

# MODEL EXAMINATION - MARCH 2021

## Mathematics (Science)

1 to 10 carry 3 scores each.

1.  $2x^2 - 24 = 2 - 20$

$$2x^2 - 24 = -18$$

$$2x^2 = 6$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

2. i)  $\text{adj } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

ii)  $A(\text{adj } A) = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 5I$$

3. LHL =  $\lim_{x \rightarrow 2^-} f(x)$

$$= \lim_{x \rightarrow 2^-} kx^2$$

$$= 4k$$

RHL =  $\lim_{x \rightarrow 2^+} f(x)$

$$= \lim_{x \rightarrow 2^+} 3$$

$$= 3$$

$f(x)$  is continuous at  $x=2$

$$\text{LHL} = \text{RHL} = f(2)$$

$$\therefore 4k = 3$$

$$k = \frac{3}{4}$$

4. The function  $f(x) = x^2 - 4x - 3$  is continuous on  $[1, 4]$  and differentiable on  $(1, 4)$

$$f(a) = f(1) = -6$$

$$f(b) = f(4) = -3$$

$$\therefore f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{-3 - (-6)}{4 - 1}$$

$$= \frac{-3 + 6}{4 - 1}$$

$$= 1$$

Theorem states that there is a point  $c \in (1, 4)$  such that

$$f'(c) = 1$$

But  $f'(x) = 2x - 4$

$$f'(c) = 1 \Rightarrow 2c - 4 = 1$$

$$2c = 5$$

$$c = \frac{5}{2} \in (1, 4)$$

5. Given that  $\frac{dr}{dt} = 5 \text{ cm/s}$

Area of a circle,

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi \times 8 \times 5$$

$$= 80\pi \text{ cm}^2/\text{s}$$

6. Unit vector,  $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

$$\vec{PQ} = \text{P.V. of } Q - \text{P.V. of } P$$

$$= 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$|\vec{PQ}| = \sqrt{27}$$

Unit vector is  $\frac{3}{\sqrt{27}}\hat{i} + \frac{3}{\sqrt{27}}\hat{j} + \frac{3}{\sqrt{27}}\hat{k}$

7. Point is  $(1, 4, 6)$ ,  
normal vector is  $\hat{i} - 2\hat{j} + \hat{k}$ .

Vector equation is,

$$(\vec{r} - \vec{a}) \cdot \vec{N} = 0.$$

$$[\vec{r} - (1\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$$

Cartesian equation is,

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

$$1(x - 1) - 2(y - 4) + 1(z - 6) = 0$$

$$x - 1 - 2y + 8 + z - 6 = 0$$

$$x - 2y + z + 1 = 0.$$

$$\underline{\underline{x - 2y + z = -1}}$$

8. i)  $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

ii)  $\tan^{-1}(1) = \frac{\pi}{4}$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \underline{\underline{\frac{3\pi}{4}}}$$

9. Equation of line joining two points

$$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

$$x(2-6) - y(1-3) + k(6-6) = 0$$

$$-4x + 2y = 0$$

$$\underline{\underline{y = 2x}}$$

10.  $\frac{dy}{dn} + \frac{y}{n} = n^2$

$$P = \frac{1}{n} \quad Q = n^2$$

$$I.F = e^{\int P dn} = e^{\int \frac{1}{n} dn}$$

$$= e^{\log n}$$

$$= n.$$

general solution is,

$$y \cdot e^{\int P dn} = \int (Q \cdot e^{\int P dn}) dn$$

$$\therefore y \cdot n = \int n^2 \cdot n dn$$

$$= \int n^3 dn$$

$$ny = \frac{n^4}{4} + c$$

$$\underline{\underline{y = \frac{n^3}{4} + c n^{-1}}}$$

11 to 22 carry 4 scores each.

