SECOND YEAR HIGHER SECONDARY EXAMINATION , MARCH 2022
ANSWER KEY

PART I

A . Answer any 5 questions , each carries 1 score.

1. a) { (1,1), (2,2), (3,3) }

2. a) $\frac{1}{2}$

3. b) $|A|^2$

If n is a non-singular square matrix of order n,

then $|adj(A)|=|A|^{n-1}$

4. $P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{1}{2}$

5. $\int_0^2 2x\,dx=\left[2.\frac{x^2}{2}\right]_0^2=[x^2]_0^2=4$

6. slope of tangent = $\frac{dy}{dx}=2x$

slope at x = 2 is 4

7. $\vec{AB}=position\ vector\ of\ B-position\ vector\ of\ A$

$=(4\hat{i}+3\hat{j}+2\hat{k})-(\hat{i}+3\hat{j}+5\hat{k})$

$=3\hat{i}-3\hat{k}$

8. c) (2, 1, -2)

9. degree= 1

B. Answer all questions , each carries 1 score.

10. $\frac{\pi}{4}$

11. b) 2

Area of triangle = $\frac{1}{2}\begin{vmatrix}0&2&1\\0&3&1\\4&6&1\end{vmatrix}=\frac{-4}{2}=-2$

12. Direction cosines are $l=\frac{x}{r},m=\frac{y}{r},n=\frac{z}{r}$

$r=\sqrt{9+4+25}=38$

Direction cosines are $l=\frac{3}{\sqrt{38}},m=\frac{-2}{\sqrt{38}},n=\frac{5}{\sqrt{38}}$

13. $\frac{dy}{dx}=\frac{1}{x^3}.3x^2=\frac{3}{x}$

PART II

A. Answer any 2 questions , each carries 2 scores.

14. $5+x=6\Rightarrow x=1$ and

$x+y=5\Rightarrow y=4$

15. Given $\frac{dx}{dt}=4\,cm/s$ and $\frac{dy}{dt}=-5\,cm/s$

Area of rectangle, A = xy

$\frac{dA}{dt}=x\frac{dy}{dt}+y\frac{dx}{dt}=10(-5)+5(4)=-30\,cm^2/s$

16. $f'(x)=3x^2+3=3(x^2+1)>0$

So f(x) is strictly increasing on R.

17. $y^2dy=2x\,dx$

integrating,

$\int y^2dy=\int 2x\,dx$

$\frac{y^3}{3}=x^2+C$

B. Answer any 2 questions ,each carries 2 scores.

18. If vectors are coplanar, then $[\vec{a}\,\vec{b}\,\vec{c}]=0$

$\begin{vmatrix}1&-1&1\\3&1&2\\1&\lambda&-3\end{vmatrix}=0$

$-3-2\lambda-11+3\lambda-1=0$

$\lambda=15$

19. taking logarithm on both sides.

$\log y=\sin x.\log x$

Differentiating w. r. t x

$\frac{1}{y}\frac{dy}{dx}=\sin x.\frac{1}{x}+\log x.\cos x$

$\frac{dy}{dx}=y\left[\sin x.\frac{1}{x}+\log x.\cos x\right]$

$\frac{dy}{dx}=x^{\sin x}\left[\sin x.\frac{1}{x}+\log x.\cos x\right]$

20. linear D.E is, $\frac{dy}{dx}-\frac{y}{x}=2x$

$P=\frac{-1}{x},Q=2x$

$I.F=e^{\int Pdx}=e^{\int\frac{-1}{x}dx}=e^{-\log x}=\frac{1}{x}$

PART III

A. Answer any 3 questions, each carries 3 scores.

21. $A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$

$$A=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}$$

$$A'=\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$$

$$A+A'=\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}$$

$$\frac{1}{2}(A+A')=\begin{bmatrix}3 & \frac{1}{2} & \frac{-5}{2}\\ \frac{1}{2} & -2 & -2\\ \frac{-5}{2} & -2 & 2\end{bmatrix} \text{ .... P}$$

$$A - A' = \begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}$$

$$\frac{1}{2}(A-A')=\begin{bmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ \frac{-5}{2} & 0 & 3\\ \frac{-3}{2} & -3 & 0\end{bmatrix} \text{ ....Q}$$

A = P + Q

22. For any $a \in Z$, a – a = 0 is an integer.

Therefore R is reflexive.

Difference between two integers is also an integer.

That is if x – y is an integer ,then y – x is an integer. So R is symmetric.

if x – y , and y – z are integers, then x – z is also an integer. So R is transitive.

Therefore R is an equivalence relation.

