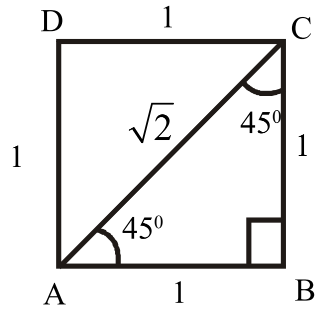
CHAPTER - 5
TRIGONOMETRY



Important Concepts

- Consider a square with side one unit. The diagonal AC divides the square into two isosceles triangles. In ΔABC

  AC = $\sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$

- The sides of an isosceles triangle with angles 45^0, 45^0, 90^0 are in the ratio 1 : 1: $\sqrt{2}$

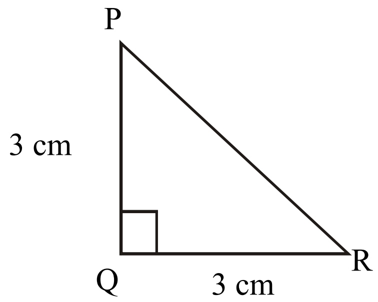
45^0	45^0	90^0
1	1	$\sqrt{2}$

eg: In an isosceles ΔPQR, QR = 3 cm. Find PR?

Ans: Angles are 45^0, 45^0, 90^0

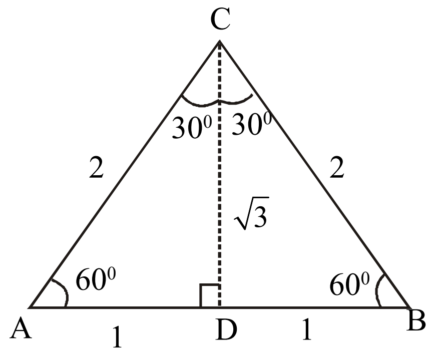
The sides are in the ratio 1:1:$\sqrt{2}$

$\therefore$ PR = $3\sqrt{2}$ cm



- Consider an equilateral triangle with sides 2 units. The perpendicular bisector CD divides the triangle into two right triangles.

  In right ΔADC, CD = $\sqrt{2^2-1^2} = \sqrt{4-1} = \sqrt{3}$

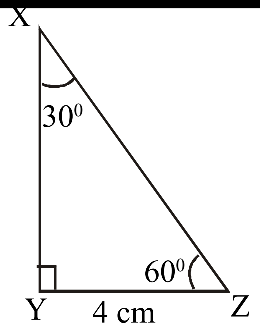


- The sides of a right triangle with angles 30^0, 60^0, 90^0 are in the ratio 1:$\sqrt{3}$:2.

30^0	60^0	90^0
1	$\sqrt{3}$	2

eg: In right triangle XYZ, YZ = 4 cm

Find the other two sides?

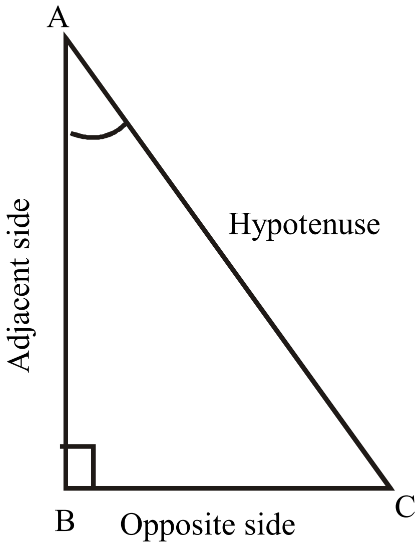


Ans: Angles are 30^0, 60^0, 90^0

Sides are in the ratio $1:\sqrt{3}:2$

$\therefore$ XY = $4\sqrt{3}$ cm, XZ = $2 \times 4 = 8$ cm

- In ΔABC, Consider $\angle A$



$$\text{Sin A} = \frac{\text{Opposite side of } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\text{CosA} = \frac{\text{Adjacent side of } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{tanA} = \frac{\text{Opposite side of } \angle A}{\text{Adjacent side of } \angle A} = \frac{BC}{AB}$$

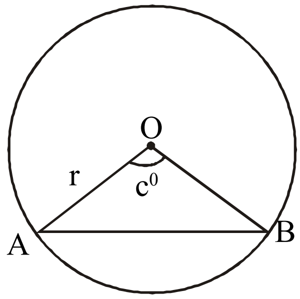
$$\frac{\text{Sin A}}{\text{Cos A}} = \text{tan A}$$

- Values of sin, cos, tan for the angles 0^0, 30^0, 45^0, 60^0, 90^0

	0^0	30^0	45^0	60^0	90^0
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	......

- The length of any chord of a circle is double the product of the radius of the circle and sin value of half the central angle.

  In a circle of radius ‘r’, length of chord of central angle C^0 is $2r \sin (C/2)^0$



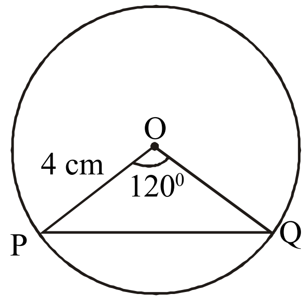
eg: Find the length of the chord PQ with central angle 120^0 and radius of the circle is 4 cms.

Ans: Length of Chord PQ = $2r \text{ Sin } (c/2)^0$

$$= 2 \times 4 \times \sin (120/2)^0$$

$$= 8 \times \sin 60^0$$

$$= 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3} \text{ cms}$$

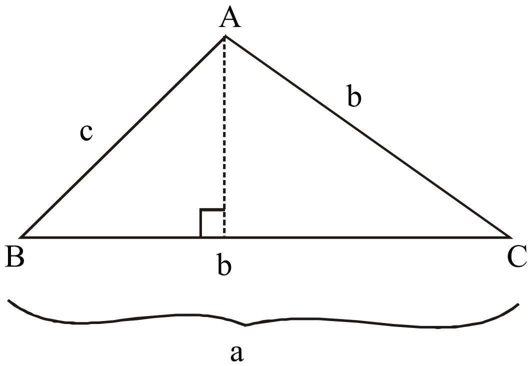


- In triangle ABC,

  AB = C, BC = a, AC = b.

  Area of $\Delta ABC = \frac{1}{2}$ ab $\text{Sin} C^0$

  or $\frac{1}{2}$ bc $\text{Sin } A^0$



or $\frac{1}{2}$ ac Sin B^0

The ratio of Sides,

BC : AC : AB = 2r Sin A : 2r Sin B : 2r Sin C.

ie, a : b : c = Sin A : Sin B : SinC

from AB = C = 2r Sin C,

2r = c/Sin C Similarly

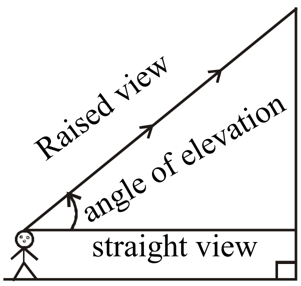
2r = b/Sin B

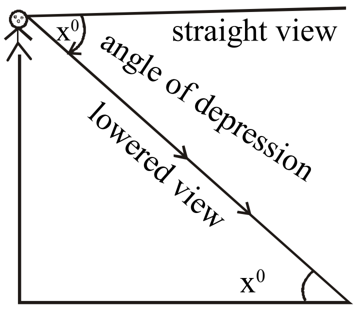
2r = a/Sin A

$$\therefore \frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}} = 2r$$

In this a, b, c are the sides of the triangle, A, B, C are the angles opposite to the sides and '2r' in the circum diameter.

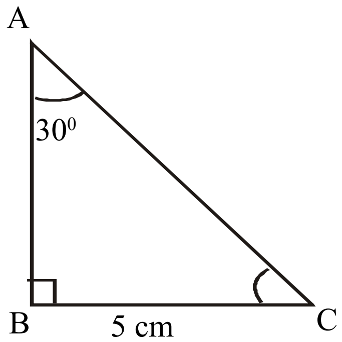
Angle of Elevation **Angle of Depression**





Part I (2 Mark Questions)

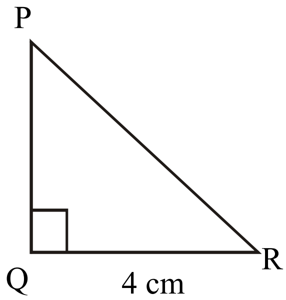
1. The smallest angle of a right triangle in 30^0. Length of its smallest side is 5 cms. Find the length of the largest side?



2. In an isosceles triangle PQR, $\angle Q = 90^0$ QR = 4 cm .

   (a) Find the length of PQ?

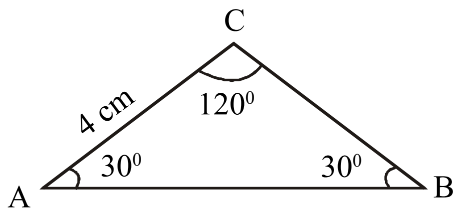
   (b) Find the length of PR?



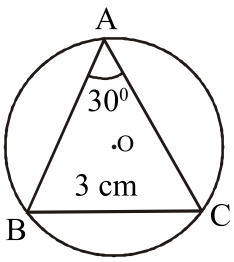
3. In ΔABC, $\angle A = \angle B = 30^0$ and AC = 4 cm

   (a) Find the length of BC.

   (b) Find the length of AB.



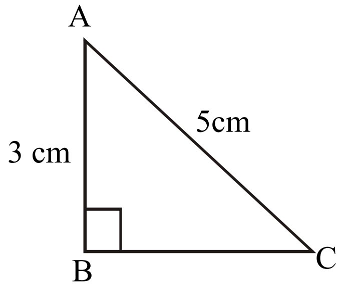
4. In figure BC = 3 cm, $\angle A = 30^0$. Find the radius of the circle?



5. In ΔABC, $\angle B = 90^0$, AB = 3 cm, AC = 5 cm.

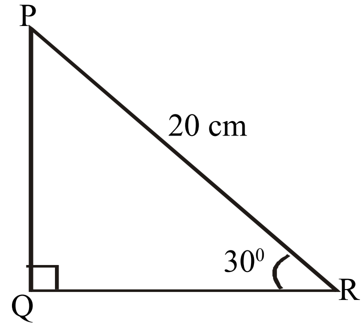
(a) Find BC ?

(b) tan A = ...



6. In right ΔPQR , $\angle R = 30^0$, PR = 20 cm.

Find the length of PQ.



7. A ladder leans against a wall. The ladder makes an angle 45^0 with the floor. The length of the ladder is 8 metre.

(a) What in the height of the top of the ladder from ground?

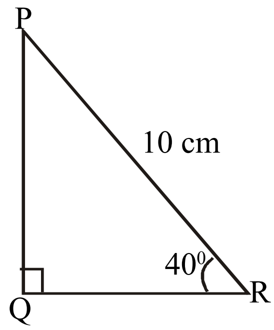
(b) How far is the foot of the ladder from the wall?

8. In right triangle PQR, $\angle R = 40^0$, PR = 10 cm.

(a) Find the length of PQ.

(b) Find the length of QR.

(Sin 40^0 = 0.64, Cos40^0 = 0.77)

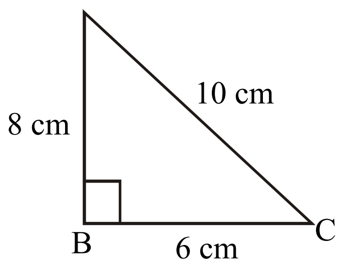


Part (II (3 mark Questions)

1. In figure $\angle B = 90^0$, AB = 8 cm, BC = 6 cm, AC = 10 cm.

(a) tan A = x/8 find x?

(b) Write Sin C, Cos C.

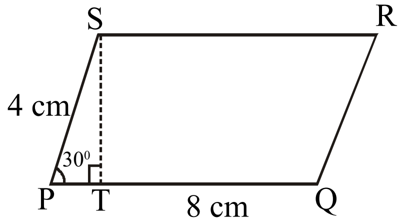


2. In figure PQRS is a parallelogram.

$\angle P = 30^0$, PQ = 8 cm, PS = 4 cm

(a) Find the length of ST?

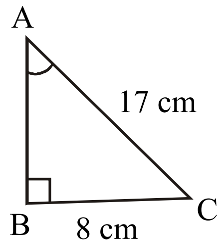
(b) Find the area of the parallelogram?



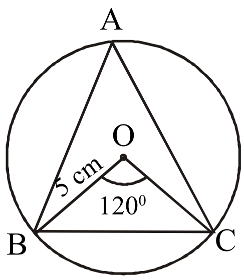
3. In ΔABC, $\angle B = 90^0$, AC = 17cm , BC = 8 cm.

(a) Find Sin A.

(b) Find Cos A.



4. In figure 'O' is the centre of the circle and radius of the circle is 5 cm. $\angle BOC = 120^0$.



(a) Find $\angle A$?

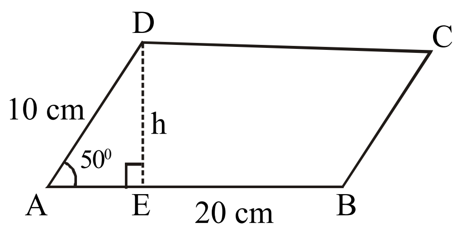
(b) Find the length of BC?

5. Two sides of a parallelogram are 20 cms and 10 cms. The angle between them in 50^0.

(a) Find the height of the parallelogram?

(b) Find the area of the parallelogram?

(Sin 50^0 = 0.77, Cos 50^0 = 0.64, tan 50^0 =1.19)



6. Consider a square of perimeter 20 cm.

(a) Find the length of one side of the square?

(b) Find the length of its diagonal?

(c) What in the area of the square drawn on its diagonal as side?

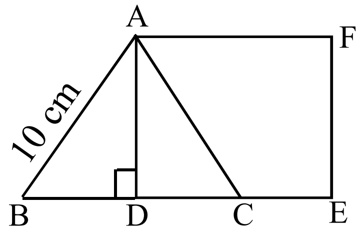
7. Consider an equilateral triangle of side 10 cms.

A square is drawn on its altitutde.

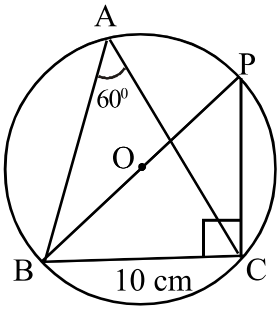
(a) Find the length of the altitude (AD).

(b) Find one side of the square?

(c) Calculate the area of the square?



8. In figure 'O' in the centre of the circle. $\angle A = 60^0$, BC = 10 cm. The diameter drawn from 'B' meets the circle at P.



(a) Find BP?

(b) Find PC?

(c) Find the radius of the circle?

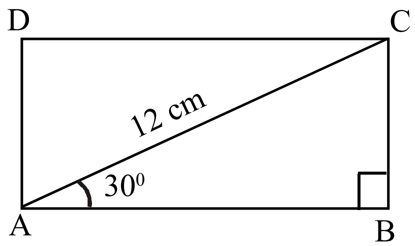
9. The diagonal of a rectangle ABCD is 12 cm.

$\angle BAC = 30^0$

(a) Find the length of AB?

(b) Find the length of BC?

(c) Calculate the area of the rectangle?

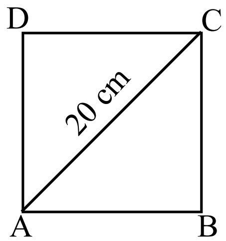


10. If the length of diagonal of a square in 20 cm.

(a) Find one side of square?

(b) Find the perimeter of square?

(c) Find its area?



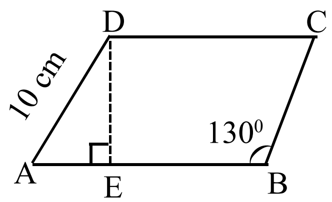
11. In figure ABCD is a rhombus

AD = 10 cm, $\angle B = 130^0$.

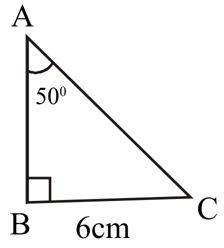
(a) Find $\angle A$?

(b) Find the length of DE?

(Sin $50^0 = 0.77$, Cos $50^0 = 0.64$, tan $50^0 = 1.19$)



12. In ΔABC, $\angle A = 50^0$, BC = 6 cm.



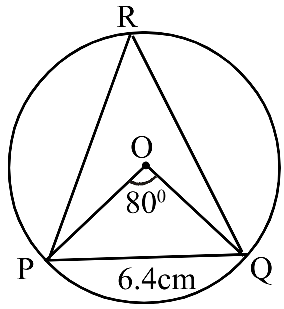
(a) Find $\angle C$?

(b) Find the length of AB?

(c) Find the area of triangle ABC?

($\sin 40^0 = 0.64$, $\cos 40^0 = 0.77$, $\tan 40^0 = 0.84$)

13. PQ is the chord of the circle with centre 'O'.



$\angle POQ = 80^0$, PQ = 6.4 cm.

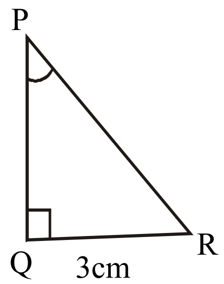
(a) Find $\angle PRQ$?

(b) Find the diameter of the circle?

($\sin 40^0 = 0.64$, $\cos 40^0 = 0.77$, $\tan 40^0 = 0.84$)

Part III (4 Marks Questions)

1. In figure $\angle Q = 90^0$, QR = 3 cm $\cos R = 3/5$.

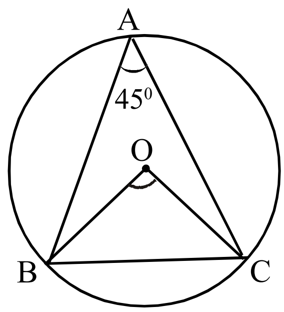


(a) Find the length of PR?

(b) Find PQ?

(c) Sin P = ...

(d) tan P = ...



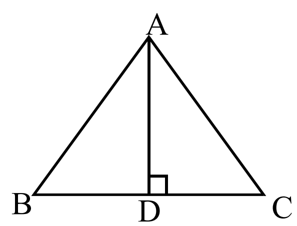
2. In figure $\angle A = 45^0$,

(a) Find $\angle BOC$?

(b) Find $\angle OCB$?

(c) If the radius of the circle is 4 cms, what is the area of ΔOBC?

3. ΔABC is an equilateral triangle. Its perimeter in 30 cm.



(a) Find its one side?

(b) Find AD.

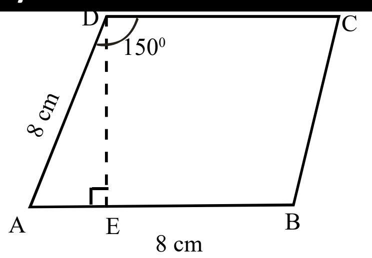
(c) Find the area of ΔABC.

4. ABCD is a rhombus with sides 8 cm. $\angle D = 150^0$

(a) Find $\angle A$?

(b) Find the distance between AB & CD.

(c) Find the area of the rhombus?



5. In figure ABC in a right triangle.

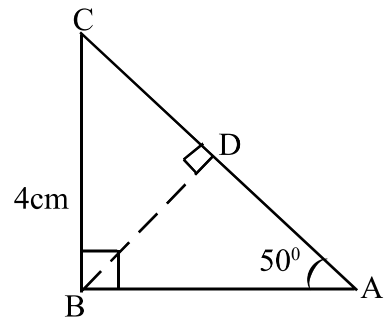
BC = 4 cm, $\angle A = 50^0$, BD perpendicular to AC.

(a) Find $\angle C$?

(b) Find the length of BD?

(c) Find the length of AC?

(sin 40 = 0.64, cos 40 = 0.77)



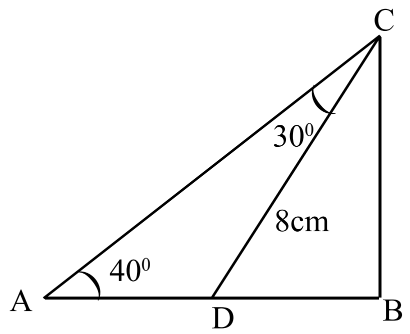
6. In figure $\angle A = 40^0$, $\angle ACD = 30^0$, CD = 8 cm.

(a) $\angle ADC$ = ...

(b) $\angle CDB$ = ...

(c) Find the length of BD?

(d) Find the length of BC? (sim70^0 = 0.97, cos 70^0 = 0.34)



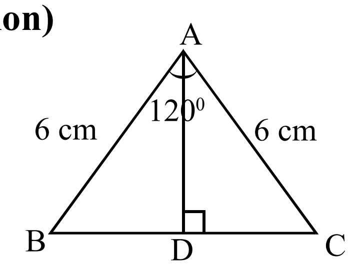
Part IV. (5 mark Question)

1. In figure AB = AC = 6 cm. $\angle A = 120^0$.

(a) $\angle B$ = ...

(b) Find the length of AD?

(c) Find the area of ΔABC ?



