC = 5x$, $PD = 3x$

$PA \times PB = PC \times PD$

$15 \times 4 = 5x \times 3x$

$15x^2 = 15 \times 4$

$x^2 = 4$

$x = 2$

$CD = 5x + 3x = 8x = 8 \times 2 = 16$ cm

9. (a) $PD = PC + CD = 6 + 6 = 12$ cm

(b) $PA = r - 3$

(c) $PB = PA + AB$

$= r - 3 + 2r = 3r - 3$

(d) $PA \times PB = PC \times PD$

$(r - 3)(3r - 3) = 6 \times 12$

$(r - 3)\, 3\, (r - 1) = 6 \times 12$

$(r - 3)(r - 1) = 24$

$r^2 - 4r - 21 = 0$

$(r - 7)(r + 3) = 0$

$r = 7, r = -3$

$\therefore$ Radius = 7 cm

CHAPTER 3

MATHEMATICS OF CHANCE

1. Mathematics of chance is helpful to analyse the probability of situations in which actual quantitative analysis is not possible.

   Probability denotes what part of total number of results is the number of favourable results.

Part A

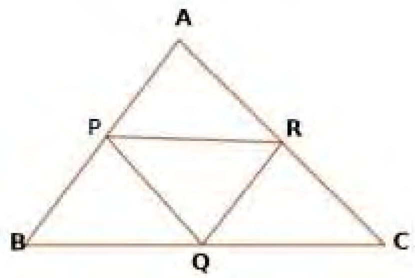
1. In a box there are 5 black balls and 4 white balls. A ball is taken from it.

   a) What is the probability that the ball taken is black?

   b) What is the probability that the ball taken is white?

2. What is the probability that there are 5 sundays in January ?

3. PQR is a triangle formed by joining the mid points of sides of triangle ABC. If a dot is put in triangle ABC what would be the probability that it may be in triangle PQR ?

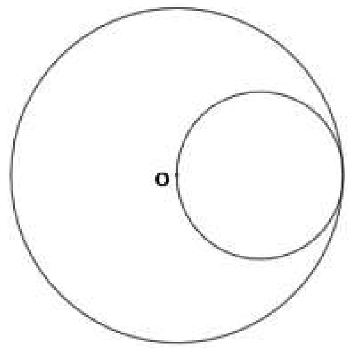


Part B

1. In figure the area of the large circle is 40π square centimetre. O is the centre of large circle.

   a) Find the area of the small circle ?

   b) If a dot is put within the large circle what is the probability that it will be in the small circle?



2. One is asked to say a two digit number.

   a) What is the probability that it is an even number ?

   b) What is the probability that the digit in the unit place is 4 ?

3. In a bag there are some red beads and some green beads and the total number of beads is 24.

   The probability of drawing a red bead from it is $\frac{2}{3}$.

   a) What is the probability that it is green ?

   b) How many red beads are there in the bag ?

4. In a box there are some red balls and blue balls and 11 balls in total.

   a) The Probability of drawing a blue ball from it is $\frac{7}{11}$. How many blue balls are there?

   b) How many are red?

5. One is asked to say a two digit number. What is the probability that

   a) getting a number with both the digits same?

   b) the number got is a multiple of 11?

   c) getting a number with digit in the unit place is 4.

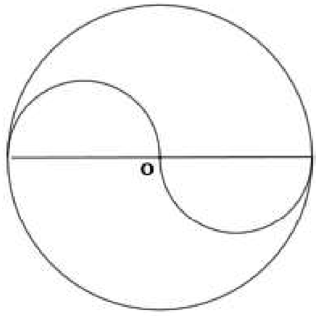
6. In a bag there are some red balls and some white balls .

   The probability of drawing a white ball from it is $\frac{2}{5}$.

   a) If the bag contains 20 balls what is the number of white balls?

   b) Find the probability of getting a red ball ?

   c) If one more white ball is added to it. What is the probability of getting a white ball ?



7. In the figure O is the centre of the circle. Two semicircles are drawn within the circle. The radius of the circle is 12 cm .

   a) What is the area of large circle ?

   b) What is the area of a semicircle ?

   c) If a dot is put within the circle, what is the probability that it may be in the semicircles?

8. If a number is selected from the two digit numbers with digits 1,2,3,4,5

   a) What is the probability that both the digits are same ?

   b) What is the probability that the number selected is a multiple of 11?

   c) What is the probability of getting a number with sum of digits 7?

9. In bag there are some red balls and white balls . If a ball is taken from it without looking it, the probability of getting a white ball is $\frac{3}{8}$.

   a) If the bag contains 40 balls, then find the number of white balls.

   b) Find the probability of getting a red ball

   c) If one white ball is removed from the bag what is the probability of getting a white ball ?

10. In a box there are 10 paper slips numbered from 1 to 10 and in another box there are 5 paper slips numbered from 1 to 5. If one paper slip is taken from both the box?

    a) What is the possible number of pairs ?

    b) What is the probability of getting both the numbers are perfect squares?

    c)What is the probability of getting sum 5 ?

11. A bag contains 8 orange balls and 7 pink balls. Another bag contains 9 orange balls and 6 pink balls.

    a) What is the probability of getting a pink ball from the first bag ?

    b) What is the probability of getting a pink ball from the second bag ?

    c) If the balls are put in a single bag , what is the probability of f getting a pink ball from it?

12. One is asked to say a two digit number. What is the probability of

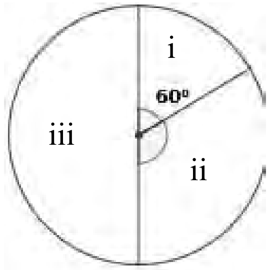
a) Both digits being different ?

b) The first digit being larger ?

c) The first digit being smaller ?