23. Bag 1 - E_1

Bag 2 - E_2

A – the event drawn ball is red

$$P(E_2/A)=\frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}X\frac{3}{10}}{\frac{1}{2}X\frac{5}{8}+\frac{1}{2}X\frac{3}{10}}=\frac{12}{37}$$

24. a) $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 1\\ 1 & 1 & 1\end{vmatrix}=-\hat{j}+\hat{k}$

b) unit vector perpendicular to $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$|\vec{a}\times\vec{b}|=\sqrt{(2)}$

Therefore unit vector is $\frac{-1}{\sqrt{2}}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$

B. Answer any 2 questions, each carries 3 scores.

25. A = I A

$$\begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

$R_2 \to R_2-2R_1$

$$\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}A$$

$R_1 \to R_1-3R_2$

$$\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}A$$

$$A^{-1}=\begin{bmatrix}7 & -3\\ -2 & 1\end{bmatrix}$$

26. a) Let e be the identity element of a.

Then $a*e=e*a=a$

$$a*e=a\Rightarrow\frac{ae}{3}=a\Rightarrow e=3$$

b) Let a^{-1} be the inverse of a.

Then $a^{-1}*a=a*a^{-1}=e$

$$a*a^{-1}=e\Rightarrow\frac{a.a^{-1}}{3}=3\Rightarrow a^{-1}=\frac{9}{a}$$

inverse of 3 is, $3^{-1}=\frac{9}{3}=3$

27.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} h[f(a)+f(a+h)+\ldots+f(a+(n-1)h)]$$

where $\quad h=\dfrac{b-a}{n}\to 0 \text{ as } n\to\infty$

$$h=\frac{2-0}{n}=\frac{2}{n}$$

$f(a)=f(0)=0$

$f(a+h)=h^2$

$f(a+2h)=(2h)^2$

$f(a+3h)=(3h)^2$

....

$f(a+(n-1)h)=((n-1)h)^2$

$$\int_0^2 x^2\,dx=\lim_{n\to\infty}\frac{2}{n}[0+h^2+(2h)^2+(3h)^2+\ldots+((n-1)h)^2]$$

$$= \lim_{n\to\infty}\frac{2}{n}[h^2(1^2+2^2+3^2+\ldots+(n-1)^2)]$$

$$= \lim_{n\to\infty}\frac{2}{n}\left[\left(\frac{2}{n}\right)^2\left(\frac{(n-1)n(2n-1)}{6}\right)\right]$$

$$= \lim_{n\to\infty}\frac{4}{3}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= \frac{8}{3}$$

PART IV

A. Answer any 3 questions ,each carries 4 scores

28. $$2\tan^{-1}\left(\frac{1}{2}\right)=\tan^{-1}\left(\frac{2.\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right)$$

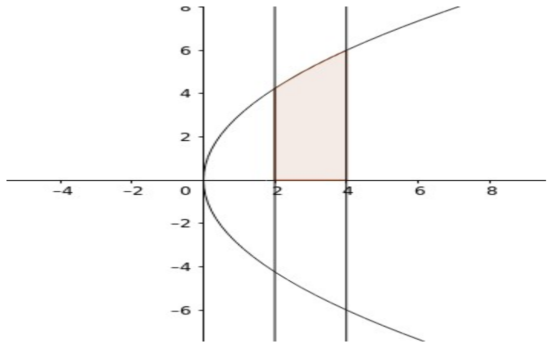
$$= \tan^{-1}\left(\frac{4}{3}\right)$$

$$2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}.\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{31}{17}\right)$$

29.



Required Area= $\displaystyle\int_2^4 y\,dx=\int_2^4 3\sqrt{x}\,dx$

$$= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$$

$$= 16-4\sqrt{2}$$

30. a) $LHL=\lim_{x\to 3}(3x+1)=10$

$RHL=\lim_{x\to 3}(x^2+1)=10$

$f(3)=10$

LHL = RHL = f(x)

Therefore f(x) is continuous .

b) f(x) is continuous on [2,4]

f(x) is differentiable on (2,4)

f(a) = f(2) = -15

f(b) = f(4) = -15

here f(a) = f(b)

$f'(x)=4x-12$

$f'(c)=0\Rightarrow 4c-12=0\Rightarrow c=3\in(2,4)$

Hence verified.

31. a) Vector form $\quad \vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$

ie, $\quad \vec{r}=2\hat{i}+\hat{j}+\lambda(2\hat{i}+3\hat{j}+3\hat{k})$

Cartesian form $\quad \dfrac{x-x_1}{x_2-x_1}=\dfrac{y-y_1}{y_2-y_1}=\dfrac{z-z_1}{z_2-z_1}$

ie, $\quad \dfrac{x-2}{2}=\dfrac{y-1}{3}=\dfrac{z}{3}$

b) Here $\quad \vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{N}=2\hat{i}+3\hat{j}+3\hat{k}$

Vector form $\quad (\vec{r}-\vec{a}).\vec{N}=0$

ie, $\quad (\vec{r}-(\hat{i}+\hat{j}+2\hat{k})).2\hat{i}+3\hat{j}+3\hat{k}=0$

Cartesian form

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$$

ie, $2(x-1)+3(y-1)+3(z-2)=0$

ie, $2x+3y+3z-11=0$

B. Answer any 1 question, carries 4 scores.

32. a) Let S = { 1, 2, 3, 4, 5, 6 }

X denote the number obtained on the throw.