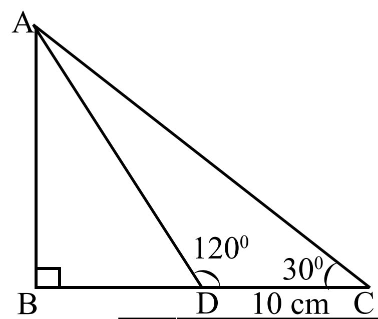
2. In figure ΔABC, $\angle B = 90^0$, $\angle C = 30^0$, $\angle ADC = 120^0$ and DC = 10 cm.

(a) Find $\angle DAC$?

(b) Find $\angle ADB$?

(c) Find the length of AD?

(d) Find the lengths of BD and AC?



3. A boy standing at the edge of a river sees the top of a tree at an angle of elevation of 30^0. Stepping 20 metres back he sees it an angle of elevation of 30^0.

   (a) Draw a rough figure?

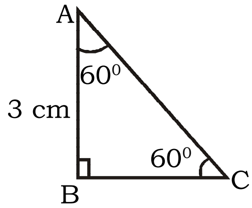
   (b) Find the width of the river?

4. In figure $\angle B = 90^0$, $\angle A = 60^0$, AB = 3 cm. If we draw a square with AC as one side,

   (a) Find the length of one side of the square?

   (b) Find the area of the square?

   (c) What is the length of one diagonal of the sqaure?



5. A man standing on the ground sees the top of 20 metre high building at an angle of elevation 45^0. He sees the mobile tower fixed on the building at an angle of a elevation 60^0.

   (a) Draw a rough figure based on the statement.

   (b) At what distance the man stands from the bottom of the building?

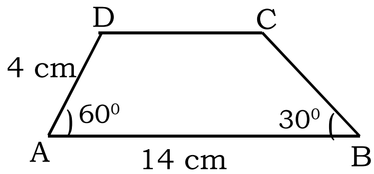
   (c) Find the height of the tower?

6. ABCD in a trapezium. $\angle A = 60^0$, $\angle B = 30^0$. AB = 14 cm, AD = 4 cm.

   (a) Find the distance between AB & CD?

   (b) Find the length of CD?

   (c) Calculate the area of the trapezium.



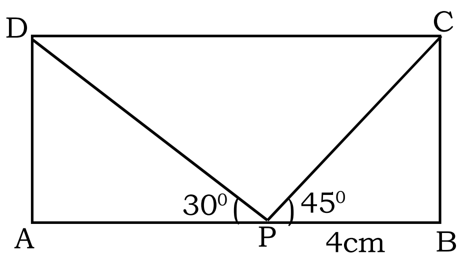
7. ABCD is a rectangle. $\angle DPA = 30^0$, $\angle CPB = 45^0$, PB = 4cm.

   (a) Find the length of BC?

   (b) Find the length of AP?

   (c) Find the length of PD and PC?

   (d) Find the area of the rectangle?

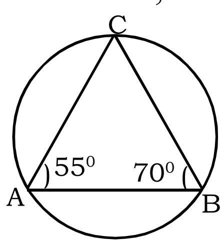


8. In figure, the circum diameter of ΔABC is 6 cm. $\angle A = 55^0$, $\angle B = 70^0$

   (a) find $\angle C$?

   (b) Find the length of AB?

   (c) Find BC?



(d) Find AC?

(Sin $55^0 = 0.82$, Cos $55^0 = 0.57$

Sin $70^0 = 0.94$, Cos $70^0 = 0.34$)

9. A boy standiang on the top of a tower sees the top of a building at an angle of depression 40^0. He sees the base of the building at an angle of depression 50^0. The distance between the tower and building in 40 metres.

(a) Draw a figure based on the given details.

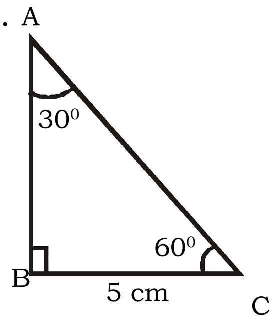
(b) Find the height of the tower?

(c) Find the height of the building?

Answers

Part I (2 Mark Questions)

1. Consider right triangle ABC, angles are $30^0, 60^0, 90^0$.

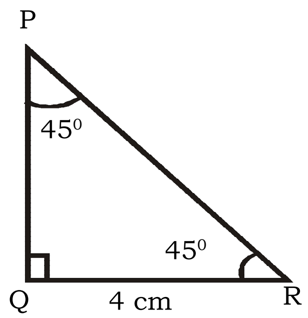


Side are in the ratio $1:\sqrt{3}:2$.

ie, $5, 5\sqrt{3}, 10$

Length of largest side = 10 cm.

2. (a) ΔPQR is an isosceles triangle.



$\therefore$ PQ = QR

ie, PQ = 4 cm.

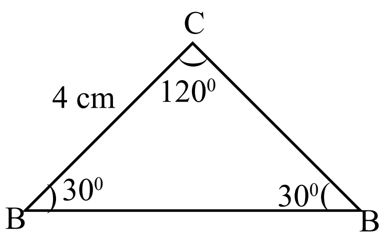
(b) In ΔPQR, angles are $45^0, 45^0, 90^0$.

sides are in the ratio $1:1:\sqrt{2}$

ie $4, 4, 4\sqrt{2}$

$\therefore$ PR = $4\sqrt{2}$ cm

3. (a) BC = 4 cm



(b) In ΔABC angles are $30^0, 30^0, 120^0$,

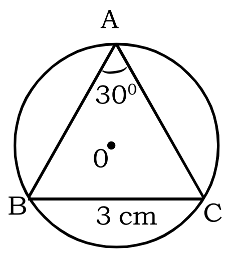
Sides are in the ratio $1 : 1 : \sqrt{3}$

ie 4, 4, $4\sqrt{3}$.

$\therefore$ AB = $4\sqrt{3}$ cm

4. BC = 2r Sin A

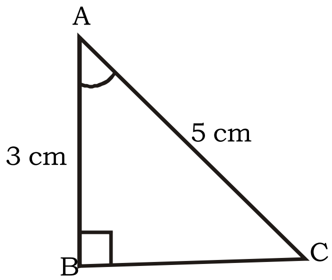
3 = 2r Sin 30^0.



$2r \times \frac{1}{2} = 3$

r = 3 cm

5. (a) BC = $\sqrt{AC^2 - AB^2}$

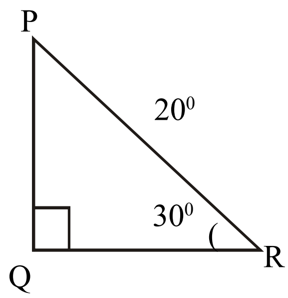


$= \sqrt{5^2 - 3^2} = \sqrt{25-9}$

$= \sqrt{16} = 4$ cm.

(b) tanA = BC/AB = 4/3

6. In triangle angles are 30^0, 60^0, 90^0.



Sides are in the ratio 1:$\sqrt{3}$:2.

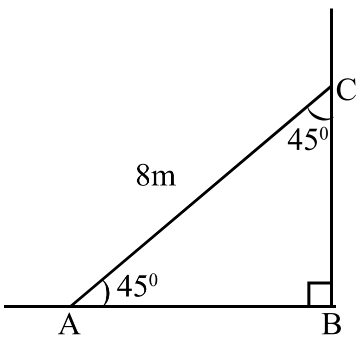
ie, 10, $10\sqrt{3}$, 20.

PQ = 10 cm.

7. Consider right triangle ABC, angles are 45^0, 45^0, 90^0.

Sides are in the ratio 1:1:$\sqrt{2}$.

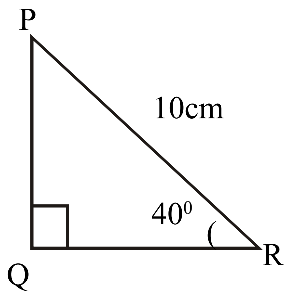
ie, $\frac{8}{\sqrt{2}}, \frac{8}{\sqrt{2}}$, 8.



(a) $\frac{8}{\sqrt{2}} = 4\sqrt{2}$ M

(b) $\frac{8}{\sqrt{2}} = 4\sqrt{2}$ M.

8. (a) $\text{Sin R} = \dfrac{PQ}{PR}$



$PQ = PR \text{ Sin } 40^0$

$= 10 \times 0.64 = 6.4$ cm.

(b) $\text{Cos R} = \dfrac{QR}{PR}$

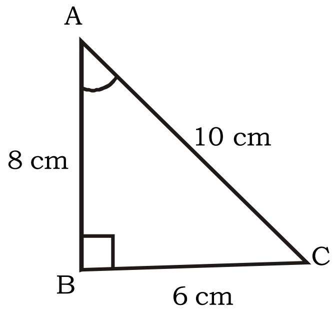
$QR = PR \times \text{Cos } 40^0$

$= 10 \times 0.77$

$= 7.7$ cm.

Part II (3 marks Questions)

1. (a) $\tan A = \dfrac{BC}{AB} = \dfrac{6}{8}$

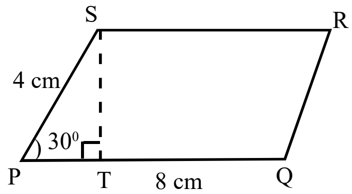


$\therefore x = 6$

(b) $\text{Sin C} = \dfrac{8}{10}, \text{Cos C} = \dfrac{6}{10}$

2. (a) Consider right triangle PTS,

Angles are $30^0, 60^0, 90^0$



Sides are in the ratio $1:\sqrt{3}:2$

ie. $2, 2\sqrt{3}, 4$

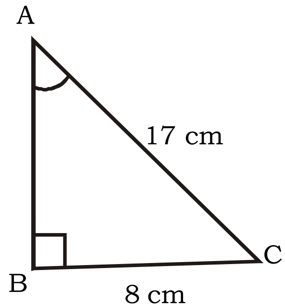
$\therefore ST = 2$ cm

(b) Areas = bh = $8 \times 2 = 16 \text{ cm}^2$

3. (a) $\text{Sin A} = \dfrac{BC}{AC} = \dfrac{8}{17}$

(b) $AB = \sqrt{AC^2 - BC^2} = \sqrt{17^2 - 8^2}$

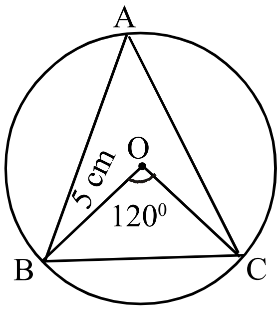
$= \sqrt{289-64} = \sqrt{225} = 15$ cm



$\text{Cos A} = \frac{AB}{AC} = \frac{15}{17}$

4. (a) $\angle A = \frac{120^0}{2} = 60^0$

(b) $BC = 2r \text{ Sin } 60^0$

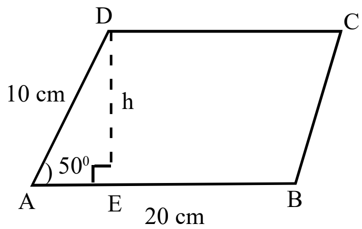


$= 2 \times 5 \times \frac{\sqrt{3}}{2}$

$= 5\sqrt{3}$ cm

5. (a) Consider right triangle AED.

$\text{Sin } 50^0 = \frac{DE}{AD}$



$\text{Sin } 50^0 = \frac{h}{10}$

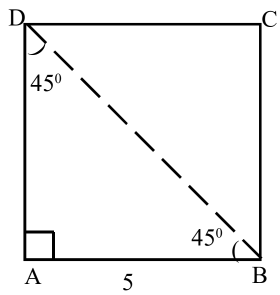
$h = 10 \times \sin 50^0$

$= 10 \times 0.77$

$= 7.7$ cm

(b) Area = bh = $20 \times 7.7 = 154$ cm^2

6. (a) $4a = 20$



$a = \frac{20}{4} = 5$ cm

(b) Consider ΔABD, angles are $45^0, 45^0, 90^0$

Sides are in the ratio $1:1:\sqrt{2}$

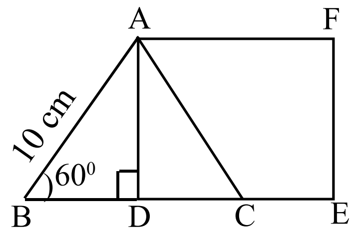
ie 5, 5, $5\sqrt{2}$

Length of diagonal = $5\sqrt{2}$ cm.

(c) Area = $a^2 = \left(5\sqrt{2}\right)^2 = 25 \times 2 = 50$ cm^2

7. (a) Consider ΔADB, Angles are 30^0, 60^0, 90^0.

Sides are in the ratio $1:\sqrt{3}:2$.



ie 5, $5\sqrt{3}$, 10

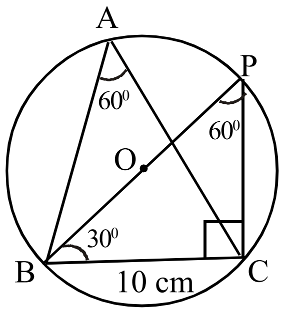
(b) One side of square (AD) = $5\sqrt{3}$

(c) Area = $a^2 = \left(5\sqrt{3}\right)^2 = 25 \times 3 = 75$ cm^2

8. (a) Consider ΔBPC, angles are 30^0, 60^0, 90^0.

Sides are in the ratio $1:\sqrt{3}:2$

ie, $\frac{10}{\sqrt{3}}$, 10, $\frac{20}{\sqrt{3}}$



BP = $\frac{20}{\sqrt{3}}$ cm.

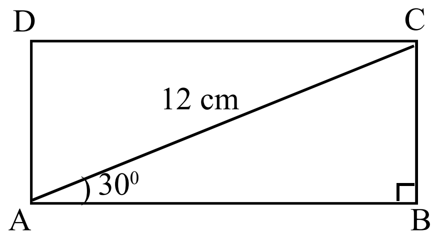
(b) PC = $\frac{10}{\sqrt{3}}$ cm

(c) Radius = $\frac{1}{2} \times BP = \frac{1}{2} \times \frac{20}{\sqrt{3}} = \frac{10}{\sqrt{3}}$ cm.

9. (a) Consider ΔABC angles are 30^0, 60^0, 90^0.

Sides are in the ratio $1:\sqrt{3}:2$

ie, $6, 6\sqrt{3}, 12$



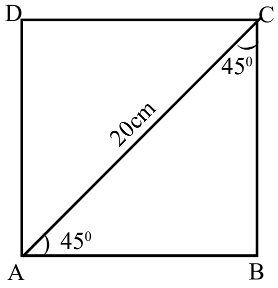
AB = $6\sqrt{3}$ cm

(b) BC = 6 cm

(c) Area = $l \times b = 6\sqrt{3} \times 6 = 36\sqrt{3}$ cm^2

10. (a) Consider ΔABC, angles are $45^0, 45^0, 90^0$.

Sides are in the ratio $1 : 1 : \sqrt{2}$



ie $\frac{20}{\sqrt{2}}, \frac{20}{\sqrt{2}}, 20$

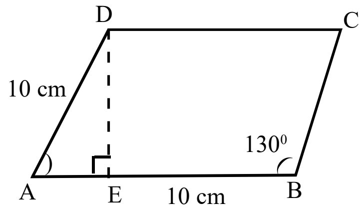
One side of square = $\frac{20}{\sqrt{2}} = 10\sqrt{2}$ cm

(b) Perimetre = 4a = $4 \times 10\sqrt{2} = 40\sqrt{2}$ cm

(c) Area = $a^2 = (10\sqrt{2})^2 = 100 \times 2 = 200$cm^2

11. (a) $\angle A = (180^0 - 130^0) = 50^0$

(b) Consider right triangle AED.



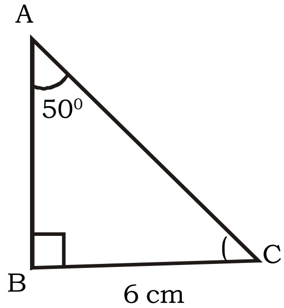
Sin 50^0 = DE / AD

DE = AD × Sin 50^0

= 10 × 0.77 = 7.7 cm.

12. (a) $\angle C = 40^0$

(b) $\tan C = \frac{AB}{BC}$



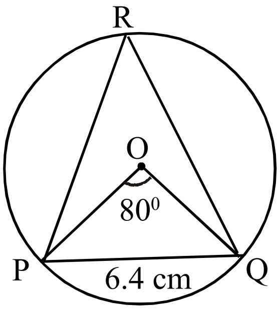
AB = BC × tan 40^0

= 6 × 0.84 = 5.04 cm

(c) Area of $\Delta ABC = \frac{1}{2}bh = \frac{1}{2} \times 6 \times 5.04 = 15.12$cm^2

13. (a) $\angle PRQ = \frac{1}{2} \times \angle POQ = \frac{1}{2} \times 80^0 = 40^0$

(b) PQ = 2r Sin R



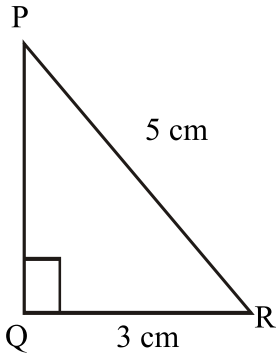
$$2\ r = \frac{PQ}{\text{Sin R}} = \frac{6.4}{\text{Sin } 40^0}$$

$$= \frac{6.4}{0.64} = 10 \text{ cm.}$$

The diametre of the circle = 10 cm

Part III (4 Mark Questions)

1. (a) given Cos R = $\frac{3}{5}$



$\therefore$ PR = 5cm

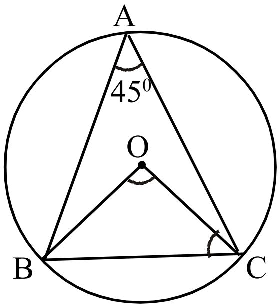
(b) PQ = $\sqrt{PR^2 - QR^2} = \sqrt{5^2 - 3^2}$

$= \sqrt{25-9} = \sqrt{16} = 4$ cm

(c) Sin P = $\frac{3}{5}$

(d) tan P = $\frac{3}{4}$

2. (a) $\angle BOC = 2 \times 45^0 = 90^0$



(b) $\angle OCB = 45^0$

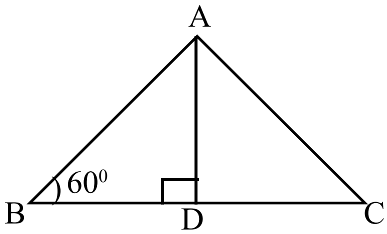
(c) r = 4 cm

Area of Δ OBC = $\frac{1}{2} \times 4 \times 4 = 8\text{cm}^2$

3. (a) $3a = 30$ cm

$$a = \frac{30}{3} = 10 \text{ cm}$$

$\therefore$ one side of equilateral triangle = 10 cm.



(b) Consider ΔADB, Angles are 30^0, 60^0, 90^0.

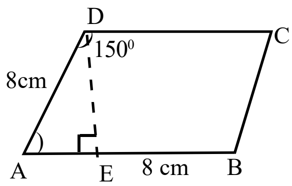
Sides are in the ratio $1 : \sqrt{3} : 2$

ie, 5, $5\sqrt{3}$, 10

AD = $5\sqrt{3}$ cm

(c) Area of $\Delta ABC = \frac{1}{2}bh = \frac{1}{2} \times 10 \times 5\sqrt{3} = 25\sqrt{3}$ cm^2

4. (a) $\angle A = (180^0 - 150^0) = 30^0$



(b) Consider the right triangle AED,

Angles are 30^0, 60^0, 90^0

Sides are in the ratio $1 : \sqrt{3} : 2$

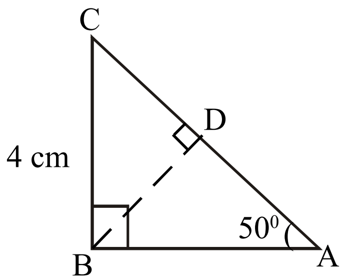
ie 4, $4\sqrt{3}$, 8

Distance between AB & CD = 4 cm

(c) Area = bh = 8 × 4 = 32 cm^2

5. (a) $\angle C = 40^0$

(b) Consider the right triangle BDC,



$$\text{Sin } 40^0 = \frac{BD}{BC}$$

BD = BC × Sin 40^0

= 4 × 0.64 = 2.56 cm.

(c) Consider right triangle ABC

$$\text{Cos } 40 = \frac{\text{Adjacent side}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{4}{AC}$$

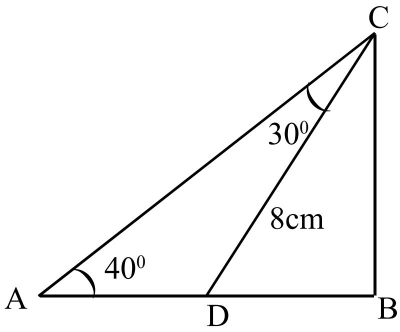
$$0.77 = \frac{4}{AC}$$

$$0.77 \times AC = 4$$

$$AC = \frac{4}{0.77} = 5.19 \text{ cm}$$

6. (a) $\angle ADC = 110^0$

(b) $\angle CDB = 70^0$



(c) $\text{Cos}70^0 = \frac{BD}{DC}$

$BD = DC \times \text{Cos } 70^0 = 8 \times 0.34 = 2.72$ m

(d) Consider $\angle CBD$

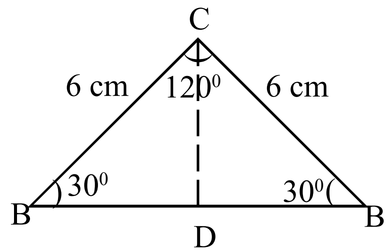
$$\text{Sin D} = \frac{BC}{DC}$$

$BC = DC \times \text{Sin } 70^0 = 8 \times 0.94 = 7.52$ m

Part IV (5 mark Questions)

1. (a) $\angle B = 30^0$

(b) Consider the right triangle ADB, angles are 30^0, 60^0, 90^0.