Part C

1. In the figure the circle is divided in to three sectors. The central angle of first sector is 60^0. If we put a dot in the circle without looking.



a) What is the probability that the dot is put in the first sector ?

b) If the probability that the dot is put in the second sector is $\frac{1}{3}$, find the central angle of the second sector.

c) What is the central angle of third sector?

d)What is the probability that the dot is put in the third sector?

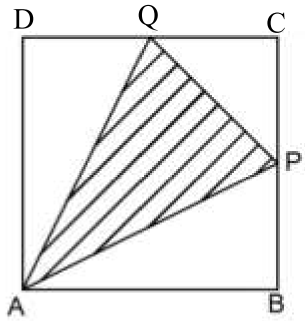
2. In the figure ABCD is a square. P and Q are the mid points of BC and CD. Triangle APQ is a triangle inside the square. The side of the square is 12 cm.

a) Find the area of the square ?

b) Find the areas of triangles which are not shaded ?

c) Find the area of triangle APQ ?

d) If we put a dot in the figure what is the probability that it may be in the triangle APQ ?



3. One is asked to say a two digit number .

   a) How many two digit numbers are there?

   b) How many are two digit perfect sqaure?

   c) What is the probability that the number being a perfect square ?

   d) What is the probability that it is not a perfect square ?

Part D

1. In a class there are 30 boys and 20 girls. In another class there are 25 boys and 15 girls . If one student is selected from each class to make a team?

   a) How many possible pairs of students are there?

   b) What is the probability that both the students are boys ?

   c) What is the probability that both the students are girls?

   d) What is the probability that the team contains at least one boy?

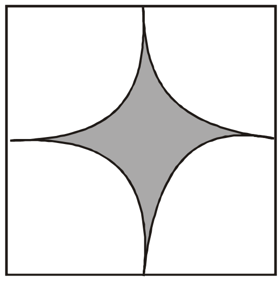
   e) What is the probability that the team contains at least one girl?

2. Four sectors are drawn in a square as shown in the figure, put a dot in the figure , without looking in to it . If the side of the square is 'a'

   a) What is the area of square ?

   b) What is the area of each sector ?

   c) What is the probability that the dot is in the shaded region ?



Answers

PART A

1. a)The probability that the ball taken is black = $\frac{5}{9}$

   b) The probability that the ball taken is white = $\frac{4}{9}$

2. First 28 days have 4 sundays .

   There remains 3 days.

   These 3 days may be

   Monday-Tuesday-Wednesday

   Tuesday-Wednesday-Thursday

   Wednesday-Thursday-Friday

   Thursday-Friday-Saturday

   Friday-Saturday-Sunday

   Saturday-Sunday-Monday

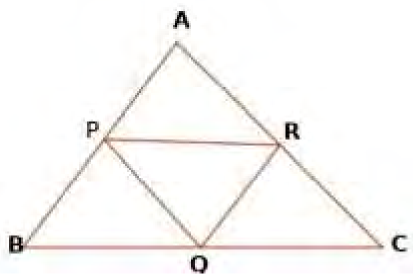
   Sunday-Monday – Tuesday

   Total there are 7 arrangements. Of these 3 arrangements have sundays.

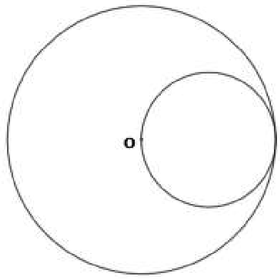
   The probability that there are 5 sundays in January = $\frac{3}{7}$

3. Area of triangle PQR is $\frac{1}{4}$ of the area of triangle ABC probability to put a dot in triangle

   PQR = $\frac{1}{4}$



Part B



1. The area of the large circle is 40π square centimetre.

   a) The area of the small circle = 10π square centimetre.

   b) Probability that it will be in the small circle = $\frac{1}{4}$

2. Total number of two digit numbers = 99-9

   = 90.

   a) Total number of even numbers = $\frac{90}{2}$ = 45

   Probability that it may be an even number = $\frac{45}{90} = \frac{1}{2}$

   b) Numbers with the digit 4 in unit place are 14,24,34,.........94

   The probability that the digit in the unit place may be 4 = $\frac{9}{90} = \frac{1}{10}$

3. In bag there are some red beads and some green beads and the Total number of beads = 24.

   probability of getting a red bead = $\frac{2}{3}$

   a) The probability of getting green bead = $1 - \frac{2}{3} = \frac{1}{3}$

   b) Number of red beads = $\frac{2}{3}$ of 24 = 16

4. Total number of balls = 11

a)Probability of getting a blue ball $= \frac{7}{11}$

Number of blue balls = 7

b) Number of red balls = 4

5. a) Probability getting a number with both the digits same $= \frac{9}{90} = \frac{1}{10}$

b) Probability of getting the number got is a multiple of 11 $= \frac{9}{90} = \frac{1}{10}$

c) Probability of getting a number with digit in the unit place is 4 $= \frac{9}{90} = \frac{1}{10}$

6. Probability of getting a white ball is $\frac{2}{5}$

a) If the bag contains 20 balls, the number of white balls $= \frac{2}{5} \times 20 = 8$

b) Probability of getting a red ball $= 1 - \frac{2}{5} = \frac{3}{5}$

c) If one more white ball is added and take a ball , the probability of getting a white ball

$$= \frac{9}{21} = \frac{3}{7}$$

7. The radius of the circle is 12 cm .

a) Area of large circle $= 12^2 = 144\pi$ square centimetre

b) Area of one semicircle = 18 π cm^2

c) If a dot is put within the circle, the probability that it may be in the semicircle $= \frac{36\pi}{144\pi} = \frac{1}{4}$