X	1	2	3	4	5	6
P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Mean = $\sum_{i=1}^{n} x_i p_i$

$$= 1.\frac{1}{6}+2.\frac{1}{6}+3.\frac{1}{6}+4.\frac{1}{6}+5.\frac{1}{6}+6.\frac{1}{6}=\frac{21}{6}$$

33. a) Angle between planes, $\cos\theta=\left|\frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|$

$\vec{n_1}=3\hat{i}-2\hat{j}+\hat{k}$ and $\vec{n_2}=2\hat{i}+\hat{j}+2\hat{k}$

$\vec{n_1}.\vec{n_2}=6-2+2=6$

$|\vec{n_1}|=\sqrt{9+4+1}=\sqrt{14}$

$|\vec{n_2}|=\sqrt{4+1+4}=\sqrt{9}=3$

$\cos\theta=\left|\frac{6}{3\sqrt{14}}\right|$

$\theta=\cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$

b) $\vec{n_1}=3\hat{i}-2\hat{j}+\hat{k}$ and $\vec{n_2}=2\hat{i}+\hat{j}+2\hat{k}$

Also $\vec{d}_1=-6$ and $\vec{d}_2=6$

The relation is $\vec{r}.(\vec{n_1}+\lambda\vec{n_2})=\vec{d_1}+\lambda\vec{d_2}$

$\vec{r}.\left((3\hat{i}-2\hat{j}+\hat{k})+\lambda(2\hat{i}+\hat{j}+2\hat{k})\right)=-6+\lambda 6$

taking $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$(3+2\lambda)x+(\lambda-2)y+(1+2\lambda)z=-6+\lambda 6$

passing through (0,0,0)

then $0=-6+\lambda 6 \Rightarrow \lambda=1$

Required Equation is $\vec{r}.(5\hat{i}-\hat{j}+3\hat{k})=0$

PART V

Answer any 2 questions, each carries 6 scores.

34. AX = B

$$\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{vmatrix}=-1$$

Co-factor matrix = $\begin{bmatrix}2 & -9 & -6\\ o & -2 & -1\\ -1 & 3 & 2\end{bmatrix}$

adj A = $\begin{bmatrix}2 & 0 & -1\\ -9 & -2 & 3\\ -6 & -1 & 2\end{bmatrix}$

$$A^{-1}=\frac{adj\,A}{|A|}$$

$$=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$$

$$X=A^{-1}B$$

$$=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$$

$$=\begin{bmatrix}0\\ 5\\ 3\end{bmatrix}$$

x = 0, y = 5, z = 3

35.a) By partial fraction

$$\frac{x}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$$

ie, $x=A(x+2)+B(x+1)$

put x = -1 we get A = -1

and put x = -2 , we get B = 2

therefore $\frac{x}{(x+1)(x+2)}=\frac{-1}{x+1}+\frac{2}{x+2}$

now $\int\frac{x}{(x+1)(x+2)}dx=\int\frac{-1}{x+1}dx+\int\frac{2}{x+2}dx$

$= -\log|x+1|+2\log|x+2|+C$

$= \log|x+2|^2-los|x+1|+C$

$= \log\left|\frac{(x+2)^2}{x+1}\right|+C$

b) $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x+\cos^4 x} dx$ I

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^4(\frac{\pi}{2}-x)}{\sin^4(\frac{\pi}{2}-x)+\cos^4(\frac{\pi}{2}-x)} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\cos^4 x+\sin^4 x} dx$$

adding them,

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^4 x+\cos^4 x}{\sin^4 x+\cos^4 x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1\, dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

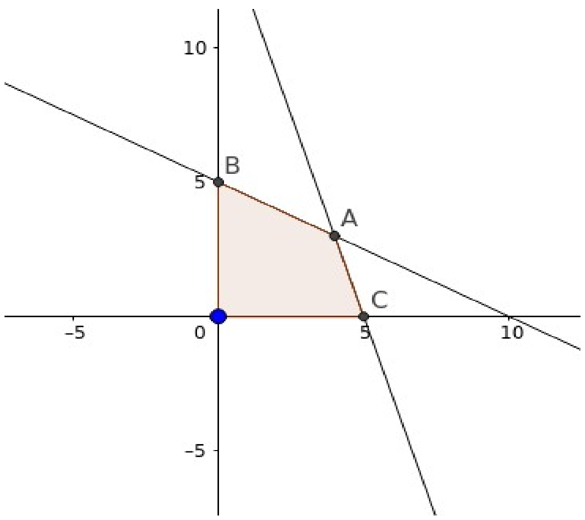
$$I = \frac{\pi}{4}$$

36. x + 2y = 10

x	y
0	5
10	0

3x + y = 15

x	y
0	15
5	0



point	Z = 3x + 2y
O(0,0)	0
A(4,3)	18
B(0,5)	10
C(5,0)	15

Maximum of Z is 18 at (4,3).

....

Prepared by ANN SABY JACOB, HSST MATHEMATICS, GTHSS POOMALA , IDUKKI