Sides are in the ratio $1:\sqrt{3}:2$

ie, $3, 3\sqrt{3}, 6$

AD = 3 cm

(c) $BD = 3\sqrt{3}$

$\therefore BC = 2 \times 3\sqrt{3} = 6\sqrt{3}$

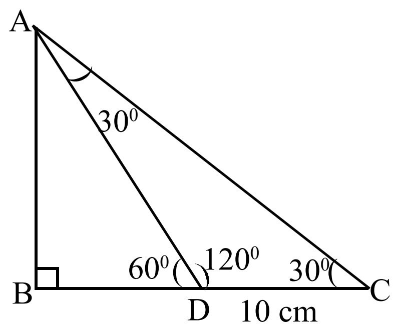
Area of $\Delta ABC = \frac{1}{2}bh = \frac{1}{2}\times 6\sqrt{3}\times 3 = 9\sqrt{3}$ cm^2

2. (a) $\angle DAC = 30^0$

(b) $\angle ADB = 60^0$

(c) $AD = 10$ cm

(d) Consider the right triangle ABD, Angles are $30^0, 60^0, 90^0$



Sides are in the ratio $1:\sqrt{3}:2$

ie, $5, 5\sqrt{3}, 10$

BD = 5 cm

Consider right triangle ABC, BC = 15 cm

Sides are in the ratio $1:\sqrt{3}:2$

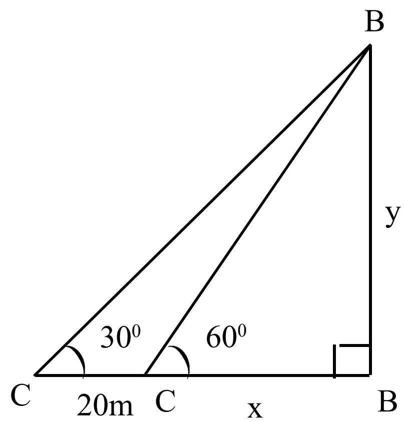
ie, $\frac{15}{\sqrt{3}}$, 15, $\frac{30}{\sqrt{3}}$

$AC = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ cm

3. (b) Let the width of river = x and

Height of tree = y

Consider ΔABC,



$\tan 60^0 = \frac{y}{x}$

$y = x \tan 60^0$ .... (1)

Consider ΔABD,

$$\tan 30^0 = \frac{y}{(x+20)}$$

$y = (x+20) \tan 30^0$ .... (2)

From (1) & (2), $x \tan 60^0 = (x + 20) \tan 30^0$.

$x \times \sqrt{3} = (x + 20) \tan 30^0$

$3x = x + 20$

$2x = 20$

$$x = \frac{20}{2} = 10$$

The width of river = 10 cm.

4. Consider the right triangle ABC,

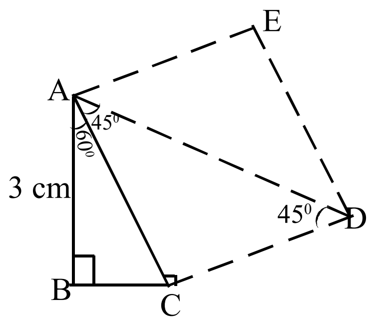
(a) Angles are $30^0, 60^0, 90^0$.

Sides are in the ratio $1:\sqrt{3}:2$

ie, 3, $3\sqrt{3}$, 6.

AC = 6 cm

One side of square = 6 cm



(b) Area of square = $6^2 = 36cm^2$

(c) Consider right triangle ADC, angles are $45^0, 45^0, 90^0$

Sides are in the ratio $1:1:\sqrt{2}$

ie, , 6, 6, $6\sqrt{2}$

Length of diagonal = $6\sqrt{2}$ cm

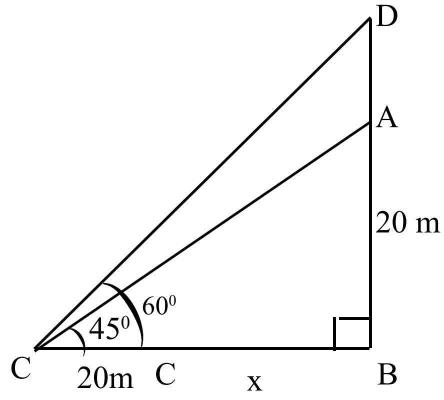
5. (b) Consider ΔABC, Angles are $45^0, 45^0, 90^0$.

Sides are in the ratio $1:1:\sqrt{2}$

ie, 20, 20, $20\sqrt{3}$

BC = 20 m.

Distance from the bottom of building = 20 m.



(c) Consider the right triangle CBD,

angles are 30^0, 60^0, 90^0.

Sides are in the ratio $1:\sqrt{3}:2$

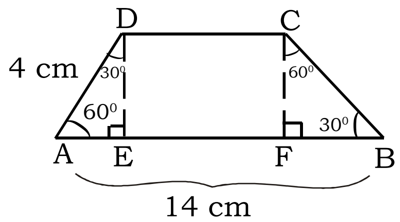
ie, 20, $20\sqrt{3}$, 40.

BD = $20\sqrt{3}$ cm

Height of tower, (AD) = $20\sqrt{3}$ – 20 = 20 $(\sqrt{3}-1)$ m.

6. (a) Consider the right triangle AED,

Angle are 30^0, 60^0, 90^0.



Sides are in the ratio $1:\sqrt{3}:2$

ie, 2, $2\sqrt{3}$, 4

Distance between AB & CD = $2\sqrt{3}$ cm

(b) Consider the right triangle BFC. Angles are 30^0,60^0, 90^0. CF = $2\sqrt{3}$.

Sides are in the ratio 1: $\sqrt{3}$:2

ie, $2\sqrt{3}$, ↓ $4\sqrt{3}$

$$2\sqrt{3}\times\sqrt{3}=6$$

ie, FB = 6 cm

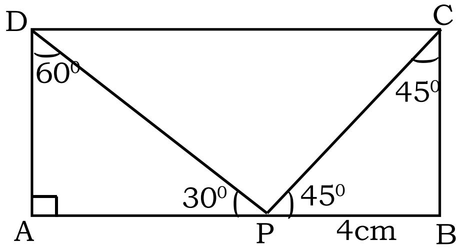
CD = FE = [14 – (6 + 2)] = 6 cm.

(c) Area = $\frac{1}{2}h(a+b)$ = $\frac{1}{2}\times 2\sqrt{3}$ (14+6)

$= 20\sqrt{3}$ cm^2

7. (a) Consider the right triangle CBP, angles are 45^0, 45^0, 90^0.

Sides are in the ratio $1:1:\sqrt{2}$



ie, 4, 4, $4\sqrt{2}$

$\therefore$ BC = 4 cm.

(b) BC = AD = 4 cm

Consider right triangle PAD, angles are 30^0, 60^0, 90^0.

Sides are in the ratio $1:\sqrt{3}:2$.

ie, 4, $4\sqrt{3}$, 8

AP = $4\sqrt{3}$ cm

(c) PC = $4\sqrt{2}$, PD = 8 cm

(d) Area = $l \times b$ = $(4 + 4\sqrt{3})\, 4 = 16\left(1+\sqrt{3}\right)$ cm^2

8. (a) $\angle C = 180^0 - (55^0 + 70^0) = 55^0$

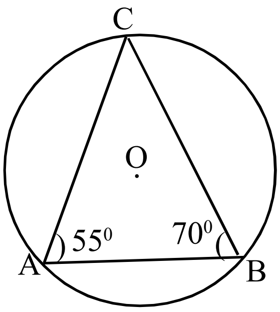
(b) Circum diameter = 6 cm

AB = 2r Sin C.

= 6 × Sin 55^0

= 6 × 0.82

= 4.92 cm



(c) $\angle A = \angle C$

$\therefore$ AB = BC

ie, BC = 4.92 cm

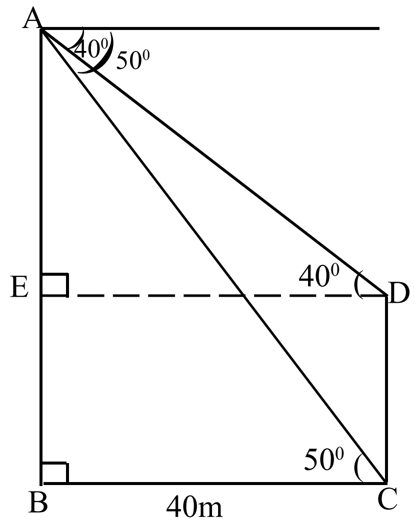
(d) $AC = 2r \sin 70^0$

$= 6 \times 0.94$

$= 5.64$ cm.

9. (a)

(b) Consider ΔABC



$\tan 50^0 = \dfrac{AB}{BC}$

$AB = BC \times \tan 50^0$

$= 40 \times 1.19 = 47.6$ m

Height of tower = 47.6 m.

(c) Consider ΔAED.

$\tan 40^0 = \dfrac{AE}{ED}$

$AE = ED \times \tan 40^0$

$= 40 \times 0.80 = 33.6$ m

Height of building = AB – EB

= 47.6 – 33.6

= 14 m

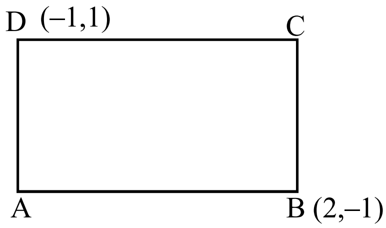
CHAPTER - 6
COORDINATES

Important facts

1. The perpendicular lines we draw to mark the positions of points are called the axes of coordinates. The horizontal line is called the x axis and the vertical line is called the y axis.
2. The point of intersection of both the axes is called the origin. The coordinates of the origin is (0,0)
3. Any point on the x axis has the y coordinate zero.
4. Any point on the y axis has the x coordinate zero.
5 All points with same y coordinate are on a line parallel to the x axis and the distance between any pair of such points is the difference of their x coordinates.
6. All points with same x coordinate are on a line parallel to the y axis and the distance between any pair of such points is the difference of their y coordinates.
7. The distance between the points with coordinates (x_1, y) and (x_2, y) is $|x_2 - x_1|$.
8. The distance between the points with coordinates (x, y_1) and (x, y_2) is $|y_2 - y_1|$.
9. The distance between the point (x, y) and origin is $\sqrt{x^2 + y^2}$.
10. The distance between the points with coordinates (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Section A (2 Scores each)

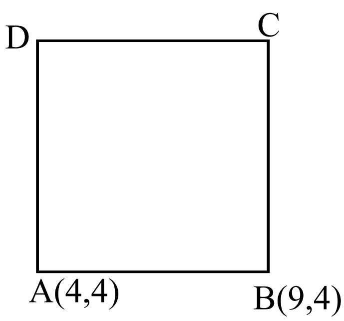
1. Find the radius of a circle with centre at the origin and passing through the point (6, 8).
2. Draw the coordinate axes and mark the points A(3, 4) and B(–3, 4)
3. The rectangle given below have sides parallel to the axes. Find the coordinates of the remaining two vertices.



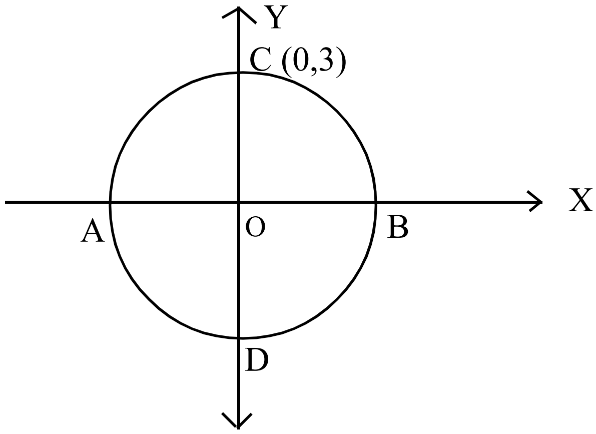
4. A circle of radius 1 is drawn with origin as centre.
   (a) Write the coordinates of the points where the circle cuts the x axis.
   (b) Write the coordinates of the points where the circle cuts the y axis.
5. (a) Write the coordinate of another point on the line passing through the points (2, 1) and (7, 1).
   (b) Write the coordinates of a point on the line perpendicular to this line and passing through (7, 1).

SECTION B (3 Scores Each)

6. (a) Find the coordinates of the point on x axis, which is at a distance 4 units from the point (3 ,4).
   (b) Find the coordinates of the point on x axis, which is at a distance 5 units from the point (3, 4).
7. In the figure, ABCD is a square. A (4, 4) and B (9, 4) are two vertices of the square.
   (a) What is the length of AB ?
   (b) Write the coordinates of C and D.



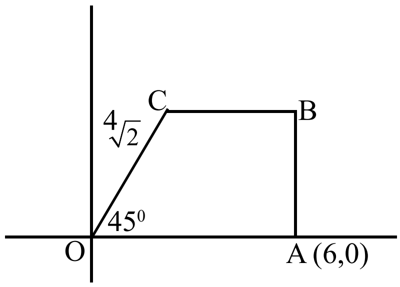
8. In the figure, AB and CD are diameters of the circle. Coordinates of C is (0, 3). Write the coordinates of A, B and D.



9. In the figure OABC is a trapezium. OC = $4\sqrt{2}$, $\angle COA = 45°$. Coordinates of A is (6, 0).

(a) What is the perpendicular distance from C to OA ?

(b) Write the coordinates of B and C.



10. (a) Check whether the circle with centre at the point (2, 4) and radius 5 passes through the point (2, 0).

(b) Write the coordinates of the points at which this circle cuts the x axis.

Section C (4 scores each)

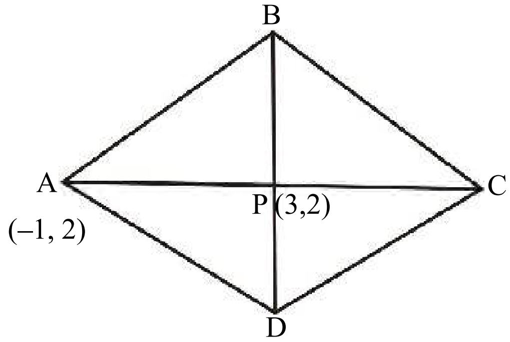
11. Draw the coordinate axes and mark the points A(-3, 0), B(3, 0) and C (0, $3\sqrt{3}$)

12. in the figure, ABCD is a rhombus. Coordinates of A are (-1, 2). The diagonals AC and BD intersect at P(3, 2).

(a) Write the coordinates of the vertex C.

(b) If the length of diagonal BD is 6. Write the coordinates of other two vertices.

(c) Calculate the length of one side of the rhombus.



13. Draw the x and y axes. Mark the point (2, 3). Draw a circle with origin as centre and passing through the point (2, 3).

14. A quadrilateral is drawn joining the points (3, 0), (8, 0), (11, 4) and (6, 4).

    (a) Find the length of each side of the quadrilateral.

    (b) What is the appropriate name of the quadrilateral ?

    (c) Find the perimeter of the quadrilateral.

15. A circle with centre at (1, 2) passes through the point (5, -1).

    (a) What is the radius of the circle?

    (b) Check whether each of the points with coordinates (0, 7), (4, 6), (5, 0) inside, outside or on the circle.

Section D (5 Scores each)

16. The vertices of a triangle are A(2, 2), B(2, 1) and C(5, 2).

    (a) What is the length of AB ?

    (b) What is the length of BC ?

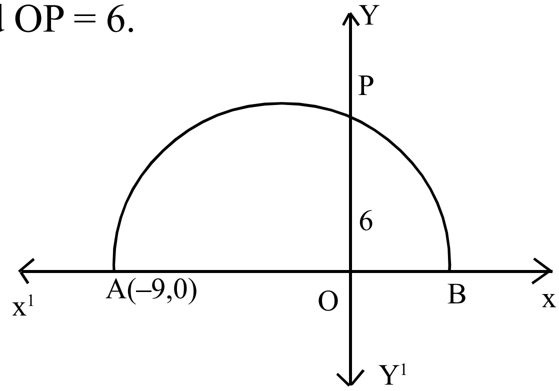
    (c) Prove that ABC is a right angled triangle.

17. In the figure, O is the origin. A semi circle with AB as diameter cuts the y axis at the point P. The coordinate of A is (-9, 0) and OP = 6.

    (a) What is the length of OA ?

    (b) What is the length of OB ?

    (c) Write the coordinates of B and P.

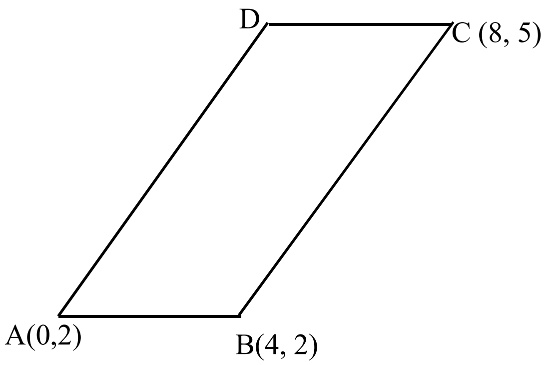


18. In the figure, ABCD is a parallelogram.

    (a) Write the coordinates of D.

    (b) What is the height of this parallelogram ?

    (c) Find the perimeter and area of it.



19. (a) Draw the coordinate axes and mark the points A(2, 3), B(2, 0), C(5, 0) and D(5, 3).

    (b) What is the suitable name of the quadrilateral ABCD so formed ?

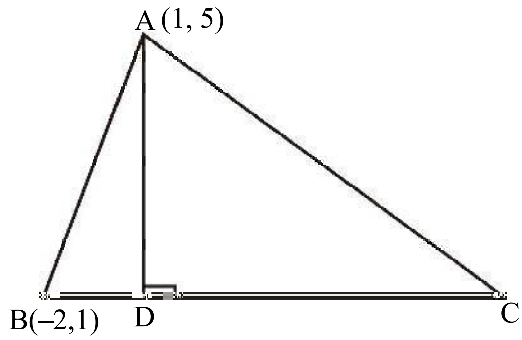
    (c) Find the area of the quadrilateral ABCD.

20. In the figure, BC is parallel to x axis. The coordinates of A is (1, 5) and B is (–2, 1).

    (a) What is the length of AB ?

    (b) If AC = $\sqrt{41}$ then write the coordinates of C.

    (c) If BC is perpendicular to AD, then write the coordinates of D.

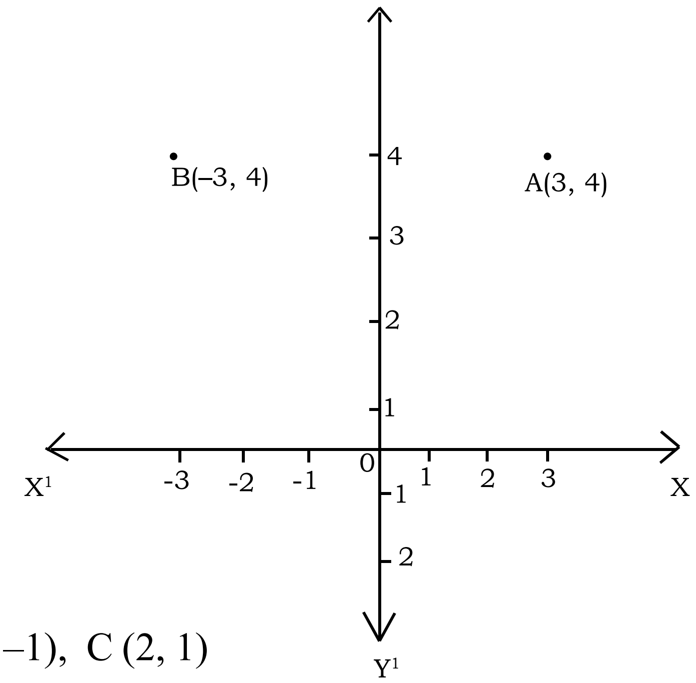


Answer key

Section A

1. radius = $\sqrt{6^2+8^2} = \sqrt{100} = 10$

2.



3. A (–1, –1), C (2, 1)

4. (a) (–1, 0), (1, 0)

   (b) (0, 1), (0, –1)

5. (a) (3, 1) or write any point whose y - coordinate is 1.

   (b) (7, 2) or write any point whose x - coordinate is 7.

Section B

6. (a) (3, 0)

   (b) Let P(x, 0) be the required point.

   A (3, 4) be the given point

   PA = 5

   $PA^2 = 25$

$(x-3)^2 = 9$

$x-3 = 3$ or $x - 3 = -3$

$x = 6$ or $x = 0$

(6, 0) and (0, 0) are the required points.