8. Total number of wo digit numbers with digits 1,2,3,4,5 = 20

a) The probability that both the digits are same = $\frac{5}{20}$

b) Probability that the number selected is a multiple of 11 = $\frac{5}{20}$

c)Probability of getting a number with sum of digits 7 = $\frac{4}{20}$

9. Probability of getting a white ball is $\frac{3}{8}$

a) If the bag contains 40 balls , the number of white balls = $40 \times \frac{3}{8} = 15$

b) probability of getting a red ball $1 - \frac{3}{8} = \frac{5}{8}$

c) If one white ball is removed from it and take a ball from the bag , the probability of getting a white ball = $\frac{14}{39}$

10. a) The possible number of pairs = $10 \times 5 = 50$

b) Probability of getting both the numbers are perfect squares = $\frac{6}{50}$

c) Probability of getting the numbers with sum 5 = $\frac{4}{50}$

11. a) Probability of getting a pink ball from the first bag = $\frac{7}{15}$

b) Probability of getting a pink ball from the second bag = $\frac{6}{15}$

c) If the balls are put in a single bag , the probability of getting a pink ball from it = $\frac{13}{30}$

12. Total number of two digit numbers = 90

a) Two digit numbers with both digits being same = 9

Two digit numbers with both digits being different = 81 Proof = 81/90 = 9/10

b) Number of two digit numbers with first digit being larger = 1 + 2 + ... + 9

= 45

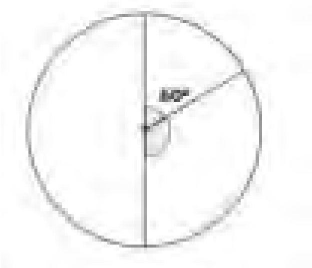
Probability that the first digit being larger = $\frac{45}{90} = \frac{1}{2}$

c) Number of two digit with first digit being smaller = 1+2+3+....+8 =36

Probability that the first digit being larger = $\frac{36}{90} = \frac{2}{5}$

Part C

1.



a) The probability that the dot is put in the

first sector = $\frac{1}{6}$

b) If the probability that the dot is put in the second sector is $\frac{1}{3}$ the central angle of second sector = 120^0

c) The central angle of third sector $=180^0$

d) The probability that the dot is put in the third sector $=\frac{1}{2}$

2. The side of the square is 12 cm

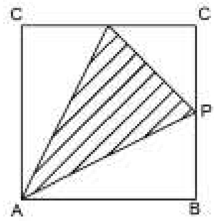
   a) The area of the square = 122 = 144 cm^2

   b) The areas of triangles which are not shaded= 36 , 36 , 18

   c) The area of triangle APQ = 54cm^2

   d) If we put a dot in the figure , the probability that it may be in the

   triangle = $\frac{54}{144}$



3. a) Total number two digit numbers = 90

   b) Two digit perfect squares = 6

   c) Probability of the number being a perfect square =$\frac{6}{90} = \frac{1}{15}$

   d) Probability that its not a perfect square = $1 - \frac{1}{15} = \frac{14}{15}$

Part D

1. Class I: 30 boys and 20 girls.

   Class II : 25 boys and 15 girls

   a) possible pairs of students = 50 × 40 =2000

   b) probability that both the students are boys $= \frac{30 \times 25}{2000} = \frac{750}{2000} = \frac{3}{8}$

   c) probability that both the students are girls $= \frac{20 \times 15}{2000} = \frac{300}{2000} = \frac{3}{20}$

   d) team contains at least one boy means 1 boy from Class I and 1 girl from Class II or 1 boy from Class II and 1 girl from Class I or 1 boy from Class I and 1 boy from Class II .

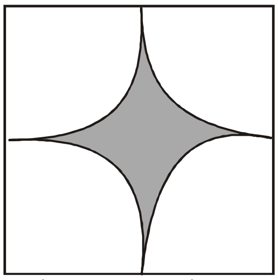
Probability of selecting at least on boy = $\frac{(30 \times 15) + (20 \times 25) + (30 \times 25)}{2000} = \frac{1700}{2000} = \frac{17}{20}$

e) team contains at least one girl means 1 boy from Class I and 1 girl from Class II or 1 boy from Class II and 1 girl from Class I or 1 girl from Class I and 1 girl from Class II .

Probability of selecting at least on girl = $\frac{(20\times 25)+(15\times 30)+(20\times 15)}{2000} = \frac{1250}{2000} = \frac{5}{8}$

2. a) Area of square = a^2

b) Area of one sector = $\pi\times\left(\frac{a}{2}\right)^2\times\frac{1}{4}$



Area of 4 sector $= \frac{\pi a^2}{4}$

c) Area of shaded region = $a^2 - \frac{\pi a^2}{4}$

Probability that put a dot in shaded region = $\frac{a^2-\frac{\pi a^2}{4}}{a^2}$

$= \frac{a^2\left(1-\frac{\pi}{4}\right)}{a^2}$

$= 1-\frac{\pi}{4} = \frac{3\pi}{4}$

CHAPTER - 4
SECOND DEGREE EQUATIONS

- An equation is of the form $ax^2+bx+c=0$, $a \neq 0$ is called second degree equations.
- To make the expression x^2+bx, as a perfect square adding $\left(\frac{b}{2}\right)^2$ to it.
- If $(x+a)^2 = 0$ then $x = -a$ and If $(x-a)^2 = 0$, then $x = a$.
- If $(ax+b)^2 = c$, then $x = \frac{\sqrt{c}-b}{a}$
- The solutions of the second degree equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- The expression '$b^2 - 4ac$' is called <u>'Discriminant'</u> of the quadratic equations.
- If $b^2 - 4ac = 0$, then the equation has only one solution.
- If $b^2 - 4ac < 0$, then the equation has no solution.
- If $b^2 - 4ac > 0$, then the equation has two different solutions.
- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(x-a)(x-b) = x^2 - (a + b)x + ab$

PART A

1. If $(x-3)^2 = 49$, what are the value of x.
2. Adding 4 to a number and take the square of the result, we get 36, Find the number.