7. (a) $AB = |9-4| = 5$

(b) C (9, 9), D (4, 9)

8. A (–3, 0), B (3, 0), D (0, –3)

9. Draw $CD \perp OA$

$CD = 4$, $OD = 4$

(a) $\perp$ distance from C to OA = 4

(b) B (6, 4), C(4, 4)

10. (a) Distance between the points (2, 4) and (2, 0) is 4 < 5 (radius)

$\therefore$ (2, 0) is not a point on the circle.

(b) Let P(x, 0) be the required point.

C (2, 4) centre

$PC = 5$

$PC^2 = 25$

$(x-2)^2 + (0-4)^2 = 25$

$(x-2)^2 = 9$

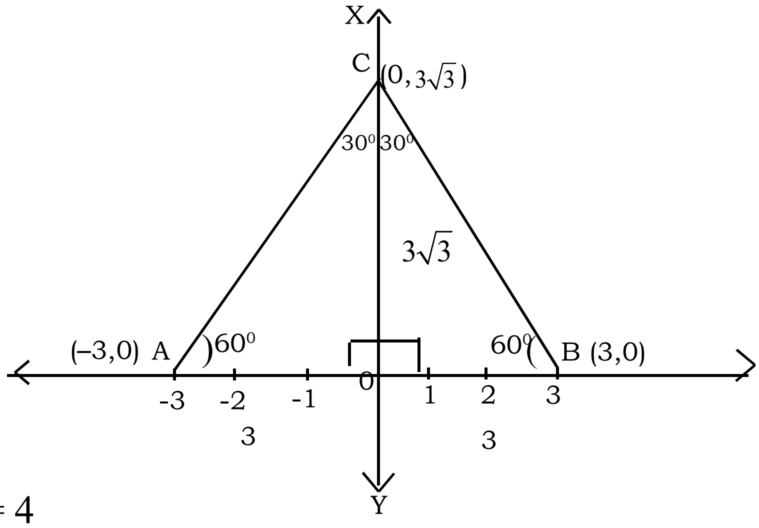
$x-2 = 3$ or $x - 2 = -3$

$x = 5$ or $x = -1$

(5, 0) and (–1, 0) are the required points.

Section C

11.



12. (a) AP = |3 – (–1)| = 4

PC = 4

Coordinates of C (7, 2)

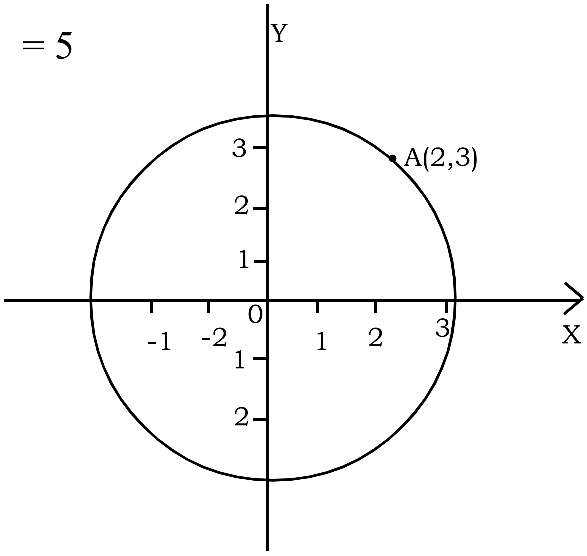
(b) B (3, 5)

D (3, –1)

(c) AB = $\sqrt{(3+1)^2 + (5-2)^2}$

$= \sqrt{25} = 5$

13.



14.. A(3, 0), B (8, 0), C (11, 4), D (6, 4)

(a) AB = 8 – 3 = 5

$BC = \sqrt{3^2 + 4^2} = 5$

C D = 11 – 6 = 5

$AD = \sqrt{3^2 + 4^2} = 5$

(b) Square

(c) Perimeter = 4 × 5 = 20

15. (a) C(1, 2), A (5, –1)

radius AC $= \sqrt{(5-1)^2 + (-1-2)^2}$

$= \sqrt{16+9} = 5$

(b) B(0, 7), D (4, 6), E (5, 0)

$CB = \sqrt{1+25} = \sqrt{26} > 5$ (radius)

B is a point outside the Circle

$CD = \sqrt{9+16} = \sqrt{25} = 5$ (radius)

D is a point on the Cirle.

$CE = \sqrt{16+4} = \sqrt{20} < 5$ (radius)

E is a point inside the circle.

Section D

16. (a) AB = 2 – 1 = 1

(b) $BC = \sqrt{3^2 + 1^2} = \sqrt{10}$

(c) AC = 5 –2 = 3

$AB^2 + AC^2 = BC^2$

$\therefore$ ΔABC is a right triangle, right angled at A.

17. (a) OA = 9

(b) $OA \times OB = OP^2$

$9 \times OB = 6^2$

$OB = \frac{36}{9} = 4$

(c) B (4, 0)

P (0, 6)

18. (a) D (4, 5)

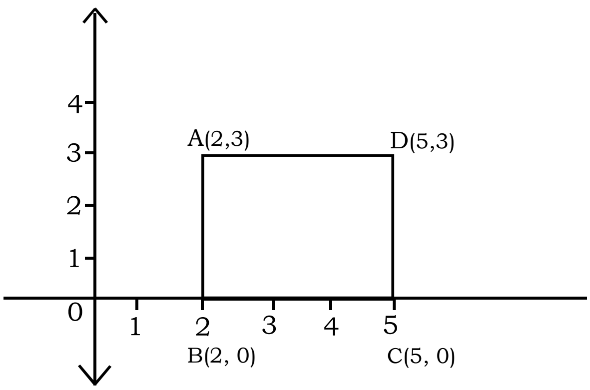
(b) $BD = 5 - 2 = 3$

(c) $AB = 4,\ BC = 5,\ CD = 4$

$AD = 5$

Perimeter = 2 (5+4) = 18 , Area = 4 × 3 = 12 Sq. units.

19. (a)



(b) Square

(c) Area = $5^2 = 25$

20. (a) $AB = \sqrt{3^2 + 4^2} = 5$

(b) coordiantes of D (1, 1)

(c) $AD = 5 - 1 = 4$

$AC = \sqrt{41}$

$CD = \sqrt{41 - 16} = \sqrt{25} = 5$

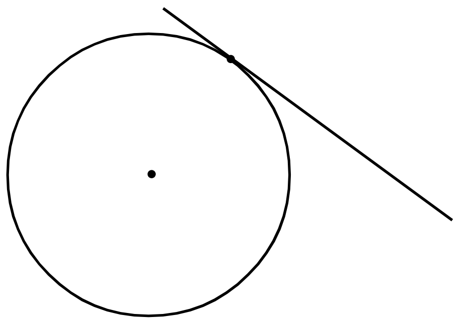
C (6, 1)

CHAPTER - 7

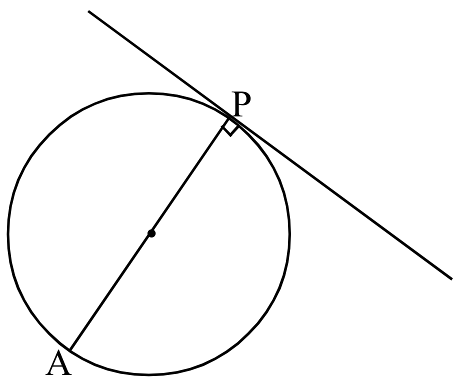
TANGENTS

Main Concepts

1. If a line touches a circle and it does not cross the circle when extends, then the line is a tangent to the circle.



2. The tangent at a point on a circle is perpendicular to the diameter through that point.

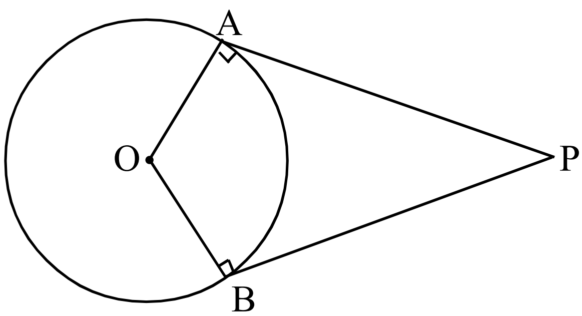


3. Two tangents from an outer point and two radii to the point where the tangents touch the circle form a cyclic quadrilateral.

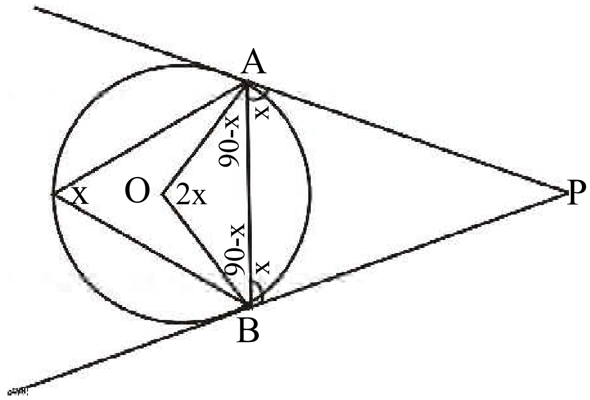
$\angle A + \angle B = 180^0$

$\therefore \angle O + \angle P = 180^0$

So the quadrilateral PAOB is cyclic

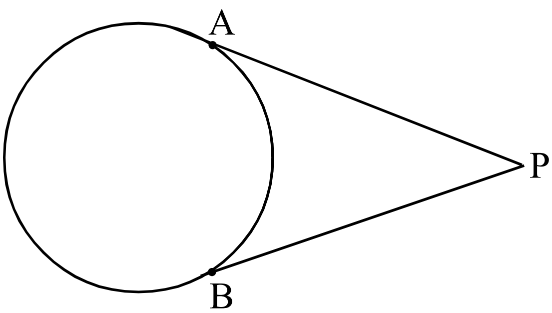


4. In a circle, the angle which a chord makes with the tangent at one end on any side is equal to the angle which it makes on the part of the circle on the other side.



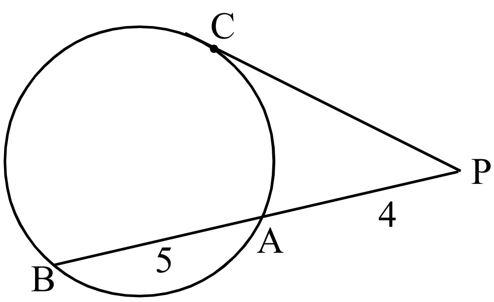
5. From a point outside a circle, two tangents can be drawn and they are of same length.

PA = PB



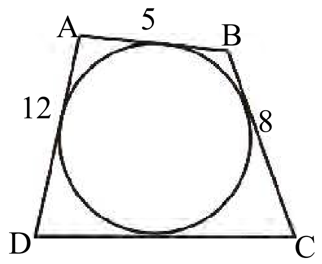
2 marks

1. The Perimeter of a triangle in 24 cm and its inradius is 2 cm. What is the Area?
2. The Perimeter and Area of a triangle are 16 cm and 80 cm^2 respectively. What is its inradius?
3. In the figure PA = 4 cm, BA = 5 cm. Find the length of PC.

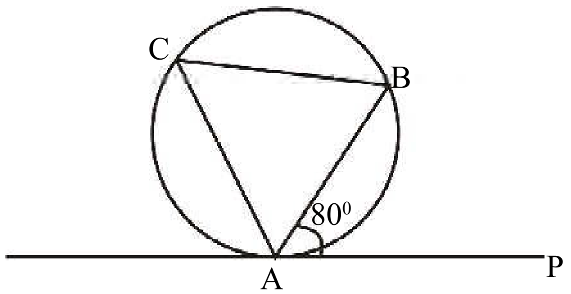


4. The area and perimeter of a triangle are equal. What would be its inradius?

5. In the figure incircle of the quadrilateral ABCD is drawn. If AD = 12 cm, BC = 8 cm, AB = 5 cm, What is the length of DC.



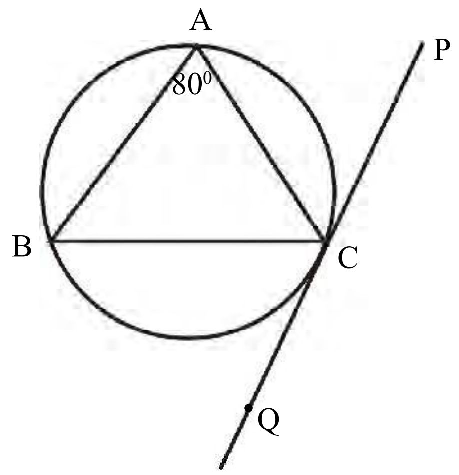
6. In the figure AP is the tangent. If $\angle PAB = 80^0$, find the measure of $\angle C$.



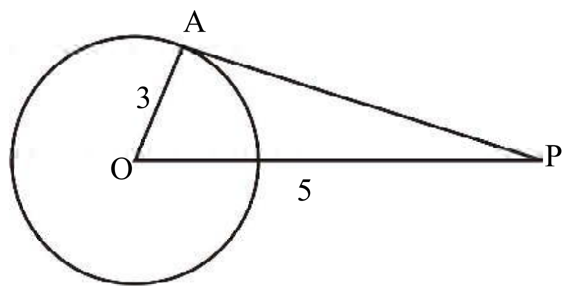
7. In the figure PQ is the tangent. If $\angle A = 80^0$ and AB = AC

(a) Find the measure of $\angle ABC$

(b) Find the measure of $\angle ACP$.



8. In the figure PA is the tangent and 'O' is the centre of the circle. If OA = 3 cm and OP = 5 cm, find the length of PA.

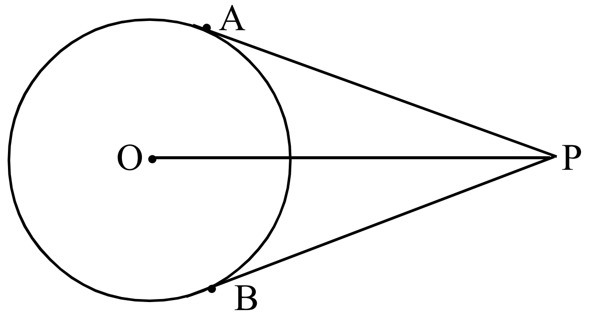


3 marks

1. In the figure, 'O' is the centre and PA, PB are tangents from the point P. If the length of OP is same as the diameter of the circle,

   (a) Find $\angle APO$

   (b) Find $\angle APB$

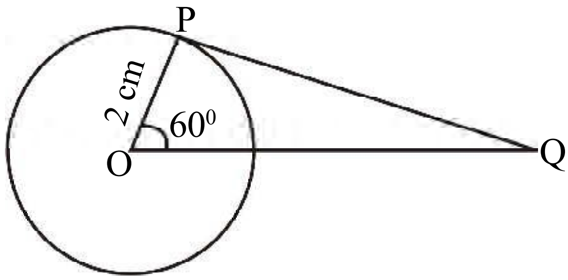


2. In the figure 'O' is the centre and 'P' is a point on the circle. PQ is a tangent.

   (a) What is the measure of $\angle OPQ$?

   (b) What is the measure of $\angle Q$?

   (c) What is the length of PQ?

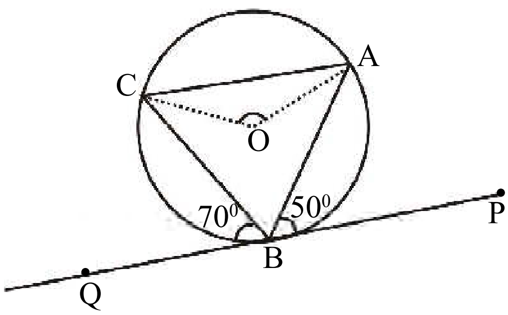


3. In the figure, 'O' is the centre of the circle and $\angle ABP = 50^0$, $\angle CBQ = 70^0$

   (a) What is the measure of $\angle C$?

   (b) What is the measure of $\angle A$?

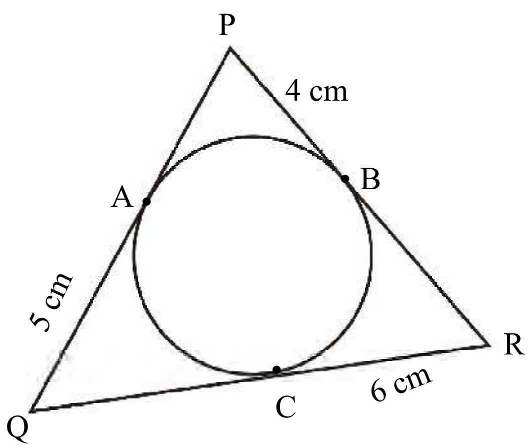
   (c) What is the measure of $\angle AOC$?



4. In the figure PB = 4 cm, CR = 6 cm QA = 5 cm

   (a) What is the length of PA?

   (b) What is the perimeter of triangle PQR?

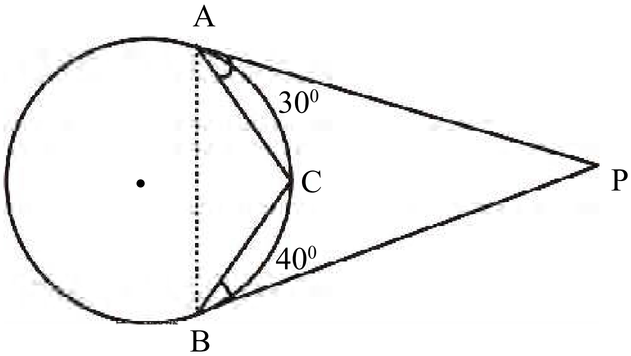


5. In the figure PA and PB an tangents.

(a) What is the measure of $\angle ABC$?

(b) What is the measure of $\angle BAC$?

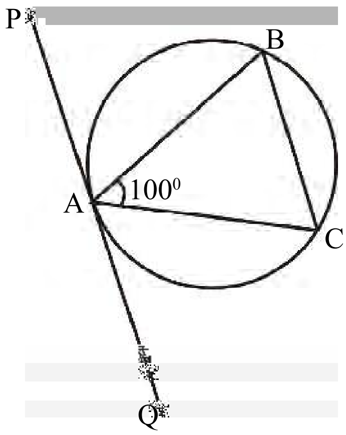
(c) What is the measure of $\angle P$?



6. In the figure, PQ is a tangent of the circle. If AB = AC and $\angle BAC = 100^0$,

(a) $\angle C$ = ...

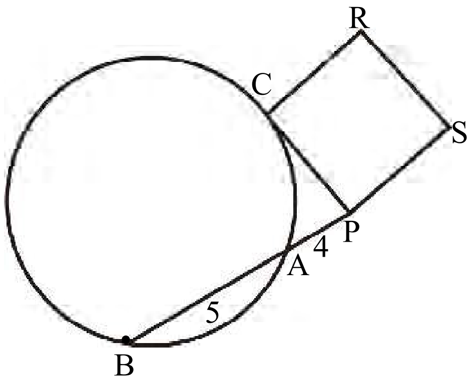
(b) What is the measure of $\angle CAQ$?



7. In the figure PC is the tangent and the quadrilateral PCRS is a square. If PA = 4 cm, BA = 5 cm.

(a) Find PA × PB

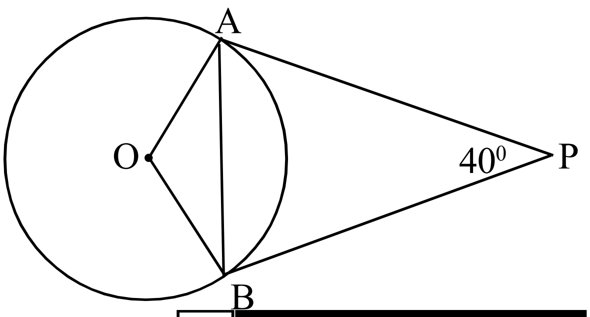
(b) What is the area of the square PCRS?



8. In the figure, 'O' is the centre, PA and PB are tangents from the point P. If $\angle P = 40^0$

(a) Find $\angle AOB$

(b) Find $\angle OAB$



4 marks

1. The perpendicular sides of right are 24 cm and 10 cm.

   (a) What is the length of its hypotenuse?

   (b) What is the perimeter of the triangle?

   (c) What is the area of the triangle?

   (d) Find the radius of the incircle of this triangle.

2. The hypotenuse of a right triangle is 13 cm and its inradius is 2 cm.

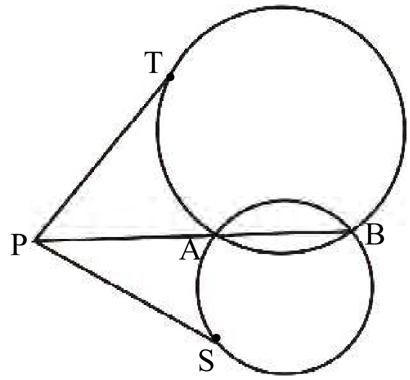
   (a) What is the perimeter of the triangle ?

   (b) What is the area of the triangle?

3. In the figure, two circles meet at the points A and B. PT is the tangent of the large circle and PS is the tangent of the small circle. If PA = 4 cm and AB = 5 cm,

   (a) What is the legnth of PB?

   (b) Find the length of PS and PT?