3. If $(3x-2)^2=169$, Find the values of x.
4. One solution of the equation $x^2+5x+k=0$ is -1, Find the value of K.
5. What number should be taken as K, so that the expression x^2+8x+k become a perfect square.
6. The product of a number and other number 8 more than that is 84.

   (a) If first number is x, what is the second number.

   (b) Form a second degree equation and find the numbers.
7. The equation $x^2+6x+k=0$ has only one solution, Find the value of K.
8. The square of a term of the Arithmetic sequence 6, 10, 14, ... is 900. Find the position of the term.
9. (a) What number added to the expression $x^2 + 16x$ to get a perfect square.

   (b) If the expression $x^2 + ax + 16$ is a perfect square what is the value of a.
10. (a) Write the algebraic expression of the sum of first 'n' odd natural numbers.

    (b) If the sum of certain number of odd numbers starting from 1 is 625, Find the number of terms.

PART B (3)

11. The product of a number and the other number 6 more than that is 160.

    (a) If first number is x, what is the second number.

    (b) Find the two numbers.
12. If the length of sides of square is increased by 5 cm its area become $100cm^2$

    (a) If the length of sides of the first square is taken as x, what is the length of sides of the second square.

    (b) Find the length of sides of the two squares.

    (c) Find the area of the first square.
13. Length of a rectangle is 4 cm more than its breadth and its Area is $572\ cm^2$.

    (a) If breadth is taken as x, what is its length.

    (b) Form a second degree equation and find length and breadth.

14. The square of a number is 96 morethan 4 times of the number.

    (a) If the number is taken as x, what is square of the number.

    (b) Find the number.

15. Perimeter of a Rectangle is 40 cm, and its Area 96 cm^2.

    (a) Length + breadth = ...

    (b) If length is x, what is breadth

    (c) Find length and breadth of the Rectangle using second degree equation.

16. Consider the Arithemetic sequence.

    9, 11, 13, ...

    (a) Find sum of first ‘n’ terms.

    (b) If the sum of first certain number of term of the given sequence is 384. Find the number of terms.

17. (a) If (x–1) is an even number, what is the next even number.

    (b) The product of two consecutive even numbers is 624, Find the numbers.

18. The base of a right angled triangle is 6 cm more than height and its Area is 36 cm^2.

    (a) If base is taken as x, find the height.

    (b) Find base and height by forming a second degree equation.

19. The difference between two numbers is 10 and their product is 200.

    (a) If first number is x, what is the second number.

    (b) Find the two numbers.

PART C (4)

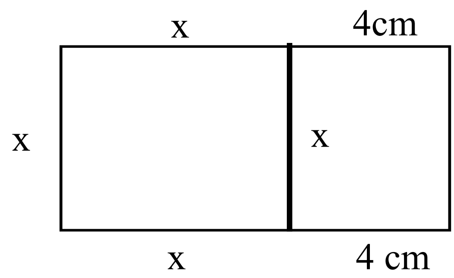
20. Perimeter of a Rectangle is 28 cm and length of diagonal 10 cm.

    (a) Length + breadth = ...?

    (b) If breadth is x, what is its length.

    (c) If $l^2 + b^2 = d^2$, then find length and breadth of the Rectangle.

21. If two parallel sides of a square extended by 4 cm as shown in the figure.



(a) If the length of sides of the square is x what is the length and breadth of the Rectangle so formed.

(b) If area of the rectangle is 396 cm^2 find length and breadth of the rectangle.

22. Length of a Rectangle is 2 cm more than 3 times the breadth and its Area is 120cm^2.

(a) If breadth is taken as x, what is its length?

(b) Form a second degree equation, then find length and breadth.

23. A rod of length 40 cm is bent and made a right angled triangle of hypotenuse 17 cm.

(a) Base + Altitude + Hypotenuse = ...

(b) If Base is x, what is its Altitude

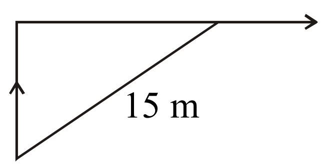
(c) Find base and Altitude using the relation $(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$

24. The common difference of an Arithmetic sequence is 3 and the product of two consecutive terms of this sequence is 54.

(a) If a term is taken as x, what is the next term.

(b) Find the two terms.

25. A boy is walking towards front then he turns towards right and walks at a distance 3 m morethan the first distance. Now he is 15 m away from the starting point.

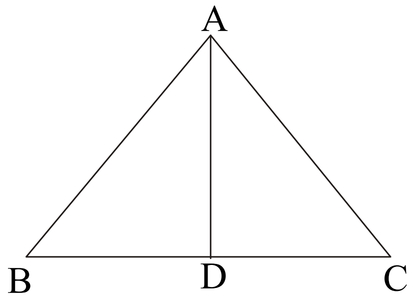


(a) If the distance he walks towards front is taken as x, what is the distance he walks towards right.

(b) Find the two distances by forming a second degree equation.

PART D (5)

26. In the figure ΔABC is an isosceles triangle with AB = AC. The height of the triangle is 8 cm less than base and its Area is 192cm^2.



   (a) If base is taken as x, what is its height?

   (b) Find base and height.

   (c) Find the length of the sides AB and AC.

27. The difference between the perimeters of two squares is 16 cm and sum of their areas 208 cm^2.

   (a) If perimeter of the first square is take as x, what is the perimeter of the second square.

   (b) Write the length of sides of each square.

   (c) Find the area of each square in terms of x.

   (d) Find length of sides of each square by forming a second degree equation.