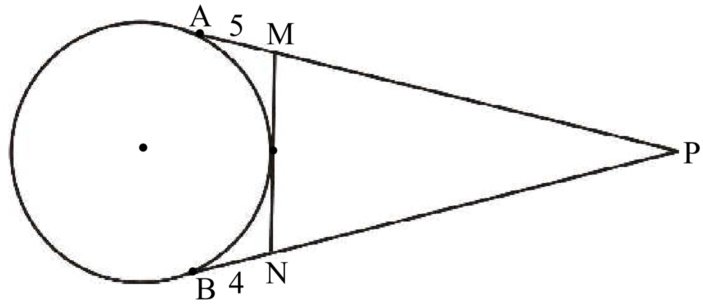
4. In the figure PA, PB, MN are the tangents.

   If AM = 5 cm, BN = 4 cm and the perimeter of the triangle PMN is 40 centimeters,

   (a) What is the length of MN?

   (b) What is the length of PA?

   (c) Find the lengths of PM and PN.

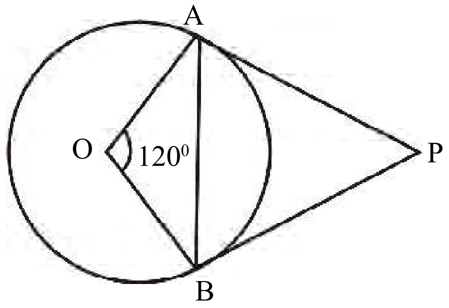


5. In the figure 'O' is the centre, PA and PB are tangents. If $\angle AOB$ 120^0.

(a) What is the measure of $\angle P$?

(b) What is the measure of $\angle PBA$?

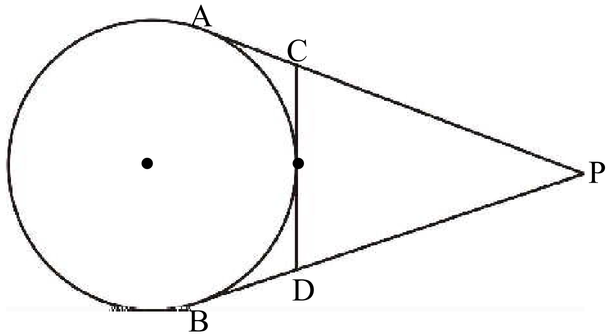
(c) If AB = 10cm, what is the perimeter of triangle PAB?



6. In the figure 'P' is a point 10 cm away from the centre and radius of the circle is 6 cm. PA, PB and CD are tangents.

(a) Find the length of PA

(b) What is the perimeter of triangle PCD?

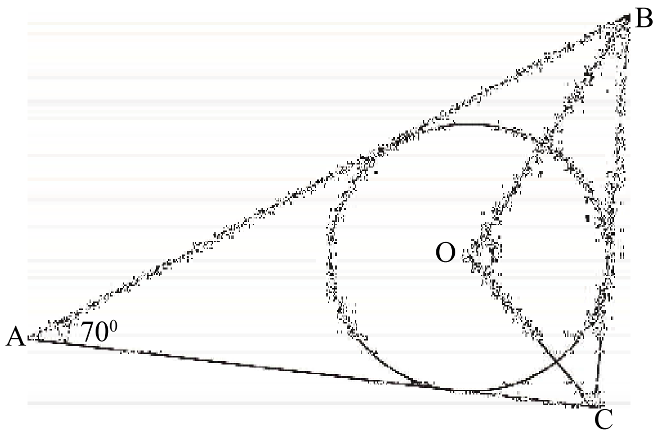


7. In the figure, two of its vertices and the centre of the circle are joined. If $\angle A = 70^0$

(a) $\angle B + \angle C$ =

(b) $\angle OBC + \angle OCB$ =

(c) $\angle BOC$ =

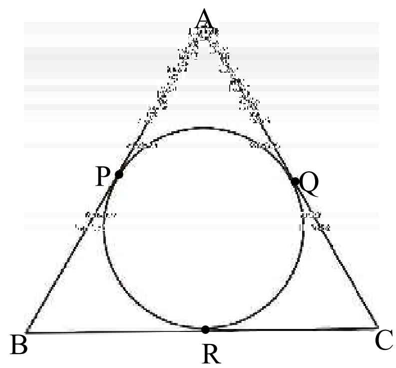


8. In the figre, if AB = 12 cm, BC = 8 cm, AC = 10 cm,

(a) Find the perimeter of the triangle ABC.

(b) What is the length of AP?

(c) AP + BR + CO = ...

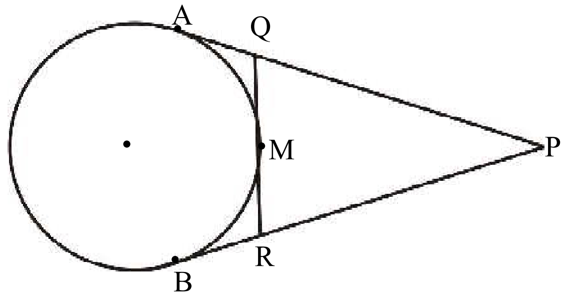


9. In the figure, PA and PB are tangents, QR is also a tangent through the point M

(a) If QA = a, QM = ...

(b) If PA = 10 cm, PB = ...

(c) Find the perimeter of the triangle PQR



5 marks

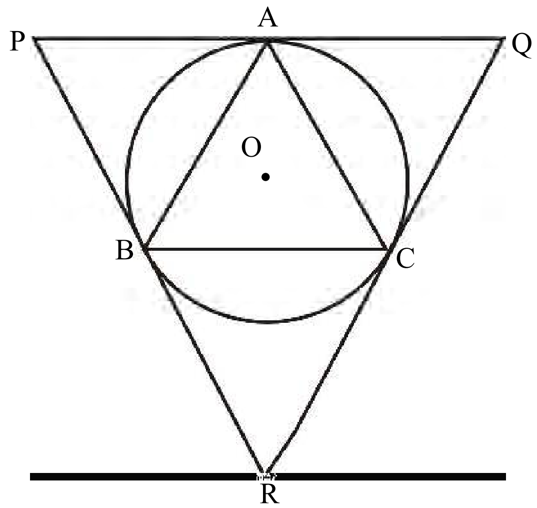
1. In the figure, triangle ABC is an equilateral triangle. The tangents of the circumcircle of triangle ABC through its vertices make the triangle PQR.

(a) If 'O' is the centre of the circle, what is the measure of $\angle AOC$?

(b) What is the measure of $\angle Q$?

(c) What are the measures of $\angle QAC$ and $\angle QCA$?

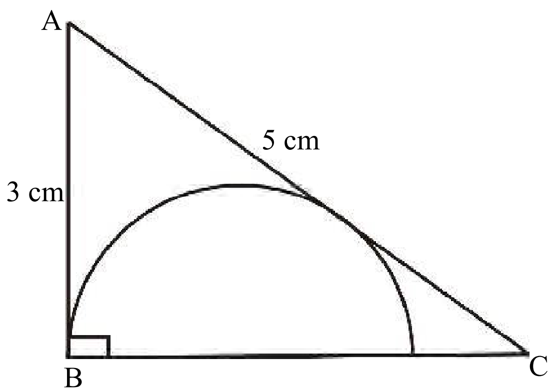
(d) If a side of the triangle ABC is 10 cm, What is the perimeter of the triangle PQR.



2. In the figure, a semicircle touching two sides of the right triangle ABC is drawn.

(a) Find the length of BC.

(b) Compute the radius of the semicircle.



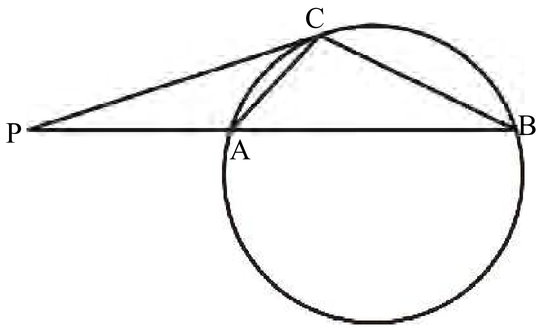
3. In the figure PA=AC and PC is a tangent. If $\angle PCA = x^0$

(a) $\angle B$ =

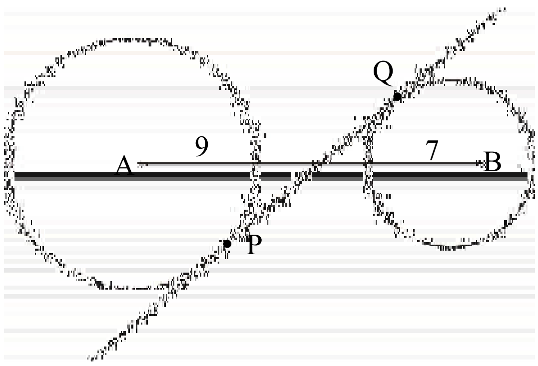
(b) $\angle P$ =

(c) PA × .... = PC^2

(d) If PA = 4 cm and AB = 5 cm, find the length of BC.



4. In the figure PQ is a common tangent to both the circles centered at A and B. radii of the circles are 9 cm and 7 cm. Distance between the centres of the circles is 20 m. What is the length of PQ?



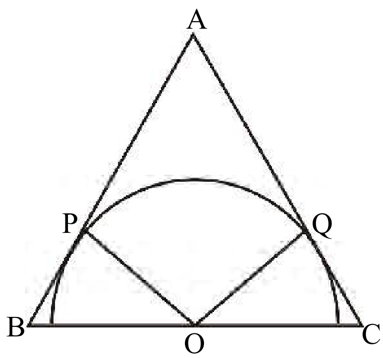
5. In the picture, triangle ABC is an equilateral triangle AP, AQ are tangents. If AB 6cm,

(a) What is the measure of $\angle BPO$?

(b) What is the measure of $\angle BOP$?

(c) What is the length of BP?

(d) Draw an equilateral triangle and draw a semicircle touches two of its sides as in figure.

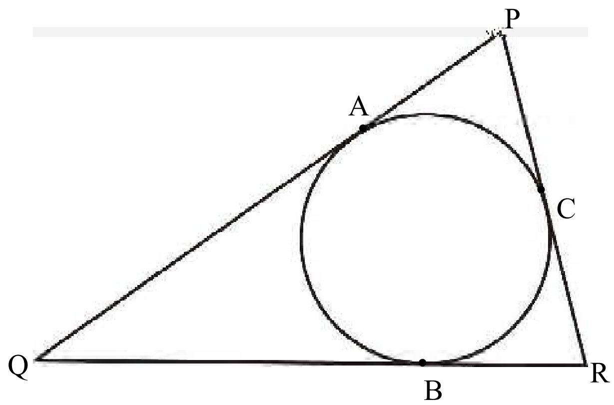


6. The sides of the triangle PQR touch the circle at the points A, B and C. If PQ =10 cm, BR = 4 cm,

(a) What is the length of CR?

(b) What is QB + PC?

(c) Find the perimeter of the triangle PQR.

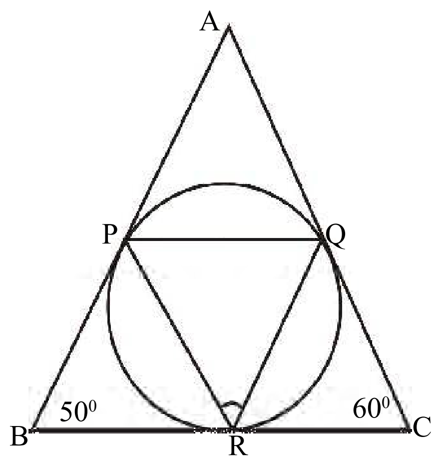


7. In the figure, the incircle of the triangle ABC touches the triangle at the points P, Q and R.

(a) $\angle A$ = ...

(b) $\angle BPR$ = ...

(c) Find all angles of ΔPQR



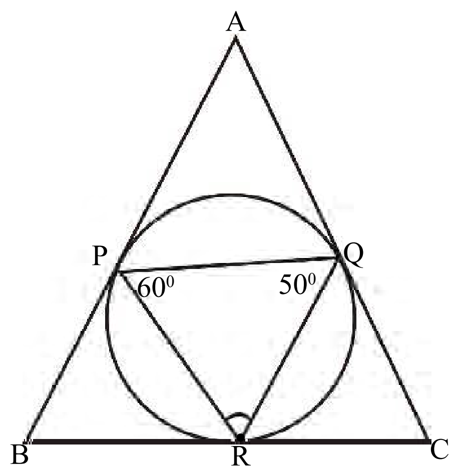
8. In the figure, the incircle of the triangle ABC touches the triangle at the points P, Q and R.

(a) $\angle PRQ$ = ...

(b) $\angle BRP$ = ...

(c) $\angle B$ = ...

(d) Find $\angle A$ and $\angle C$ =



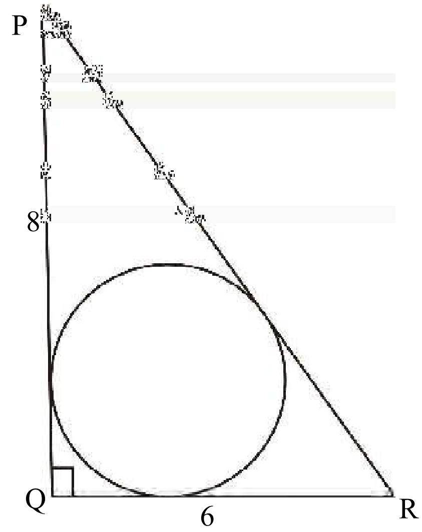
9. In the figure $\angle Q = 90^0$ PQ = 8 cm, QR = 6 cm

(a) Find the length of PR.

(b) What is the perimeter of triangle PQR?

(c) What is the area of the triangle PQR?

(d) Find the radius of the incircle.



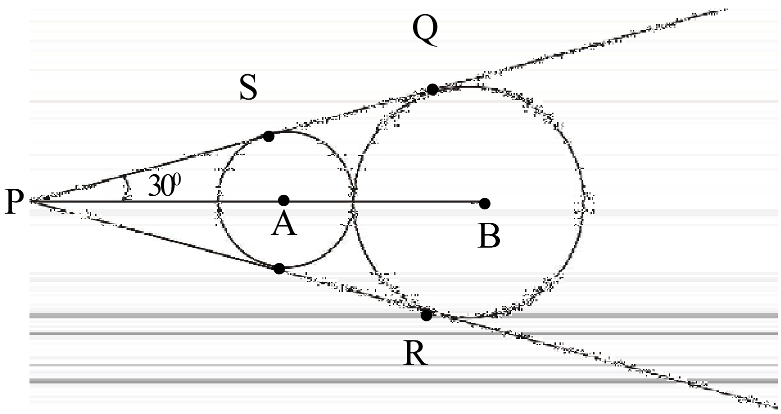
10. In the figure PQ and PR are the common tangents to both the circles centred at A and B. If $\angle BPQ = 30^0$ and radius of the small circle is 3 cm,

(a) What is the measure of $\angle PAS$?

(b) What is the length of PA?

(c) If we take the radius of the large circle as 'r', write PB in terms of 'r'

(d) Find the radius of the large circle.



11. In the figure 'PQ' is a tangent.

If $\angle BCP = 60^0$, $\angle DCQ = 40^0$ and $\angle ABD = 55^0$

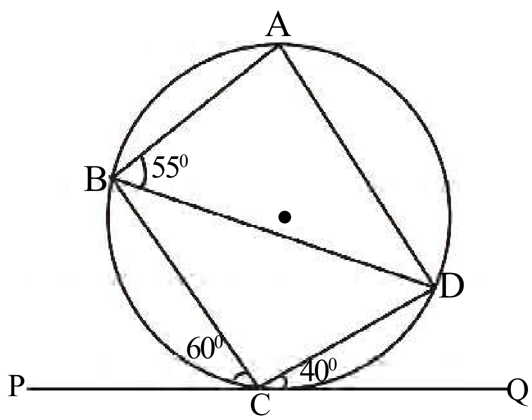
(a) $\angle CBD$ =

(b) $\angle BDC$ =

(c) $\angle BCD$ =

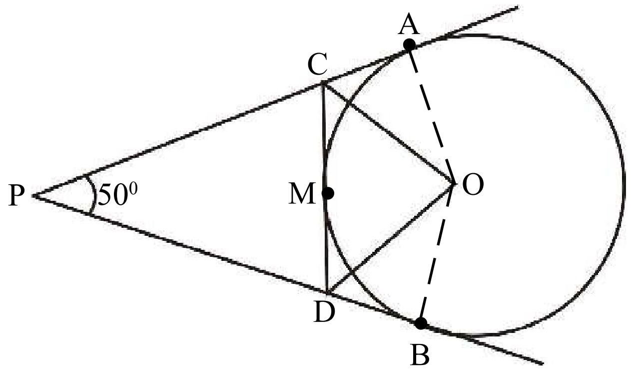
(d) $\angle A$ =

(e) $\angle BDA$ =



12. In the figure PA, PB and CD are tangents.

If $\angle P = 50^0$



(a) $\angle PCD + \angle PDC =$ ....

(b) $\angle ACD + \angle BDC =$ ....

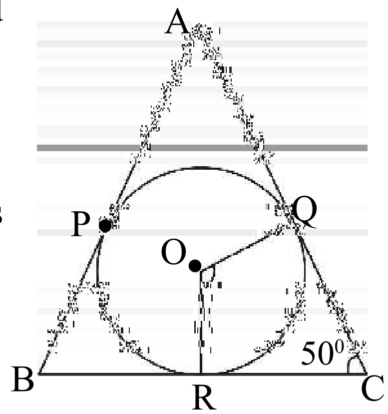
(c) $\angle OCD + \angle ODC =$ ....

(d) What is the measure of $\angle COD$

(e) If $\angle AOC = 30^0$, What is the measure of $\angle BOD$?

Drawing

1. Draw a circle of radius 2.5 cm and mark a point 'P' on it. Draw tangent through 'P'.
2. Draw a circle of radius 3 cm. Mark a point 'P which is 8 cm away from the centre. Draw tangents from 'P' to the circle.
3. In the figure, circle touches the sides of the triangle at P, Q and R. If $\angle C = 50^0$,



(a) What is the measure of $\angle QOR$?

(b) Draw a circle of radius 2 cm. Draw a triangle with two of its angles are 50^0, 60^0 and sides are tangents to the circle.

4. Draw a triangle of angles 70^0, 60^0 and the radius of its incircle is 3 cm.
5. Draw the incircle of a triangle with sides 5 cm, 6 cm and 7 cm.
6. In the triangle ABC, AB = 6cm, $\angle A = 70^0$ and AC = 8 cm.

(a) Draw the triangle ABC with these measures.

(b) Draw its incircle and measure the inradius.

Answers

2 marks

1. $A = r \times s = 2 \times 12 = 24$

2. $r = \frac{A}{S} = \frac{80}{8} = 10$

3. $PC^2 = 4 \times 9$, $PC = 6$

4. 2

5. $DC + AB = AD + BC$, $DC + 5 = 12 + 8$

   $DC = 15$ cm

6. $\angle C = 80^0$

7. (a) $\angle ABC = 50^0$

   (b) $\angle ACP = 50^0$

8. $PA = 4$ cm

3 marks

1. (a) $\angle APO = 30^0$

   (b) $\angle APB = 60^0$

2. (a) $\angle OPQ = 90^0$

   (b) $\angle Q = 30^0$

   (c) $PQ = 2\sqrt{3}$

3. (a) $\angle C = 50^0$

   (b) $\angle A = 70^0$

   (c) $\angle AOC = 120^0$

4. (a) PA = 4

(b) 2 (4 + 5 + 6) = 30 cm

5. (a) $\angle ABC = 30^0$

(b) $\angle BAC = 40^0$

(c) $\angle P = 180 - (70 + 70) = 40^0$

6. (a) $\angle C = 40^0$

(b) $\angle CAQ = 40^0$

7. (a) $PA \times PB = 4 \times 9 = 36$

(b) 36 cm^2

8. (a) $\angle AOB = 140^0$

(b) $\angle OAB = 20^0$

<u>4 mark</u>

1. (a) 26 cm

(b) 60 cm

(c) 120 cm^2

(d) $r = \frac{A}{S} = \frac{120}{30} = 4$ cm

2. Perimeter = 2 (r + h) = 2 (2 + 13) = 30 cm

Area = r (r + h) = 2 (2 + 13) = 30 cm^2

3. PB = 4 + 5 = 9 cm

$PA \times PB = PT^2 = PS^2 = 4 \times 9 = 36$

$\therefore$ PT = PS = 6 cm

4. (a) 5 + 4 = 9 cm

(b) $\frac{40}{2} = 20$ cm

(c) PM = 20 – 5 = 15

PN = 20 – 4 = 16

5. (a) $\angle P = 60^0$

(b) $\angle PBA = 60^0$ (c) 3 × 10 = 30 cm.

6. (a) PA = 8 cm

(b) Perimeter of the triangle PCD = 2 × 8 = 16 cm

7. (a) $\angle B + \angle C = 110^0$

(b) $\angle OBC + \angle OCB = \frac{110}{2} = 55^0$

(c) $\angle BOC = 180 - 55 = 125^0$

8. (a) Perimeter of the triangle = 30 cm

(b) AP = S – BC = 15 – 8 = 7

(c) $AP + BR + CQ = \frac{30}{2} = 15$

9. (a) QM = a

(b) PB = 10 cm

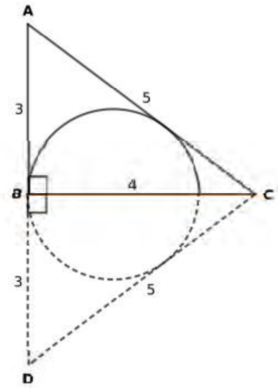
(c) Perimeter of the triangle PQR = 2 × 10 = 20 cm

<u>5 marks</u>

1. (a) $\angle AOC = 120^0$ (b) 60^0

(c) $\angle QAC = 60^0$, $\angle QCA = 60^0$

(d) 3 × 20 = 60 cm



2. (a) 4 cm

(b) $r = \frac{A}{S} = \frac{12}{8} = 1.5$ cm

3. (a) $\angle B = x^0$

(b) $\angle P = x^0$

(c) $PA \times PB = PC^2$

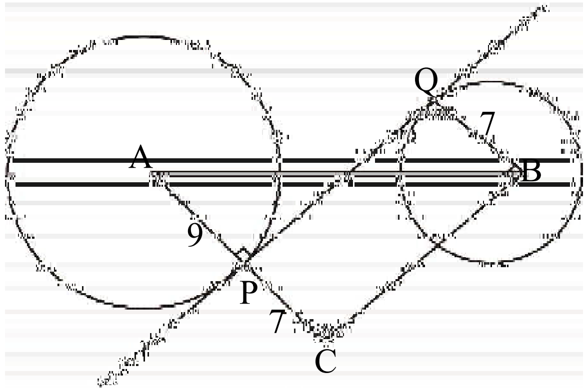
(d) $PC^2 = PA \times PB$, $PC^2 = 4 \times 9 = 36$

$PC = 6$, $BC = 6$ cm

4. Consider the right triangle ACB

AC = AP + PC = 9 + 7 = 16 (Sum of radii)

AB = 20 (distance between centres)



$\therefore CB = \sqrt{AB^2 - AC^2}$

$= \sqrt{20^2 - 16^2} = \sqrt{400 - 256} = \sqrt{144}$

$= 12$ cm

$\therefore$ PQ = 12 cm

5. (a) $\angle BPO = 90^0$

(b) $\angle BOP = 30^0$

(c) $BP = \frac{3}{2} = 1.5$ cm

(d) Draw

6. (a) CR = 4 cm

(b) QB + PC = 10 cm

(c) Perimeter of the triangle PQR = 10 + 10 + 2 × 4 = 28 cm

7. (a) $\angle A = 70^0$

(b) $\angle BPR = 65^0$

(c) $\angle Q = 65^0$, $\angle P = 60^0$, $\angle R = 55^0$

8. (a) $\angle PRQ = 70^0$

(b) $\angle BRP = 50^0$

(c) $\angle B = 80^0$

(d) $\angle A = 40^0$, $\angle C = 60$

9. (a) 10 cm

(b) 24 cm

(c) 24 cm

(d) 2 cm

10. (a) $\angle PAS = 60^0$

(b) PA = 6 cm

(c) PB = $6 + 3 + r = 9 + r$

(d) $2r = 9 + r$

$r = 9$ cm

11. (a) $\angle CBD = 40^0$

(b) $\angle BDC = 60^0$

(c) $\angle BCD = 80^0$

(d) $\angle A = 100^0$

(e) $\angle BDA = 25^0$

12. (a) $\angle PCD + \angle PDC = 18 - 50 = 130^0$

(b) $360 - 130 = 230$

(c) $\frac{230}{2} = 115^0$

(d) $\angle COD = 180 - 115 = 65^0$

(e) $\angle BOD = \frac{130 - 60}{2} = 35^0$

CHAPTER - 8

SOLIDS

Square Pyramid

- Length of the diagonal = $\sqrt{2}a$
- Perimeter of the base = 4a
- For a square pyramid; h^2, l^2, e^2 form an arithmetic sequence with common difference $\left(\frac{a}{2}\right)^2$
- Base area = a^2
- Area of one lateral face = $\frac{1}{2}a\,l$ Lateral surface area (L S A) = $2al$
- Total surface area (T S A) = $a^2 + 2\,al$
- Volume = $\frac{1}{3}a^2h$
- Total length of all edges = 4a + 4e
- If the lateral faces are equilateral triangles,

  LSA = $\sqrt{3}\,a^2$;   Total surface area = $a^2\left(\sqrt{3}+1\right)$

Cone

- If a sector is rolled up to make a cone , then the radius of this sector becomes the slant height of the cone ($R = l$)
- Arc length of this sector = Base perimeter of the cone.

- $\frac{r}{l} = \frac{x}{360}$ (x = central angle of the sector)
- Curved surface area = $\pi r l$
- Total surface area = $\pi r^2 + \pi r l = \pi r(r+1)$
- Volume = $\frac{1}{3}\pi r^2 h$

Sphere

- For a sphere with radius "r"
- Surface ara = $4\pi r^2$
- Volume = $\frac{4}{3}\pi r^3$
- If r_1 and r_2 are the radius of the two spheres then,
- the ratio of the surface area = $r_1^2 : r_2^2$
- the ratio of the volumes = $r_1^3 : r_2^3$

Hemisphere

- For a hemisphere with radius "r"
- Base area = πr^2 Curved surface area = $2\pi r^2$
- Total surface area = $3\pi r^2$ Volume = $\frac{2}{3}\pi r^3$
- If r_1 and r_2 are the radius of the two hemispheres then,
- the ratio of the surface areas = $r_1^2 : r_2^2$
- the ratio of the volumes = $r_1^3 : r_2^3$

SECTION A (2 MARKS)

1 In a sqaure pyramid all edges are equal, the total length of all edges is 80 cm.

a) What is the length of the base edge?

b) What is the slant height ?

2. The base of a square pyramid is 16cm, slant height is 10cm

a) What is the height?

b) What is the length of a lateral edge?

3. A cone of height 16 cm and base radius 12 cm

a) Find the slant height of the cone.

b) What is the central angle of the sector used to make the above cone?

4. A sector of radius 30 cm with central angle 60^0 is rolled to make a cone.

a) What is the slant height of the cone?

b) What is the radius of the cone?

5. The ratio of the radius of two hemispheres are in the ratio 2: 3. If the volume of the first is $100 cm^3$. What is the volume of the second?

6. The total surface area of a sphere is $100\pi \, cm^2$

a) What is the radius ?

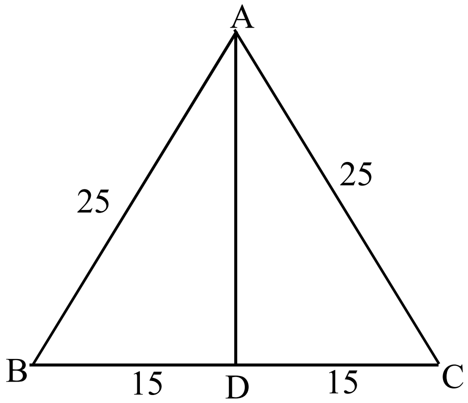
b) What is the volume of the sphere?

SECTION B (3 MARKS)

7. The given is the lateral face of a square pyramid. AB = AC = 25 cm and BD = DC = 15 cm.

a) What is the length of the base edge ?

b) Find the lateral surface area of the pyramid.



8. All edges of a square pyramid are of the same length and the sum of all edges is 80 cm.
   a) What is the length of the lateral edge?
   b) What is the measure of the slant height?
   c) What is the height?
   d) Find the ratio of one edge, slant height and height?
9. A sector of angle 120^0 is bent to form a cone.
   a) What is the ratio of radius to the slant height of the cone?
   b) If the curved surface area is 108π cm^2 What is the slant height and radius?
10. The base radius and slant height of a cone are 6 cm and 10 cm respectively.
   (a) What is its height?
   (b) Find its volume.
11. The diameters of two spheres are in the ratio 2 : 3.
   (a) What is the ratio of their radii ?
   (b) Find the ratio of their surface areas.
   (c) If the surface area of the first sphere is 16π cm^2. Find the surface area of the second sphere.
12. A hemisphere of radius 9 cm is melted and recast into cones of radius 3 cm and height 6 cm.
   a) What is the volume of the hemisphere?
   b) What is the volume of the cone?
   c) How many cones can be made ?

SECTION C (4 MARKS)

13. For a square pyramid with all edges equal, the base edge is 20 cm.
   a) Find the total surface area
   b) Find the volume of the pyramid?
14. For a square pyramid the slant height is 20 cm and the total surface area is 624 cm^2.
   a) Find the base edge.

b) Find the height.

c) Find the volume of the square pyramid.

15. A circular sheet of paper is cut into two sectors. Central angle of one of them is 160^0.

(a) What is the central angle of the other sector?

(b) These sectors are bent into cones of maximum volume. If the radius of the small cone is 8 cm, what is the radius of the other?

(c) What is the slant height of the cones ?

16. A circular sheet with radius 36 cm is divided into four equal sectors and one of them bent into a cone.

(a) What is the slant height of the cone?

(b) What is the radius of the cone?

(c) What is the curved surface area?

17. A vessel is in the shape of a hemisphere with diameter 14 m.

a) What is the volume of the vessel in litres? [Take $\pi = 3.14$)

b) What is the cost of painting its outer curved surface at the rate of ₹ 100/m^2.

18. A cylindrical bucket 32 cm high and with radius of the base 18 cm is titled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. find the radius and slant height of the heap.

SECTION D (5 MARKS)

19. A cone of maximum volume is carved out from a solid metal cylinder of base radius 12 cm and height 15 cm.

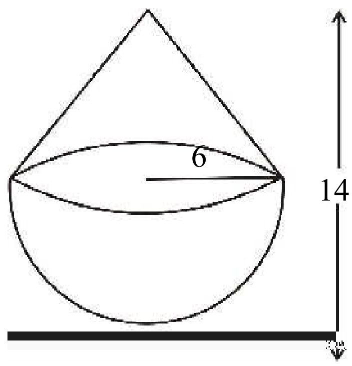
a) What is the volume of the cylinder?

b) What is the volume of the cone?

c) What is the volume of the remaining portion of the cylinder?

d) The remaining portion of the cylinder is melted and recast into small cones of base radius 4cm and height 6 cm. What is the number of small cones obtained?

20. A solid is formed by joining a hemisphere and a cone of same radius; as shown in the figure. Radius of the hemisphere is 6 cm and total height of the solid is 14 cm.



(a) What is the height of the conical part?

(b) Find the volume of the cone and hemisphere.

(c) Find the volume of the solid.

21. A solid metal cone has base radius 2 cm and height 8 cm.

(a) Compute the volume of the cone.

(b) If this cone is melted and recast into a sphere of maximum size what will be its radius ?

22. The base radius and height of a cylindrical block of wood are 8 cm and 15 cm. A cone of maximum size is carved out of this.

(a) What are the radius and height of the cone?

(b) Find its slant height.

(c) Find the curved surface area of this cone.

Additional Questions (Try Yourself)

23. A hemispherical tank is full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second.

How much time will it take to empty the tank, if its 3m in diameter. [Use $\pi = \frac{22}{7}$]

24. Metallic spheres of radius 6cm 8cm and 10 cm are melted to form a single solid sphere. Find the radius of the resulting sphere.

ANSWER

SECTION A

1. a) base edge =10 [given 8 a = 80]

   b) $l = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}$ cm

2. a) h = $\sqrt{10^2-8^2}$ = 6 cm

   b) e = $\sqrt{10^2+8^2} = \sqrt{164}$ cm

3. a) $l = \sqrt{16^2+12^2} = \sqrt{400}$ =20 cm

   b) $\frac{x}{360} = \frac{12}{20}$; x = $\frac{12 \times 360}{20} = 216^0$

4. a) Slant height of the cone = radius of the sector = 30 cm

   b) r = $\frac{60}{360} \times 30 = 5$ cm

5. $r_1 : r_2 = 2 : 3$ $v_1 : v_2 = 8 : 27$ $\frac{100}{v_2} = \frac{8}{27}$

   $V_2 = \frac{2700}{8}$

6. (a) $4\pi r^3 = 100\pi$ $r^2 = \frac{100\pi}{4\pi} = 25$

   r = 5 cm

   (b) $V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3}$ cm^3

SECTION B

7. (a) base edge = 30cm

(b) $l = \sqrt{25^2 - 15^2} = \sqrt{400} = 20$ cm LSA = 2 × 30 × 20 = 1200cm^2

8. a) $e = \frac{80}{8} = 10$ cm

b) $l = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}$ cm

c) h = $\sqrt{75-25} = \sqrt{50} = 5\sqrt{2}$ cm

d) ratio = $10:5\sqrt{3} : 5\sqrt{2} = 2 : \sqrt{3} : \sqrt{2}$

9. A sector of angle 120^0 is bent to form a cone.

a) r : 1 = 120 : 360 = 1 : 3

b) Let r = x, 1 = 3x

$3\pi x^2 = 108\pi$

x = 6 cm

r = 6 cm 1 = 18 cm

10. (a) h = $\sqrt{100-36} = \sqrt{64} = 8$ cm

(b) V = $\frac{\pi}{3} \times 6^2 \times 8 = 96\ \pi \text{cm}^3$

11. (a) $r_1 : r_2 = d_1 : d_2 = 2 : 3$

(b) $A_1 : A_2 = 2^2 : 3^2 = 4 : 9$

(c) $16\pi : A_2 = 4 : 9$

$A_2 = \frac{16\pi \times 9}{4} = 36\pi$

12. (a) Volume of hemisphere = $\frac{2\pi}{3}(9)^3 = 486\pi$

(b) Volume of cone = $\frac{\pi}{3}(3)^2 \times 6 = 18\pi$

(c) number of cones = $\frac{486\pi}{18\pi} = 27$

SECTION C

13. a = e = 20 cm $l = \frac{a}{2}\sqrt{3} = 10\sqrt{3}$, h = $\frac{a}{2}\sqrt{2} = 10\sqrt{2}$

a) TSA = $(1+\sqrt{3})\,a^2 = 400\,(1+\sqrt{3})\text{cm}^2$

b) Volume = $\frac{1}{3}a^2h = \frac{4000\sqrt{2}}{3}\ \text{cm}^3$

14. TSA = 624 cm^2

a) $a^2 + 40a = 624$ on solving we get a = 12 cm

b) h = $\sqrt{400-144} = \sqrt{256}$ = 16cm

c) V = $\frac{1}{3} \times 12^2 \times 16 = 768\ \text{cm}^3$

15. (a) Central angle = 360 – 160 = 200^0

(b) 8 : R = 160 : 200 Radius of the other sector = 20 cm

(c) $l = \frac{8\times360}{160}$ = 18 cm

16. (a) Slant height of the cone = radius of the sector = 36

(b) r = $\frac{90}{360} \times 36$ = 9 cm

(c) CSA = $\pi rl = \pi \times 9 \times 36 = 324\pi$

17. (a) $V = \frac{2}{3} \times \pi \times 7^3 = \frac{2156}{3}$

(b) Cost of painting = $\frac{2 \times 22}{7} \times 7^2$ = Rs. 30800

18. Volume of the cone = Volume of the cylinder

$$\frac{1}{3}\pi r^2 h = \pi R^2 H$$

$$\frac{1}{3}\pi \times r^2 \times 24 = \pi \times 18^2 \times 32$$

$$8\pi \times r^2 = \pi \times 18^2 \times 32$$

$$r^2 = \frac{\pi \times 18 \times 18 \times 32}{8\pi}$$

$$r = \sqrt{1296} = 36 \text{ cm}$$

$$l = \sqrt{36^2 + 24^2} = \sqrt{1872} = 12\sqrt{13} \text{ cm}$$

SECTION D

19. (a) Volume of cylinder = $\pi r^2 h = \pi \times 12^2 \times 15 = 2160\ \pi cm^3$

(b) Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 12^2 \times 15 = 720\pi\ cm^3$

(c) Volume of the remaining portion of the Cylinder = $2160\ \pi$ cm – $720\pi = 1440\pi$

$$n \times \frac{1}{3}\pi r^2 h = 1440\ \pi$$

(d) $n \times \frac{1}{3}\pi \times 4^2 \times 6 = 1440\pi$

$n \times 32\pi = 1440\pi$

$n = 45$

20. (a) Height of the conical part = 14 – 6 = 8

(b) Volume of the cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 6^2 \times 8 = 96\pi \text{ cm}^3$

Volume of the hemisphere

$= \frac{2}{3}\pi r^2 \qquad = \frac{2}{3}\pi \times 6^2 = 144\pi \text{ cm}^3$

(c) Volume of the solid

$= 96\pi + 144\pi = 240\pi \text{ cm}^3$

21. A solid metal cone has base radius 2 cm and height 8 cm.

(a) Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 2^2 \times 8 = \frac{32\pi}{3}$

(b) Volume of the sphere - Volume of the cone = $\frac{4}{3}\pi r^3 = \frac{32\pi}{3}$

$r^3 = 8 \qquad r = \sqrt[3]{8} = 2$ cm

22. (a) radius of the cone = radius of cylinder = 8 cm

height of the cone = height of cylinder = 15 cm

(b) $l = \sqrt{225+64} = \sqrt{289} = 17$ cm

(c) Curved surface area = $\pi r l = \pi \times 8 \times 17 = 136\pi \text{cm}^2$

CHAPTER 9

Geometry and Algebra

PREVIOUS KNOWLEDGE

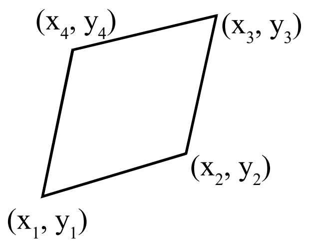
- The x coordinate of the line parallel to the y axis is equal.
- The y coordinate of the line parallel to the x axis is equal.
- The distance between the points $(x_1\ y)$ and (x_2, y) are $|x_1 - x_2|$
- The distance between the points (x, y_1) and (x, y_2) are $|y_1 - y_2|$
- The distance between the origin and the point (x, y) are $\sqrt{x^2+y^2}$
- The distance between the points (x_1, y_1) and (x_2, y_2) are $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

Important Concept

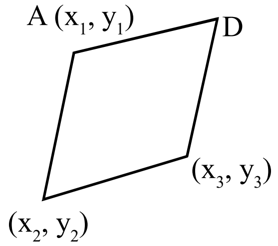
- The coordinates of four vertices of a parallelogram are (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) then

  $x_1 + x_3 = x_2 + x_4$ and

  $y_1 + y_3 = y_2 + y_4$

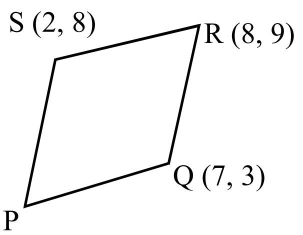


- The coordinates of three vertices of a parallelogram are (x_1, y_1), (x_2, y_2), (x_3, y_3) then the coordinates of 4th vertex is D $(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$



eg: In the figure three vertices of a parallelogram are given. Find the fourth vertex

Ans: The coordinates of P are (7 + 2 – 8, 3 + 8 – 9 $\Rightarrow$ (1, 2)



- The point (x, y) which divides the line joining the points (x_1, y_1) and (x_2, y_2) are in the ratio m : n. Then the coordinates of (x, y) are

$$x = x_1 + \frac{m}{m+n}(x_2 - x_1)$$

$$y = y_1 + \frac{m}{m+n}(y_2 - y_1)$$

OR

$$P(x, y) = P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

eg: Find the point which divides the line joining the points (4, 2) and (9, 7) are in the ratio 2:3

$$x = x_1 + \frac{m}{m+n}(x_2 - x_1) \quad = 4 + \frac{2}{5}(5) \; = \mathbf{6}$$

$$y = y_1 + \frac{m}{m+n}(y_2 - y_1) \quad = 2 + \frac{2}{5}(5) = \mathbf{4}$$

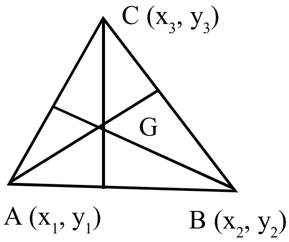
$(x, y) \Rightarrow (6, 4)$

- The midpoint of the linejoining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Eg: P (-3, 4) and Q (7, 8) are the end points of the diameter of a circle. What are the coordinates of the centre.

Ans : $\left(\frac{-3+7}{2},\frac{4+8}{2}\right) \Rightarrow (2, 6)$

- The centroid of a triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) are G $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$



eg: The vertices of a triangle are (3, 4), (-7, -2), (10, -5). What is the centroid of the triangle.