28. The floor of a Rectangular room has length 25 m and breadth 20m. All around it there is a border of equal width inside the floor. The area of the floor except the boarder is 456 m^2

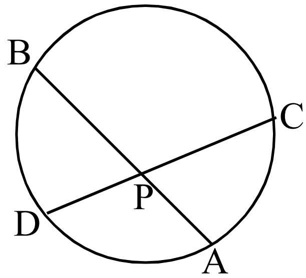
   (a) If the width of the boarder is x, write the length and breadth of the floor except the boarder.

   (b) Find the area of the floor except the boarder.

   (c) Find width of the boarder.

29. In the figure the chords AB and CD of the circle meet at the point P.

   AB = 22 cm, CD = 20 cm and PA : PB = 8 : 3



   (a) Find the length of PA and PB

   (b) If PC = x, then PD = ... ?

   (c) PC × PD = ... × .... ?

   (d) Find length of PC and PD by forming a second degree equation.

30. In the figure the chord AB of the circle extended and mark the point P.

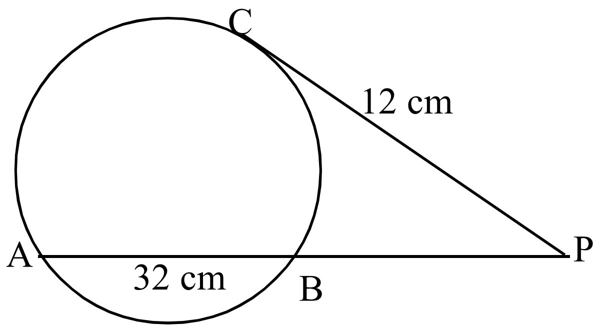
PC is a tangent to the circle.

AB = 32 cm and PC = 12 cm

(a) PA × PB = ...?

(b) If PB = x , what is PA

(c) Find PB and PA.



31. From each corners of a rectangular sheet of paper, small squares of equal size are to be cut off. A box is to be made by folding the remaining portion.

Length of the base of the box is 21 cm. Breadth is 10 cm less than height. Volume of the box is 2499 cm^3.

(a) If the height is taken as x, what is its breadth

(b) Length × breadth × height = ...

(c) Form a second degree equation then find the length and breadth.

Answers PART-A

1. $(x-3)^2 = 49$

   $\therefore x-3 = \sqrt{49} = \pm 7$

   $x-3 = 7$

   $x = 7+3 = 10$

   $x-3 = -7$

   $x = -7+3$

   $x = -4$

2. Let the number = x

   $\therefore (x+4)^2 = 36$

   $x+4 = \sqrt{36}\ \pm 6$

   $x+4 = 6$

   $x = 6-4$

   $x = 2$

   $x+4 = -6$

   $x = -6-4$

   $x = -10$

3. $(3x-2)^2 = 169$

   $\therefore 3x-2 = \sqrt{169} = \pm 13$

   $3x-2 = 13$

   $3x = 13+2$

   $3x = 15$

   $x = \frac{15}{3} = 5$

   $x = 5$

   $3x-2 = -13$

   $3x = -13+2$

   $3x = -11$

   $x = \frac{-11}{3}$

4. $x^2+5x+k=0$

Given that –1 is a solution

$\therefore\ (-1)^2+5\times-1+k=0.$

$1+-5+k=0$

$-4+k=0$

$\therefore\ k=4$

5. $x^2+8x+k=0$

$$k=\left(\frac{8}{2}\right)^2$$

$k=4^2$

$k=16$

$\therefore\ x^2+8x+16=0$

$\therefore\ (x+4)^2=0.$

6. Let the number = x

$\therefore$ Second Number = x + 8.

$\therefore\ x(x+8)=84$

$x^2+8x=84$

$x^2+8x+4^2=84+4^2$

$(x+4)^2=84+16=100.$

$\therefore\ x+4=\sqrt{100}=\pm10.$

$x+4=10$	$x+4=-10$
$x=10-4$	$x=-10-4$
$x=6$	$x=-14$

7. $x^2+6x+k=0.$

Given that it has only one solution.

$\therefore\ b^2 - 4ac = 0.$

$6^2 - 4\times1\times k = 0$

$36 - 4k = 0.$

$36 = 4k.$

$k = \frac{36}{4}$

$k = 9$

8. $x_1 = 6$

$x_2 = 10$

$\therefore\ d = 10 - 6 = 4\cdot$

$\therefore\ x_n = dn + (f - d)$

$= 4n + (6 - 4)$

$= 4n + 2$

$\therefore\ (4n+2)^2 = 900.$

$4n+2 = \sqrt{900} = \pm30$

$4n+2 = 30$

$4n = 30 - 2$

$4n = 28$

$\therefore\ n = \frac{28}{4}$

$n = 7$

9. (a) the number be added $= \left(\frac{16}{2}\right)^2 = 8^2 = 9$

(b) $x^2+ax+16$ is a perfect square

$\therefore$ a = $2\times\sqrt{16} = 2\times(\pm 4) = \pm 8$

10. a) Sum of odd natural numbers = n^2

b) $\therefore n^2 = 625$

$n = \sqrt{625}$

$\therefore n=25$

PART B (Score-3)

11. (a) Let first number = x

$\therefore$ Second number = x + 6

(b) $x(x+6) = 160$

$x^2+6x=160$

$x^2+6x+3^2 = 160+ 3^2$

$(x+3)^2 = 160+9$

$(x+3)^2 = 169$

$\therefore x + 3 = \sqrt{169} = \pm 13$

$x + 3=13$	$x+3= -13$
$x = 13-3$	$x = -13-3$
$x = 10$	$x = -16$

when x = 10 the next number is 16

when x = –16 the next number is –10

12. (a) Length of sides of the first square = x

$\therefore$ Length of sides of the second square = x + 5

b) Area of second square = $(x+5)^2$

$\therefore (x+5)^2 = 100$

$x+5=\sqrt{100} = \pm 10$

$x+5=10$

$x=10-5$

$x = 5$

(c) Area of first square $= 5^2 = 25\ cm^2$

13. a) breadth = x

$\therefore$ length = x+4

b) Area of rectangle = l×b.