Ans:

$$\left(\frac{3+(-7)+10}{3},\frac{4+(-2)+(-5)}{3}\right) \Rightarrow (2,-1)$$

- In any line not parallel to either axis the change in y coordinate is the product of the change in x coordinate with a fixed number (In otehrwords, in any line not parallel to either axis the change in y is proportional to the change in x. This constant of proportionality is called the slope of the line)

The slope of the line joining the points (x_1, y_1) and (x_2, y_2) is Slop $= \frac{y_2-y_1}{x_2-x_1}$

Eg: What is the Slope of the line joining the points (1, 4) and (3, 2)

slope $= \frac{y_2-y_1}{x_2-x_1} = \frac{2-4}{3-1} = -1$

- Equation of a line joining the point (x_1, y_1) and (x_2, y_2) and it passes through the point (x, y) is

$y - y_1 = \text{slope}\,(x - x_1)$

eg: What is the equation of the line joining points (3, 4) and (5, 7)

Ans : Slope $= \frac{7-4}{5-3} = \frac{3}{2}$

Equation $y - 4 = \frac{3}{2}(x-3)$

$2(y-4) = 3(x-3)$

$3x - 2y - 1 = 0$

- Slope of a line is unique.

  eg: Prove that (3, 2), (8, 5), (13, 8) are lie on a line

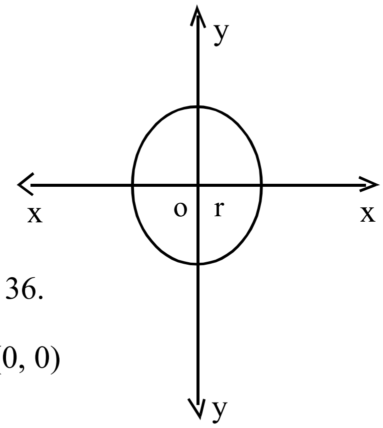
  Slope of (3, 2) and (8, 5) = $\frac{5-2}{8-3} = \frac{3}{5}$

  Slope of (8, 5) and (13, 8) = $\frac{8-5}{13-8} = \frac{3}{5}$

  Slope are equal

  $\therefore$ (3, 2), (8, 5), (13, 8) are lie on a line.

- If two lines are parallel, the slopes are equal.
- If two lines are perpendicular, the product of their slopes are -1.
- Equation of a circle with centre as origin and radius r is given by



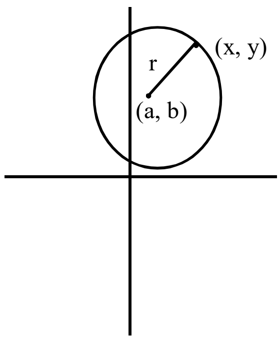
$x^2 + y^2 = r^2$

Eg: The equation of a circle is $x^2 + y^2 = 36$.

a) What is the centre of the circle $\rightarrow$(0, 0)

b) What is the radius of the circle 6.

- Equation of a circle with centre (a, b) and radius 'r' is given by $(x - a)^2 + (y - b)^2 = r^2$



Eg : What is the equation of a circle with centre (4, 3) and radius 2.

Ans: $(x - 4)^2 + (y - 3)^2 = 2^2$

$x^2 + y^2 - 8x - 6y + 21 = 0$

PART : A

(2 Scores)

1. The equation of a circle is $x^2 + y^2 = 81$

   a) What is the centre of the circle?

   b) What is the radius of the circle?

2. If the points A(m, 3), B(3, -5), C(4, 3) are lie on a line. Find the value of m?

3. Write two more points on the line joining the points (5, -2) and (7, 4)

4. An equation of a circle is $(x-7)^2 + (y-4)^2 = 4^2$

   a) What is the centre of the circle?

   b) What is the radius of the circle?

5. The slope of the line joining the points P(4, 7) and Q (8, a) is 3. What is the value of ‘a’?

PART : B

(3 Scores)

6. Prove that the points (4, -3), (7, 2), (10, 7) are lie on a line.

7. Prove that the line joining the points (2, 3) and (3, -1) passes through the point (5, -9)

8. The slope of a line passes through the point (6, -3) is 2.

   a) Write the equation of the line.

   b) Check whether (4, -7) is a point on this line.

9. A (3, 0), B (7, 0), C (9, 3) are the three vertices of a parallelogram.

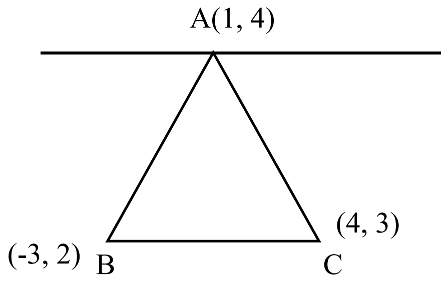
   a) Find the midpoint of AC

   b) Find the fourth vertex of the parallelogram.

10. In the figure A(1, 4), B(-3, 2), C(4, 3) are the vertices of a triangle.

    a) Write the slope of BC

    b) Write an equation of a line parallel to BC through A?



PART : C

(4 Scores)

11. The vertices of a triangle are A(-4, 6), B(6, -2), C(4, 5)

    a) Find the centroid of the triangle.

    b) Find the midpoint of AB.

    c) What is the length of the median throughC.

12. The equation of a circle is $(x-8)^2 + (y-6)^2 = 100$

    a) Write the coordinates of the centre of the circle.

    b) What is the radius of the circle.

    c) Check whether (16, 12) is a point on this circle.

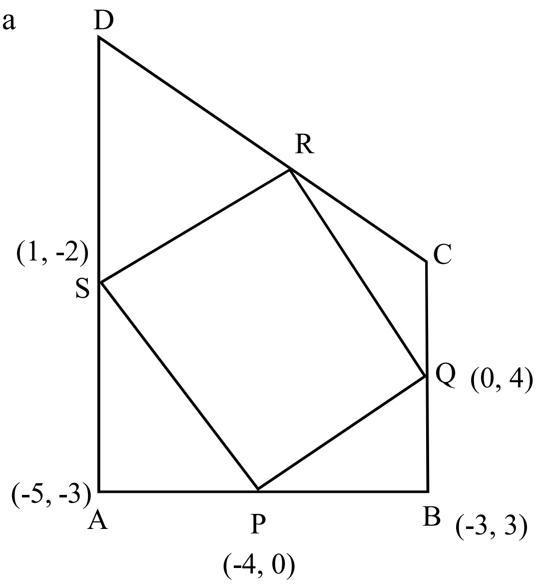
13. The line joining the points are (1, 3) and (2, 7)

    a) What is the slope of the line?

    b) Write an equation of the line?

    c) If m, n are lie on this line. Prove that (m+1, n+4) is also lie on this line.

14. In the figure P, Q, R, S are the mid point of the sides of a quadrilateral ABCD.



a) Which type of quadrilateral is PQRS.

b) Write the coordinates of the vertex R.

c) Write the coordinates of the vertices C and D of the quadrilateral ABCD

15. A circle is drawn with (8, 3) as centre. (8, 6) is a point on the circle.

a) What is the radius of the circle.

b) Write the equation of the circle.

c) What is the distance from the centre of the circle to the x-axis.

PART : D

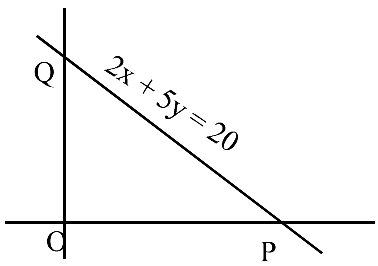
(5 Scores)

16. In the figure PQ is the line cut the x axis at P and y axis at Q. The equation of the line PQ is $2x + 5y = 20$.

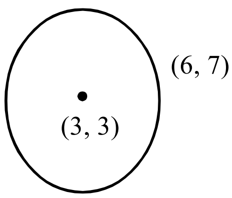
a) Write the coordinates of P and Q.

b) What are the lengths of OP and OQ.

c) What is the slope of the line PQ.



17. A circle is drawn with (3, 3) as centre. (6, 7) is a point on the circle.



a) Find the radius of the circle?

b) Write the equation of the circle?

c) The x- cordinate of the point on the circle is 6. What is the y coordinate of that point.

18. P(-4, -8) R(8, 8) are the two points on a line. Q is a point lies between P and R. PQ : PR = 2:5

a) What is the length of PR

b) What is the length of PQ

c) Write the coordinates of Q.

19. a) What is the slope of the line joining the points A(-3, 5) and B (4, -9)

b) At what ratio the point P(2, -5) divide the above line.

c) What is the equation of the line AB.

20. Equation of a line is y = 2x

a) 'A' is a point on the line. If the 'x' coordinate of A is -2. Find its y coordinate.

b) Verify that whether a circle of radius 5 centred at A passes through the point B(5, 5)

c) Radius of a circle passing through B is 5 and its centre is on the above mentioned line. Find the co-ordinates of its centre.

ANSWERS

PART - A

1. a) (0, 0) b) 9

2. Slopes are equal

$$\therefore \frac{-5-3}{3-m} = \frac{3-(-5)}{4-3}$$

$-8 = 24 - 8m$ **m = 4**

3. Slopes of all points lie on a line are equal. The slope of (5, -2) and (7, 4) = $\frac{4-(-2)}{7-5} = 3$

$(5 + 3, -2 + 3)\ (7 + 3,\ 4 + 3) \Rightarrow (8, 1), (10, 7)$

$\text{Slope} = \frac{7-1}{10-8} = \frac{7-1}{10-8} = \frac{6}{2} = 3$

Other two points are (8, 1), (10, 7)

4. a) (7, 4) b) 4

5. Slope = 3

$\frac{a-7}{8-4} = 3$ $a = 19$

PART - B

6. A(4, -3), B(7, 2), C(10, 7)

$\text{Slope of AB} = \frac{2-(-3)}{7-4} = \frac{5}{3}$

$\text{Slope of BC} = \frac{7-2}{10-7} = \frac{5}{3}$ Slopes are equal

$\therefore$ These points are lie one line.

7. (2, 3), (3, 1) $\Rightarrow$ slope = -4

(3, -1), (5, -9) $\Rightarrow$ slope = -4

Slopes are equal

$\therefore$ (5, -9) is lie on the line joining the point (2, 3) and (3, -1)

8. Slope (m) = 2

a) Equation of a line $\Rightarrow y - y_1 = m(x - x_1)$

$y - (-3) = 2(x - 6)$

$2x - y - 15 = 0$

b) Substitute x = 4, y = -7 in above equation

LHS $2 \times 4 - (-7) - 15 = 8 + 7 - 15 = 0 = \text{RHS}$

LHS = RHS

$\therefore$ (4, -7) is a point on this line

a) Midpoint of AC $\left(\frac{3+9}{2},\frac{0+3}{2}\right) \Rightarrow (6,1.5)$

b) (3 + 9 – 7, 0 + 3 – 0) $\Rightarrow$ (5, 3)

10. a) Slope of BC = $\frac{3-2}{4-(-3)} = \frac{1}{7}$

b) Slope of a line parallel to BC also $\frac{1}{7}$

$\therefore$ $y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{7}(x - 1)$

7y – 28 = x – 1

x – 7y + 27 = 0

PART - C

11. a) $\left(\frac{-4+6+4}{3},\frac{6+(-2)+5}{3}\right) \Rightarrow (2,3)$

b) $\left(\frac{-4+6}{2},\frac{6+-2}{2}\right) \Rightarrow (1,2)$

c) $\sqrt{(4-1)^2+(5-2)^2} = \sqrt{3^2+3^2} = 3\sqrt{2}$

12. a) 8, 6 b) 10

c) Put x = 16, y = 12

LHS $(16-8)^2 + (12-6)^2 = 8^2 + 6^2 = 100$ = RHS

LHS = RHS

$\therefore$ (16, 12) is a point on the circle.

13. a) Slope = $\frac{7-3}{2-1}$ = 4

b) $y - 3 = 4(x-1)$ $\qquad$ $4x - y - 1 = 0$

c) Since m, n are the points on a line

$\therefore$ $4m - n - 1 = 0$

Substitute $m = m + 1$, $n = n+4$ in the equation.

LHS $4(m + 1) - (n + 4) - 1 = 4m - n - 1 = 0 =$ RHS

LHS = RHS

$\therefore$ $(m + 1, n + 4)$ are the points on this line.

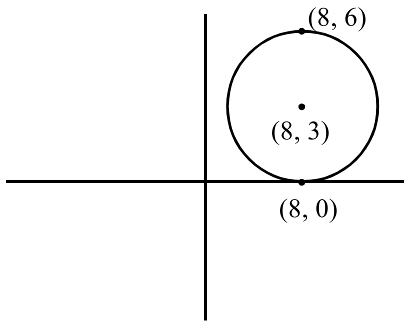
14. a) Line joining the midpoints of the quadrilateral is a parallelogram (9th standard)

b) Coordinates of R $(1 + 0 - (-4), -2 + 4 - 0) \Rightarrow (5, 2)$

c) Coordinates of C $(2 \times 0 + 3, 2 \times 4 - 3) = (3, 5)$

Coordinates of D $(2 \times 1 + 5, 2 \times -2 + 3) = (7, -1)$

15. a) Radius $= |3 - 6| = 3$



b) $(x - x_1)^2 + (y - y_1)^2 = r^2$

$(x - 8)^2 + (y - 3)^2 = 3^2$

$x^2 + y^2 - 16x - 6y + 64 = 0$

c) Distance = Radius = 3

PART - D

16. a) Since P is a point lie on x - axis

$\therefore$ y co-ordinate = 0

$2x = 20 \Rightarrow x = 10$

$\therefore$ Co-ordinates of P (10, 0)

Since Q is a point lie on y axis

$\therefore$ x co-ordinate = 0

$5y = 20 \Rightarrow y = 4$

$\therefore$ Co-ordinates of Q (0, 4)

b) OP = 10 $\qquad$ OQ = 4

c) Slope = $\frac{4-0}{0-10} = \frac{4}{-10} = \frac{-2}{5}$

17. a) Radius = $\sqrt{(3-6)^2+(3-7)^2} = \sqrt{25} = 5$

b) $(x-3)^2 + (y-3)^2 = 25$

$x^2 + y^2 - 6x - 6y - 7 = 0$

c) Substitute $x = 6$

$(6-3)^2 + (y-3)^2 = 25 \quad (y-3)^2 = 16$

$y - 3 = \pm 4 \qquad y = 7$ or $y = -1$

18. a) PR = $\sqrt{(-4-8)^2+(-8-8)^2} = \sqrt{144+256} = \sqrt{400}$

b) PQ : PR = 2:5 (given)

$\frac{PQ}{20} = \frac{2}{5} \Rightarrow PQ = 8$

c) PQ : QR = 8 : 12= 2 : 3

The coordinates of Q are

$x = x_1 + \frac{m}{m+n}(x_2 - x_1) = -4 + \frac{2}{5}(12) = \frac{4}{5}$

$y = y_1 + \frac{m}{m+n}(y_2 - y_1) = -8 + \frac{2}{5}(16) = \frac{-8}{5}$

Q $\left(\frac{4}{5}, \frac{-8}{5}\right)$

19. a) Slope of AB = $\frac{-9-5}{4-(-3)} = \frac{-14}{7} = -2$

b) $x = x_1 + \frac{m}{m+n}(x_2 - x_1)$

$2 = -3 + \frac{m}{m+n}(7)$

$5 = \frac{7m}{m+n}$

$5m + 5n = 7m$

$\frac{m}{n} = \frac{5}{2} \Rightarrow m:n = 5:2$

$y = y_1 + \frac{m}{m+n}(y_2 - y_1)$

$-5 = 5 + \frac{m}{m+n}(-14)$

$-10 = \frac{-14m}{m+n}$

$-10m - 10n = -14m$

$\frac{m}{n} = \frac{10}{4} = \frac{5}{2} \Rightarrow m:n = 5:2$

c) $y - y_1 = m(x - x_1)$

$y - 5 = -2(x - (-3))$

$y - 5 = -2x - 6$

$2x + y + 1 = 0$

20. a) $y = 2x$

If $x = -2$, $y = -4$

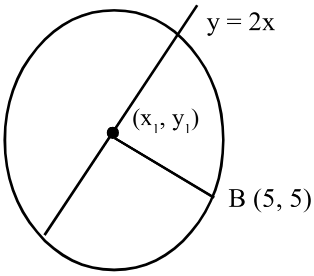
b) Co-ordinates of A (-2, -4)

AB = 5

$(5 - (-2))^2 + (5 - (-4))^2 = 5^2$

$49 + 81 \neq 25$

$\therefore$ The circle does not passes through B(5, 5) c)



Centre is $(x_1\ y_1)$

$(x_1 - 5)^2 + (y_1 - 5)^2 = 5^2$

Put $y_1 = 2x_1$

$5x_1^2 - 30x_1 = -25$

$x_1^2 - 6x_1 = -5$

$x_1^2 - 6x_1 + (3)^2 = (3)^2 + (-5)$

$(x_1 - 3)^2 = 4$

$\Rightarrow x_1 = 5 \quad \text{or } x_1 = 1$

$\therefore (x_1, y_1) \Rightarrow (5, 10) \text{ or } (1, 2)$

CHAPTER - 10
POLYNOMIALS

PREVIOUS KNOWLEDGE

Look at the algebraic expression given below.

- $2.5x - 24x^2 + 4x^3$
- $x^2 + 7x + 12$
- $20 - 9.8x$
- $x^2 + x$

In all cases, the only mathematical operations involved are multiplying different powers of the number 'x' by various numbers and adding or substracting them. A definite number is also sometimes added or subtracted. An algebraic expression involving such operations is called a polynomial.

Note :

x is a changing number (variable)

Features of Polynomials

- ♦ Various powers of the changing numbers.
- ♦ All powers are natural numbers
- ♦ Powers of the changing number are multiplied by definite numbers.
- ♦ Sum or difference of the products are found
- ♦ A definite number is added or subtracted.

Note

Algebraic expression involving the operations of taking reciprocal or square root of the changing number are not polynomials.

Eg :- $x+\sqrt{x}$

$x^2 + \frac{4}{x}$

$\sqrt[3]{x} - 1$

Degree of a Polynomial

In a polynomial, we take powers of the changing numbers. The largest power used is called the degree of the polynomial

eg : $x^2 + 5x + 6 \Rightarrow$ Degree 2

$x^3 + 6x^2 + 11x + 6 \Rightarrow$ Degree 3

$49 - 9.8x \Rightarrow$ Degree 1

General form of a polynomial

Based on the degree, we can write the general form of the polynomials.

Polynomial	General Form	
First degree polynomial	$ax + b$	$a \neq 0$
Second degree polynomial	$ax^2 + bx + c$	
Third degree polynomial	$ax^3 + bx^2 + cx + d$	

Here the letters a ,b, c and d are called coefficients of the polynomial.

Eg:- Consider the polynomial

$x^2 + 5x + 6$, coefficient of x^2 is 1,

Coefficient of x is 5, constant term is 6.

Activity

Find P(0), P(1) and P(-1) in - the following polynomials

a. $P(x) = 2x + 3$

b. $P(x) = x^2 + 5x + 6$

Answer

(a) $P(x) = 2x + 3$

$P(O) = 2 \times 0 + 3 = 0 + 3 = 3$

$P(1) = 2 \times 1 + 3 = 2 + 3 = 5$

$P(-1) = 2\ (-1) + 3 = -2 + 3 = 1$

(b) $P(x) = x^2 + 5x + 6$

$P(0) = 0^2 + 5 \times 0 \times 6 = 0 + 0 + 6$

$= 6$

$P(1) = 1^2 + 5 \times 1 + 6 = 1 + 5 + 6 = 12$

$P(-1) = (-1)^2 + 5 \times (-1) + 6 = 1 = 1 - 5 + 6 = 2$

IMPORTANT CONCEPT

- If a polynomial P(x) can be expressed as p(x) = q(x) × r(x) then q(x) and r(x) are factors of p(x)
- The remainder got by dividing a polynomial P(x) by (x-a) is P(a) and P(-a) is the remainder when it is divided by (x+ a)
- If P(a) = 0 then (x-a) is a factor of P(x) and if P (-a) = 0 then (x+ a) is a factor of P(x)
- If P(a) $\neq$ 0, , then (x-a) is not a factor of P(x).and if P (-a) $\neq$ 0 then (x+a) is not a factor of P(x).
- The remainder got by dividing the polynomial P(x) by (ax+b) is P $\left(\frac{-b}{a}\right)$
- If P and q are the solutions of the second degree equation $ax^2 + bx + c = 0$ then the polynomial $ax^2 + ax + c$ can be expressed as a(x-p) (x-q).
- In the polynomial P(x), P(1) will be the sum of coefficient of the variables and P(0) will be constant term.
- If P(x) is polynomial and a is any number then (x-a) will always be a factor of P(x) – P(a).

PART A (2 marks)

1. If $P(x) = x^2 - 8x + 15$ then what is P (0) and P(1) ?
2. If $P(x) = 3x^3 - 2x^2 - 3x + 2$ then what is P(0) and P(–1) ?
3. If $P(x) = 6x^2 - 11x + 3$ then what is P(2)
4. If $P(x) = x^2 - 9x + 6$ find the number to be added to P(x) to get a polynomial for which (x-1) is a factor?
5. If $P(x) = x^2 - kx + 18$ then what is the value of K if (x-3) is a factor of P(x) ?

PART B (3 MARKS)

1. If $P(x) = x^2 + 2x - 15$
   a. Find P (5) ?
   b. Write P(x) as the product of two first degree polynomials?

2. If $P(x) = x^2 - kx + 10$
   a. What is the value of K if (x-2) is a factor of P(x)
   b. Write P(x) as the product of two first degree polynomials if one of the factor is (x-2) ?