$\therefore (x+4)\,x = 572$

$x^2+4x = 572$

$x^2+4x+2^2 = 572 + 2^2$

$(x+2)^2 = 572+4$

$(x+2)^2 = 576$

$\therefore x+2= \sqrt{576} = \pm 24$

$x+2= 24.$

$x= 24-2$

$x = 22.$

$\therefore$ breadth = 22cm

length = 22+4

= 26cm

14. (a) Let number = x

$\therefore$ Square of the number $= x^2$

Four times of the number = 4x

$\therefore x^2 = 4x + 96$

$\therefore x^2 - 4x = 96$

$x^2 - 4x + 2^2 = 96 + 2^2$

$(x-2)^2 = 100$

$x - 2 = \sqrt{100} = \pm 10$

$x - 2 = 10$ $\quad\quad$ $x-2 = -10$

$x = 10 + 2$ $\quad\quad$ $x = -10 + 2$

$x = 12$ $\quad\quad$ $x = -8$

15. Perimeter of rectangle = $2\,(l + b) = 40$

(a) $l + b = \frac{40}{2} = 20$

(b) Let, Breadth = x, then length = 20 – x

Area of rectangle = $l \times b$

$\therefore\ x\,(20-x) = 96$

$20x - x^2 = 96$

$-x^2 + 20x = 96$

$\therefore\ x^2 - 20x = -96$

$x^2 - 20x + 10^2 = -96 + 10^2$

$(x-10)^2 = -96 + 100$

$(x-10)^2 = 4$

$x-10 = \sqrt{4} = \pm 2$

$x - 10 = 2$

$x = 2 + 10$ $\quad\quad$ $\therefore$ length = 12 cm

$x = 12$ $\quad\quad$ breadth = 20 – 12 = 8 cm

16. (a) $x_1 = 9$

$x_2 = 11$

$\therefore\ d = 11 - 9 = 2$

$S_n = \frac{n}{2}\ (2f + (n-1)d)$

$= \frac{n}{2}\ (2 \times 9 + (n-1)2)$

$= \frac{n}{2}(18 + 2n - 2)$

$= \frac{n}{2}\ (2n + 16)$

$= \frac{n}{2} \times 2\ (n + 8)$

$= n^2 + 8n$

(b) $\therefore\ n^2 + 8n = 384$

$n^2 + 8n + 4^2 = 384 + 4^2$

$(n + 4)^2 = 384 + 16 = 400$

$n + 4 = \sqrt{400}\ = \pm\ 20$

$n + 4 = 20$

$n = 20 - 4$

$n = 16$

17. (a) Given that $(x-1)$ is an even number

$\therefore$ next even number $= x - 1 + 2 = x + 1$

(b) Product of the two numbers = 624

$\therefore\ (x-1)\ (x + 1) = 624$

$\therefore\ \ x^2 - 1^2 = 624$

$x^2 - 1 = 624$

$x^2 = 624 + 1 = 625$

$x = \sqrt{625} = \pm 25$

18. (a) Let base = x

$\therefore$ Altitude = x + 6

Area of triangle = $\frac{1}{2}bh = 36$

$\therefore \frac{1}{2} \times x\,(x + 6) = 36$

$x\,(x + 6) = 36 \times 2$

$x^2 + 6x = 72$

$x^2 + 6x + 3^2 = 72 + 3^2$

$(x + 3)^2 = 72 + 9 = 81$

$x + 3 = \sqrt{81} = \pm 9$

$x + 3 = 9$

$x = 9 - 3$

$x = 6$

$\therefore$ base = 6 cm

Altitude = 6 + 6 = 12 cm

19. (a) Let first number = x

$\therefore$ Second number = 10 + x

(b) Product of two number = 200

$\therefore x\,(10 + x) = 200$

$10\,x + x^2 = 200$

$x^2 + 10x = 200$

$x^2 + 10x + 5^2 = 200 + 5^2$

$(x + 5)^2 = 200 + 25 = 225$

$\therefore x + 5 = \sqrt{225} = \pm 15$

$x + 5 = 15 \qquad x + 5 = -15$

$x = 15 - 5 \qquad x = -15 - 5$

$x = 10 \qquad x = -20$

When $x = 10$

first number = 10

Second number = 10 + 10 = 20

When $x = -20$

First number = –20

Second number = –20 + 10 = –10

Part C (Score 4)

20. (a) perimeter of rectangle = $2(l + b) = 28$

$$\therefore l + b = \frac{28}{2} = 14$$

(b) Let, breadth = x

$\therefore$ length = 14 – x

(c) $l^2 + b^2 = d^2$

$\therefore (14 - x)^2 + x^2 = 10^2$

$14^2 - 2 \times 14 \times x + x^2 + x^2 = 100$

$196 - 28x + x^2 + x^2 = 100$

$2x^2 - 28x + 196 = 100$

$2x^2 - 28x + 196 - 100 = 0$

$2x^2 - 28x + 96 = 0$

$$\frac{\cancel{2}x^2}{\cancel{2}} - \frac{\overset{14}{\cancel{28}}x}{\cancel{2}} + \frac{96}{2} = 0$$