3. If $P(x) = x^2 + 2x - 15$
   a. Find P(3) ?
   b. Is (x-3) is a factor of P(x) why?

4. If $P(x) = x^2 - 7x + 9$
   a. Find P (2) ?
   b. Find the number to be added to P(x) to get a polynomial for which (x-1) is a factor ?

5. $P(x) = x^2 - 5x - 6$

   a. Express P(x) as the product of two first degree polynomials

   b. Find the solutions of the equation $P(x) = 0$.

6. If $P(x) = (x-1)(x-5)$

   a. Find P (5) ?

   b. Find the number added to P(x) to get a perfect square?

PART C (4 marks)

1. If $P(x) = (x-2)(x-8) + 5$

   a. Find P (5) ?

   b. Check whether (x-7) is a factor of P(x) or not.

   c. Write p(x) as the product of two first degree polynomial

2. If $P(x) = x^{99}+1$

   a. Find P(1) ?

   b. Check whether (x+1) is a factor of P(x) or not

   c. Is (x-1) is a factor of P (x) or not ?

3. If $P(x) = 4x^2 + 9x + 2$

   a. Find P(2) ?

   b. Write a factor of $P(x) - P(2)$?

   c. Write $p(x) - P(2)$ as a product of two first degree polinomial.

4. If $P(x) = x^2 - 11x + 30$ then

a. Find P(3) ?

b. Write a factor of $P(x) - P(3)$?

c. Write $P(x) - P(3)$ as a product of two first degree polynomial

5.

a. If $p(0) = -5$ in $P(x) = ax^2 + bx + c$ then find the value of c

b. If (x-1) is a factor of P(x). prove that a+b= 5

Part D (5 Marks)

1. If $x^2 - 10x + 16 = (x-a)(x-b)$

a. What is the value of (a+b) ?

b. What is the value of ab ?

c. Write $x^2 - 10x + 16$ as the product of two first degree polynomial

d. Find the solution of the equation $P(x) = 0$

2. Consider the polynomial $P(x) = ax^2 - 2bx + c$

a. If x-1 is a factor of P(x). Prove that a, b, c are in an arithmetic sequence.

b. Write two polynomial in the form $ax^2 - 2bx + c$ such that a, b, c are in an arithmetic sequence.

c. If $x^2 - 1$ is a factor of P(x) then what is a + c ?

3. If $P(x) = x^3 - 4x^2 + 6x - k$ then

a. Find K such that (x-1) is a factor of P(x)

b. Check whether (x + 1) is a factor of P(x)

4. Write the following second degree polynomial as the product of first degree polynomials

a. $x^2 + 4x + 3$

b. $x^2 + 14x + 48$

c. $x^2 - 8x + 12$

EXTRA QUESTIONS

1. If $P(x) = lx^2 + mx + n$

a. Find P(1) ?

b. If x +1 is a factor of P(x), Prove that m= l +n ?

c. Write second degree polynomial whose factor is x+1 ?

2. If x is a natural number

a. What number is to be added to $x^2 - 8x$ to get a perfect square ?

b. If $x^2 - mx + 36$ is a perfect square, which number is 'm'

c. If $x^2 - mx + n$ is a perfect square, prove that $m^2 = 4n$

d. Write a second degree polynomial which is a perfect square and having a factor x – 3 ?

3. The solutions of the question of the question P(x) = 0 are -3 and -7.

a. Write one factor of P(x) ?

b. Write P(x) as the product of two first degree polynomials ?

4. $P(x) = 2x^2 - 5x + 8$

a. Which number should be subtracted from p(x) so that (x-2) is a factors of the resulting polynomial.

b. Find the second factors of the resulting polynomial.

ANSWER PART A

1. $P(x) = x^2 - 8x + 15$

   $P(0) = 15$

   $P(1) = 1-8+15 = 16-8 = 8$

2. $P(0) = 2$

   $P(-1) = 3 \times (-1)^3 - 2 \times (-1)^2 - 3 \times (-1) + 2$

   $P(-1) = -3 - 2 + 3 + 2$

   $P(-1) = 0$

3. $P(2) = 6 \times 2^2 - 11 \times 2 + 3$

   $P(2) = 24 - 22 + 3$

   $P(2) = 2+3 = 5$

4. $P(1) = 1-9+6$

   $P(1) = 7-9 = -2$

   $P(1) + 2 = 0$

   The number should be added to P (x) is +2

5. f(x–3) is a factor of P(x)

   i.e : $P(3) = 0 \Rightarrow 3^2 - k \times 3 + 18 = 0$

   $\Rightarrow 9 - 3k + 18 = 0$

   $\Rightarrow 3k = 27$

   $\Rightarrow K = \frac{27}{3} = 9$

Part B

1. (a) $P(5) = 5^2 + 2 \times 5 - 15$

   $P(5) = 25 + 10 - 15$

   $P(5) = 35 - 15 = 20$

   $P(5) = 20$

(b) $x^2 + 2x - 15 = x^2 + (a+b)x + ab$

$x^2 + 2x - 15 = (x + a)(a + b)$

$x^2 + 2x - 15 = (x + 5)(x - 3$

2. If $(x-2)$ is a factor of $P(x)$, $P(2) = 0$

$P(2) = 0 \Rightarrow 2^2 - K \times 2 + 10 = 0$

$\Rightarrow 4 - 2k + 10 = 0$

$\Rightarrow 14 - 2k = 0$

$\Rightarrow 2k = 14$

$\Rightarrow K = \frac{14}{2} = 7$

$\Rightarrow K = 7$

(b) $x^2 - 7x + 10 = (x-5)(x-2)$

$x^2 - 7x + 10 = x^2 - (a+b)x + ab$

$a + b = -7$

$ab = 10$

$a = -5$ and $b = -2$

$x^2 - 7x + 10 = (x - 5)(x - 2)$

3. $P(3) = 3^2 + 2 \times 3 - 15$

$P(3) = 9 + 6 - 15$

$P(3) = 15 - 15 = 0$

$(x-3)$ is a factor of $P(x)$

4. $P(2) = 2^2 - 7 \times 2 + 9$

$P(2) = 4 - 14 + 9$

$P(2) = -1$

$P(2) + 1 = 0$

The number should be added to P(x) is +1.

5. a) $x^2 - 5x + 6 = x^2 - (a+b)x + ab$

$a + b = -5$

$ab = 6$

$a = -3, b = -2$

b) $P(x) = 0$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3, x = 2$

6. a) $P(5) = (5-1)(5-5) = 4 \times 0 = 0$

$P(5) = 0$

$P(x) = (x-1)(x-5)$

$P(x) = x^2 - 6x + 5$

A should be added for this polynomial to be a perfect square.

$x^2 - 6x\ 5 + 4 = x^2 - 6x + 9$

$= (x-3)^2$

PART C

1. $P(x) = (x - 2)(x - 8) + 5$

a) $P(5) = (5-2)(5-8) + 5$

$P(5) = 3x - 3 + 5$

$P(5) = -9 + 4 = -4$

$P(7) = (7-2)(7-8)$

$P(7) = 5x - 1 = -5 \neq 0$

(x-7) is not a factor of P(x)

b) $P(x) = (x-2)(x-8) + 5$

$P(x) = x^2 - 10x + 16 + 5$

$P(x) = x^2 - 10x + 21$

$P(x) = x^2 - (a+b)x + ab$

$a + b = -10$

$ab = 21$

$a = -7, b = -3$

$x^2 - 10x + 21 = (x-7)(x-3)$

2. a) $P(1) = 1^{99} + 1 = 1 + 1 = 2$

b) $P(-1) = (-1)^{99} + 1 = -1 + 1 = 0$

$P(-1) = 0, \therefore x + 1$ is a factor of P(x)

c) $P(1) = 2 \neq 0 \therefore x - 1$ is not a factor of P(x)

3. $P(x) = 4x^2 + 9x + 2$

a) $P(2) = 4 \times 2^2 + 9 \times 2 + 2$

$P(2) = 16 + 18 + 2 = 36$

$P(x) - P(2) = 4x^2 + 9x + 2 - 36$

$P(x) - P(2) = 4x^2 + 9x - 34$

$a = 4$

$b = 9$

$c = -34$

$4x^2 + 9x - 34 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-9 \pm \sqrt{81 + 4 \times 4 \times 34}}{8}$$

$$x = \frac{-9 \pm \sqrt{625}}{8}$$

$$x = \frac{-9 \pm 25}{8}$$

$$x = \frac{-9 + 25}{8}, \quad x = \frac{-9 - 25}{8}$$

$$x = \frac{16}{8}, \quad x = \frac{-34}{8}$$

$$x = 2, \quad x = \frac{-17}{4}$$

$x - 2 = 0$, $\quad 4x + 17 = 0$

OR

$4x^2 + 9x - 34 = P(x) - P(2)$

For any second degree polynomial P(x) and for any number 2, the polynomial (x-2) is a factor of the polynomial P(x) – P(2).

$lx^2 + mx + n = 4x^2 + 9x - 34$

$p(x) – P(a) = (x-a) (lx + la – m)$

$p(x) – P(2) = (x-2) (4x + 4 x 2 + 9)$

$P(x) – P(2) = (x-2) (4x + 17)$

$4x^2 + 9x= 34 = (x-2) (4x + 17) = (x–2) (4x + 17)$

4. $P(x) = x^2 – 11x + 30$

$P(3) = 3^2 – 11 x 3 + 30$

$P(3) = 9 – 33 + 30 = 6$

$P(3) = 6$

$P(x) – P(3) = x^2 – 11x + 30 – 6$

$P(x) – P(3) = x^2 – 11x + 24$

$P(x) – P(3) = x^2 – (8+3) x + 8 \times 3$

$= (x–8) (x–3)$

5. $P(x) = ax^2 + bx + c$

$P(0) = -5$

$P(0) = -5 \quad \Rightarrow \quad C = -5$

b) $P(1) = 0 \quad \Rightarrow \quad a + b + c = 0$

$\Rightarrow \quad a + b = 5$

PART D

1. $x^2 – 10x +16 = (x-a) (x-b)$

$x^2 – (a+b) x +ab = (x –a) (x-b)$

a) $a+b = -10$

$ab = 16$

b) $x^2 - 10x + 16 = (x-8)(x-2)$

$P(x) = 0 \Rightarrow (x-8)(x-2) = 0$

$x = 8$ ad $x = 2$

2. $P(x) = ax^2 - 2bx + c$

$P(1) = 0$

$a-2b + c = 0$

$a + c = 2b$

a , b, c are in an arithmetic sequence

b $a+c = 2b$

$a-c = 0$ then $b = 0$

$a = c = 1$ then $b = 1$

$a = c = 2$ then $b = 2$

$ax^2 - 2bx + c \Rightarrow x^2 - 2x + 1$

$ax^2 - 2bx + c \Rightarrow 2x^2 - 4x + 2$

c) $x^2 - 1$ is a factor of P(x)

$P(1) = 0$ and $P(-1) = 0$

$P(x) = ax^2 - 2b + c = 0$

$P(1) - 0 \Rightarrow a-2b + c = 0$

$\Rightarrow a+c = 2b$

$P(-1) = 0 \Rightarrow a+2b + c = 0$

$\Rightarrow a + c = -2b$

3. $P(x) = x^3 - 4x^2 + 6x - k$

$x - 1$ is a factor of $P(x)$, $P(1) = 0$

$P(1) = 0 \quad \Rightarrow \quad 1 - 4 + 6 - k = 0$

$\Rightarrow \quad k = 3$

4. $P(x) = x^3 - 4x^2 + 6x - 3$

$P(-1) = (-1)^3 - 4 \text{ x } (-1)^2 + 6 + (-1) - 3$

$P(-1) = -14 \neq 0$

$P(-1) \neq 0$, $(x+1)$ is not a factor of $P(x)$

a) $x^2 + 4x + 3 = x^2 + (3+1)x + 3$

$x^2 + 4x + 3 = (x + 3)(x + 1)$

b) $x^2 + 14x + 48 = x2 + (6+8) x + 6 \text{ x } 8$

$x^2 + 14x + 48 = (x+6)(x+8)$

c) $x^2 - 8x + 12 = x^2 - (6+2)x + 6 \times 2$

d) $x^2 + 12 = (x-6)(x-2)$

CHAPTER - 11
STATISTICS

Important Points

- A single number which represents a group of measures is called average.

Arithmetic mean

- Arithmetic mean = $\frac{\text{Sum}}{\text{Number}}$

Median

- Median is the middle most number when the numbers are arranged in increasing or decreasing order.
- If the number of terms (n) is odd then $\left(\frac{n+1}{2}\right)^{th}$ number is the median.
- If the number of terms (n) is even, then median is the average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ numbers.

2 Mark Questions

1. Scores of 10 students is a class are given below.

   11, 35, 32, 33, 39, 45, 41, 49, 48, 47

   (a) Find the mean score

   (b) Find the median score.

2. Scores of 7 students are given below 37, 43, 24, 42, 33, 44, 29.

   (a) Find the mean score.

   (b) Find the median score

3. The arithmetic mean of 10, 14, 17, 21, K is 16. Find K.

4. The numbers 10, 15, 17, x, x + 2, 24, 29, 31 are in ascending order and its median is 20. Find the value of x?

5. The mean of 10 scores in a class group is 13. What is the change in the mean when two students whose scores 14 and 12 are included int the same class group.

3. Mark Questions

1. The height of student in a class is given below.

Height (cm)	Number of Students
140	4
145	7
150	18
155	11
160	6
165	5

Find the median height?

2. Households in a neighbourhood sorted according to their monthly income in the table below.

Monthly income (Rs)	Number of workers
6000	6
7000	7
8000	8
9000	9
10000	10
15000	5
20000	4
25000	3

What is the median monthly income.

3. The table below shows the workers in a factory sorted according to their daily wages.

Daily wages (Rs)	Number of workers
500	5
600	6
700	8
800	5
900	4
1000	3

What is the median daily wage?

5. Mark Questions

1. The table below shows the members in an area sorted according to their ages.

Age	Number of members
20 - 30	4
30 - 40	8
40 - 50	10
50 - 60	7
60 - 70	4
70 - 80	2

(a) If the members are arranged in increasing order of ages, the age of the member at what position is taken as the median?

(b) What is assumed to be age of the member at the 13^{th} Position?

(c) Find the median of the ages?

2. The table below shows the classification of students in class 10 according to their weight?

Weight (kg)	Number of students
30 - 35	3
35 - 40	15
40 - 45	5
45 - 50	9
50 - 55	6
55 - 60	3

(a) If the students are arranged in ascending order according to their weight, the weight of the student at what position is taken as the median?

(b) Identify the median class and also find the number of students in the median class?

(c) Find the median weight?

3. The table below shows the workers in a factory sorted according to their daily wages.

Daily wages (Rs)	Number of Workers
300 - 400	5
400-500	7
500 - 600	8
600 - 700	10
700 - 800	13
800 - 900	7
900 - 1000	3

(a) If the workers are arranged in increasing order of daily wages. What is the position of the workers whose daily wage is median?

(b) What is assumed to be the daily wages of 21^{st} worker?

(c) What is the median daily wage?

Answers (2 marks)

1. (a) Mean score = $\frac{11+35+32+33+39+45+41+49+48+47}{10}$

$$= \frac{380}{10} = 38$$

(b) Writing the score in ascending order.

11, 32, 33, 35, **39, 41,** 45, 47, 48, 49

Median score = $\frac{39+41}{2} = \frac{80}{2} = 40$

2. (a) Mean = $\frac{37+43+24+42+33+44+29}{7}$

$$= \frac{252}{7} = 36$$

(b) 24, 29, 33, 37, 42, 43, 44

Median = 37

3. $\frac{10+14+17+21+K}{5} = 16$

$\frac{62+K}{5} = 16$

62 + K = 80

K = 80 – 62 = 18

4. $\frac{x+x+2}{2} = 20$

2x + 2 = 40

$2x = 38$

$x = 19$

5. $\text{Mean} = \dfrac{\text{Sum}}{\text{Number}}$

$\dfrac{\text{Sum}}{10} = 13$

Sum = 130

14 and 12 scores are included then

sum = 130 + 14 + 12 = 156

$\text{Mean} = \dfrac{156}{12} = 13$

3 Marks (Answer)

Height (cm)	**Number of Students**	**height**	**Number of students**
140	4	up to 140	4
145	7	up to 145	11
150	18	up to 150	29
155	11	up to 155	40
160	6	up to 160	46
165	5	up to 165	51

Total number of students = 51

Median height = Height of $\left(\dfrac{51+1}{2}\right)$th student

= Height of 26^{th} student = 150 cm

2.

Monthly Income (Rs)	No. of workers	Monthly Income	No. of workers
6000	6	up to 6000	6
7000	7	up to 7000	13
8000	8	up to 8000	21
9000	9	up to 9000	30
10000	10	up to 10000	40
15000	5	up to 15000	45
20000	4	up to 20000	49
25000	3	up to 25000	52

Total number of workers = 52

Median monthly income

$$= \text{average income of } \left(\frac{52}{2}\right)^{th} \text{ and } \left(\frac{52}{2}+1\right)^{th} \text{ worker}$$

= Average income of 26^{th} and 27^{th} worker

$$= \frac{9000+9000}{2} = 9000 \text{ Rs.}$$

3.

Monthly (Rs)	No. of workers	Daily wages	No. of workers
500	5	up to 500	5
600	6	up to 600	11
700	8	up to 700	19
800	5	up to 800	24
900	4	up to 900	28
1000	3	up to 1000	31

Median daily wage

$= \text{Daily wage of } \left(\frac{31+1}{2}\right)^{\text{th}} \text{ worker}$

= Daily wage of 16th worker

= 700 Rs.

5 Marks (Answers)

1.

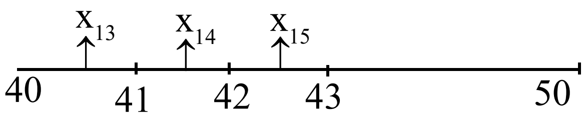
Age group	Number of members
below 30	4
below 40	12
below 50	22
below 60	29
below 70	33
below 80	35

(a) $\frac{35+1}{2} = \frac{36}{2} = 18$

Median is the age of 18th member.

(b) There are 10 members between 40 and 50.

$d = \frac{10}{10} = 1$



Age of the member of 13th position $(x_{13}) = \frac{40+41}{2} = 40.5$

(c) Median age = $x_{18} = x_{13} + 5d$

$= 40.5 + 5 \times 1$

$= 45.5$

2.

Weight (Kg)	No. of Students
below 35	3
below 40	18
below 45	23
below 50	32
below 55	38
below 60	41

(a) Position of the median weight

$= \text{weight of } \left(\frac{41+1}{2}\right) \text{student}$

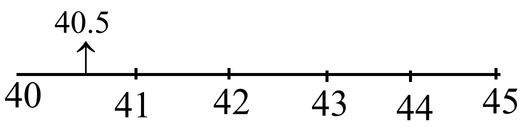
$= \text{weight of } 21^{st} \text{ student}$

(b) Median class = 40 – 45

No. of student = 5

(c) Median weight = weight of 21^{st} student

$$d = \frac{45-40}{5} = 1$$



Median weight = 40.5

3.

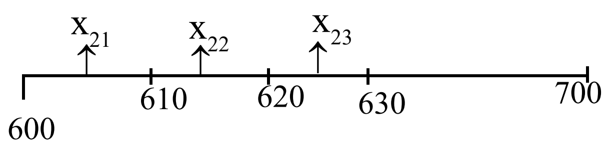
Daily wages	No. of Workers
below 400	5
below 500	12
below 600	20
below 700	30
below 800	43
below 900	50
below 1000	53

(a) Median = Daily wage of the $\left(\frac{53+1}{2}\right)^{th}$ worker

= Daily wage of 27^{th} worker

(There are 10 workers between 600 and 700)

(b) $d = \frac{700-600}{10} = \frac{100}{10} = 10$



Daily wages of 21^{st} worker.

$(x_{21}) = \frac{600+610}{2} = 605$

(c) Median $= x_{27} = x_{21} + 6d = 605 + 60$

$= 665$ Rs.

4. The table below shows, children of class sorted according to their scores in an examination.

Score	No.of children
0 - 10	5
10 -20	8
20 - 30	10
30 - 40	13
40 - 50	9

(a) If the children are arranged in the ascending order of their scores, then what will be the assumed score of the 14^{th} child?

(b) The table below shows the classification of students according to their marks. Find the median mark?

Mark	No.of Students
10	4
20	7
30	10
40	12
50	8

Do your self

1. The strength of 14 classes in a school are given below.

   55, 35, 48, 52, 39, 48, 51, 62, 59, 36, 47, 50, 43, 33

   (a) What is the mean strength?

   (b) What is the median strength?

2. The number of pictures drawn by the arts club members of a school are given below 15, 39, 31, 42, 27, 33, 24, 18, 36, 21, 40 What is the median of the number of pictures.
3. The table below shows the scores of 50 students in a class.

Score	No.of Students
below 10	3
below 20	7
below 30	13
below 40	22
below 50	32
below 60	40
below 70	46
below 80	50

Find the median score?