$x^2 - 14x + 48 = 0$

$x^2 - 14x = -48$

$x^2 - 14x + 7^2 = -48 + 7^2$

$(x-7)^2 = -4 + 49$

$(x-7)^2 = 1$

$x - 7 = \sqrt{1} = \pm 1$

$x = 1 + 7$

$x = 8$

breadth = 8 cm

length = 14 – 8 = 6 cm

21. (a) length of sides of the square = x

$\therefore$ length of rectangle = x + 4

breadth of the rectangle = x

Area of rectangle = $l \times b = 396\ cm^2$

$\therefore (x + 4)\, x = 396$

$x^2 + 4x = 396$

$x^2 + 4x + 2^2 = 396 + 2^2$

$(x + 2)^2 = 396 + 4 = 400$

$x + 2 = \sqrt{400} = \pm 20$

$x + 2 = 20$

$x = 20 - 2$

Breadth of rectangle x = 18cm, Length of rectangle = 22 cm

length of sides of the square = 18 cm

22. (a) Let, breadth = x

$\therefore$ length = 3x + 2

(b) Area of rectangle = $l \times b = 120\ cm^2$

$\therefore (3x + 2)\ x = 120$

$3x^2 + 2x = 120$

$3x^2 + 2x - 120 = 0$

a = 3

b = 2

c = –120

$$\sqrt{b^2 - 4ac} = \sqrt{2^2 - 4 \times 3 \times -120}$$

$$= \sqrt{4 - 12 \times -120}$$

$$= \sqrt{4 + 1440}$$

$$= \sqrt{1444} = 38$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm 38}{2 \times 3}$$

$x = \frac{-2+38}{6}$	$x = \frac{-2-38}{6}$
$x = \frac{36}{6}$	$x = \frac{-40}{6}$
$x = 6$	

$\therefore$ breadth = 6 cm

length = $3 \times 6 + 2$

$= 18 + 2 = 20$ cm

23. (a) Length of rod = 40 cm

$\therefore$ base + Altitude + hypotenuse = 40

(b) Given, hypotenuse = 17 cm

Let base = x

$\therefore$ x + Altitude + 17 = 40

$\therefore$ Altitude = 40 – 17 – x

Altitude = 23 – x

(c) $(\text{base})^2 + (\text{Altitutde})^2 = (\text{hypotenuse})^2$

$x^2 + (23 - x)^2 = 17^2$

$x^2 + 23^2 - 2 \times 23 \times x + x^2 = 289$

$x^2 + 529 - 46x + x^2 = 289$

$2x^2 - 46x + 529 - 289 = 0$

$2x^2 - 46x + 240 = 0$

$$\frac{2x^2}{2} - \frac{46x}{2} + \frac{240}{2} = 0$$

$x^2 - 23x + 120 = 0$

a = 1

b = –23

c = 120

$$\sqrt{b^2 - 4ac} = \sqrt{(-23)^2 - 4 \times 1 \times 120}$$

$$= \sqrt{529 - 4 \times 120}$$

$$= \sqrt{529 - 480}$$

$= \sqrt{49} = 7$

$\therefore\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-23) \pm 7}{2 \times 1}$

$x = \dfrac{23+7}{2}$	$x = \dfrac{23-7}{2}$
$x = \dfrac{30}{2}$	$x = \dfrac{16}{2}$
$x = 15$	$x = 8$

$\therefore$ base = 15

Altitude = 8

24. (a) Common difference = 3

Let one term = x

$\therefore$ the next term = x + 3

(b) Product of terms = 54

$\therefore\ x(x + 3) = 54$

$x^2 + 3x = 54$

$x^2 + 3x - 54 = 0$

a = 1

b = 3

c = 54

$\sqrt{b^2 - 4ac} = \sqrt{3^2 - 4x \times 1 \times -54}$

$= \sqrt{9 - 4 \times -54}$

$= \sqrt{9 + 216}$

$= \sqrt{225} = 15$

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-3 \pm 15}{2 \times 1}$

$x = \dfrac{-3+15}{2} \qquad x = \dfrac{-3-15}{2}$

$x = \dfrac{12}{2} \qquad x = \dfrac{-18}{2}$

$x = 6 \qquad x = -9$

When $x = 6$

one term = 6

next term = 6 + 3 = 9

when $x = -9$

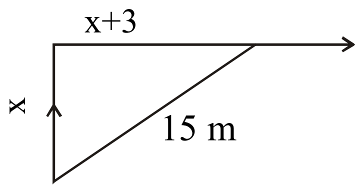
one term = –9

next term = –9 + 3 = –6

25. (a) Let the distance he walk towards front = x

$\therefore$ The distance he walks towards right = x + 3

(b) The distance between two place = 15 m



$\therefore x^2 + (x+3)^2 = 15^2$

$x^2 + x^2 + 2 \times x \times 3 + 3^2 = 225$

$2x^2 + 6x + 9 = 225$

$2x^2 + 6x + 9 - 225 = 0$

$2x^2 + 6x - 216 = 0$

$a = 2$

$b = 6$

$c = -216$

$$\sqrt{b^2 - 4ac} = \sqrt{6^2 - 4 \times 2 \times -216}$$

$$= \sqrt{36 - 8 \times -216}$$

$$= \sqrt{36 + 1728}$$

$$= \sqrt{1764} = 42$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 + 42}{2 \times 2}$$

$$x = \frac{36}{4} = 9$$

$\therefore$ The distance he walks towards front = 9 m

$\therefore$ The distance he walks towards right = 9 + 3 = 12 m

Part D (Score - 5)

26. (a) Let base = x

$\therefore$ height = x – 8

(b) Area of triangle = $\frac{1}{2}$bh = 192 cm^2

$\therefore \frac{1}{2} \times x\,(x-8) = 192$

$x\,(x-8) = 192 \times 2$

$x\,(x-8) = 192 \times 2$

$x^2 - 8x = 384$

$x^2 - 8x + 4^2 = 384 + 4^2$

$(x-4)^2 = 384 + 16 = 400$

$x - 4 = \sqrt{400} = \pm 20$

$x - 4 = 20$

$\therefore\ x = 20 + 4$

$x = 24$

$\therefore$ base = 24 cm

height = 24 – 8 = 16

$\therefore\ BD = \frac{24}{2} = 12$ cm

AD = 16 cm

$\therefore\ AB = \sqrt{BD^2 + AD^2}$

$AB = \sqrt{12^2 + 16^2}$

$= \sqrt{144 + 256}$

$= \sqrt{400} = 20$ cm

$\therefore$ AC = 20 cm

27. (a) Perimeter of first square = x

$\therefore$ Perimeter of second square = 16 + x

(b) length of sides of first square = $\frac{x}{4}$

length of sides of second square = $\frac{16+x}{4}$

(c) Area of first square = $\left(\frac{x}{4}\right)^2$

Area of second square = $\left(\frac{16+x}{4}\right)^2$

(d) Sum of Areas = 208

$$\therefore \left(\frac{x}{4}\right)^2 + \left(\frac{16+x}{4}\right)^2 = 208$$

$$\frac{x^2}{16} + \frac{16^2 + 2\times16\times x + x^2}{16} = 208$$

$$\therefore \frac{x^2 + 256 + 32x + x^2}{16} = 208$$

$2x^2 + 32x + 256 = 208 \times 16$

$2x^2 + 32x + 256 = 3328$

$2x^2 + 32x + 256 - 3328 = 0$

$2x^2 + 32x - 3072 = 0$

$$\frac{2x^2}{2} + \frac{32x}{2} - \frac{3072}{2} = 0$$

$x^2 + 16x - 1536 = 0$

$x^2 - 16x = 1536$

$x^2 - 16x + 8^2 = 1536 + 8^2$

$(x-8)^2 = 1536 + 64 = 1600$

$x - 8 = \sqrt{1600} = \pm 40$

$x - 8 = 40$

$x = 40 + 8$

$x = 48$

$\therefore$ Perimeter of first square = 48

$\therefore$ length of sides = $\frac{48}{4}$ = 12 cm

Perimeter of second square $= 16 + x$

$= 16 + 48$

$= 64$ cm

length of sides = $\frac{64}{4}$ = 16 cm

28. (a) Width of the border = x

$\therefore$ length of the floor except the border = 25 – x

breadth of the floor except the border = 20 – x

(b) Area of the floor except the border = (25 – x) (20 – x)

(c) $\therefore$ $(25 - x)(20 - x) = 456$

$25 \times 20 - 25 \times x - x \times 20 + x \times x = 456$

$500 - 25x - 20x + x^2 - 456 = 0$

$x^2 - 45x + 500 - 456 = 0$

$x^2 - 45x + 44 = 0$

$a = 1$

$b = -45$

$c = 44$

$\sqrt{b^2 - 4ac} = \sqrt{(-45)^2 - 4 \times 1 \times 44}$

$= \sqrt{2025 - 4 \times 44}$

$= \sqrt{2025 - 176} = \sqrt{1849} = 43$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-45) \pm 43}{2 \times 1}$$

$x = \frac{45 + 43}{2}$	$x = \frac{45 - 43}{2}$
$x = \frac{88}{2}$	$x = \frac{2}{2}$
$x = 44$	$x = 1$

$\therefore$ width of the border = 1 meter

29. (a) AB = 22 cm

PA : PB = 8 : 3

$\therefore$ $PA = \frac{8}{11} \times 22$

PA = 8 × 2 = 16 cm

$PB = \frac{3}{11} \times 22$

PB = 3 × 2 = 6 cm

(b) CD = 20 cm

Let PC = x

PD = 20 – x

(c) PC × PD = PA × PB

$\therefore$ x (20 – x) = 16 × 6

$20x - x^2 = 96$

$\therefore -x^2 + 20x - 96 = 0$

$x^2 - 20x + 96 = 0$

$x^2 - 20x = -96$

$x^2 - 20x + 10^2 = -96 + 10^2$

$(x - 10)^2 = -96 + 100$

$(x - 10)^2 = 4$

$\therefore x - 10 = \sqrt{4} = \pm 2$

$\therefore$ $x - 10 = 2$	$x - 10 = -2$
$x = 10 + 2$	$x = -2 + 10$
$x = 12$	$x = 8$

PC = 8 cm

PD = 12 cm

30. (a) $PA \times PB = PC^2$

(b) AB = 32 cm

Let, PB = x

$\therefore PA = 32 + x$

(c) $PA \times PB = PC^2$

$(32 + x)\,x = 12^2$

$32x + x^2 = 144$

$x^2 + 32x = 144$

$x^2 + 32x + 16^2 = 144 + 16^2$

$(x + 16)^2 = 144 + 256$

$(x + 16)^2 = 400$

$x + 16 = \sqrt{400} = \pm 20$

$x + 16 = 20$

$x = 20 - 16$

$x = 4$

$\therefore$ PB = 4 cm

PA = 32 + 4 = 36

31. (a) Let height = x

$\therefore$ breadth = 10 + x

length = 21 cm

Volume of the box $= l \times b \times h = 2499 \text{ cm}^3$

$\therefore \quad 21 \times (10 + x)\, x = 2499$

$21\,(10x + x^2) = 2499$

$210\,x + 21x^2 = 2499$

$$\frac{21x^2}{3} + \frac{210\,x}{3} = \frac{2499}{3}$$

$7x^2 + 70\,x = 833$

$$\frac{7x^2}{7} + \frac{70x}{7} = \frac{833}{7}$$

$x^2 + 10\,x = 119$

$x^2 + 10x + 5^2 = 119 + 5^2$

$(x + 5)^2 = 119 + 25 = 144$

$x + 5 = \sqrt{144} = \pm 12$

$x + 5 = 12$

$x = 12 - 5$

$x = 7$

$\therefore$ length of the box = 7 cm

breadth of the box = 10 + 7 = 17 cm