11.  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$a_{ij} = \frac{(1+j)^2}{2}$$

$$a_{11} = \frac{(1+1)^2}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = 8$$

$$\therefore A = \underline{\underline{\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}}}$$

$$\text{ii) } a - b = -1$$

$$2a - b = 0$$

$$d = 4$$

$$c = 5$$

$$a - b = -1$$

$$2a - b = 0$$

$$\hline -a = -1$$

$$a = 1$$

$$\therefore a - b = -1$$

$$b = a + 1$$

$$= 1 + 1$$

$$= \underline{\underline{2}}$$

$$a = 1, b = 2, c = 5, d = 4$$

$$12. \text{ i) } 3A = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix}$$

$$3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

$$\text{ii) } AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

$$13. \text{ i) } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$$

$$\text{ii) } \tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24}$$

$$= \tan^{-1} \left( \frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \times \frac{7}{24}} \right)$$

$$= \tan^{-1} \left( \frac{\frac{125}{264}}{\frac{250}{264}} \right)$$

$$= \tan^{-1} \left( \frac{125}{250} \right) = \tan^{-1} \left( \frac{1}{2} \right)$$

14.

$$\text{i) } y = \sin(\cos(x^2))$$

$$\frac{dy}{dx} = \cos(\cos(x^2))$$

$$\times -\sin(x^2) \times 2x$$

$$= -2x \sin(x^2) \cos(\cos(x^2))$$

$$\text{ii) } x^2 + xy + y^2 = 100$$

differentiating w.r.t.  $x$ .

$$2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$$

$$(x + 2y) \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

15.

$$\text{i) } x = 0, f(x) = \sin 0 + \cos 0 = 1$$

$$x = \frac{\pi}{6}, f(x) = \sin \frac{\pi}{6} + \cos \frac{\pi}{6}$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1 + \sqrt{3}}{2} \approx 1.366$$

$$x = \frac{\pi}{4}, f(x) = \sin \frac{\pi}{4} + \cos \frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}}$$

$$= \sqrt{2} \approx 1.414$$

$$x = \frac{\pi}{3}, f(x) = \sin \frac{\pi}{3} + \cos \frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{1 + \sqrt{3}}{2}$$

$$x = \frac{\pi}{2}, f(x) = \sin \frac{\pi}{2} + \cos \frac{\pi}{2} = 1$$

$\therefore$  Maximum value of  $f(x)$  in  $[0, \frac{\pi}{2}]$  is  $\sqrt{2}$

OR

$$(\sin n + \cos n)^2$$

$$= \sin^2 n + \cos^2 n + 2 \sin n \cos n$$

$$= 1 + \sin 2n$$

Maximum value of

$$\sin 2n \text{ is } 1$$

$$\therefore (\sin n + \cos n)^2 = 1 + 1$$

$$= 2$$

$$\sin n + \cos n = \sqrt{2}$$

maximum value is  $\sqrt{2}$

ii)  $f(x) = x^2 + 2x - 5$

$$f'(x) = 2x + 2$$

$$f'(x) = 0 \Rightarrow 2x + 2 = 0$$

$$x = -1$$

There are two intervals

$$(-\infty, -1) \text{ and } (-1, \infty)$$

$$f'(0) = 2 > 0$$

$f(x)$  is increasing in  $(-1, \infty)$

$$f'(-2) = -2 < 0$$

$f(x)$  is decreasing in  $(-\infty, -1)$ .

16. i) Order = 2

ii)  $\sec^2 x \tan x dx + \sec^2 y \tan y dy = 0$

$$\sec^2 x \tan x dx = -\sec^2 y \tan y dy$$

$$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$$

$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$$

Put  $u = \tan x$

$v = \tan y$

$$du = \sec^2 x dx$$

$$dv = \sec^2 y dy$$

$$\therefore \int \frac{1}{u} du = -\int \frac{1}{v} dv$$

$$\log |u| = -\log |v| + c$$

$$\log |u| + \log |v| = c$$

$$\log (uv) = \log c$$

$$\log |\tan x \tan y| = \log c$$

$$\tan x \tan y = c$$

17.  $\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$

$$\vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

Vector  $\perp$  to both  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$

is  $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ .

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$$= \hat{i}(16 - 0) - \hat{j}(16 - 0) + \hat{k}(0 - 8)$$

$$= 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = \sqrt{256 + 256 + 64}$$

$$= \sqrt{576}$$

$$= 24$$

Unit vector  $\perp$  to  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$

is  $\frac{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})}{|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|}$

$$= \frac{16}{24}\hat{i} - \frac{16}{24}\hat{j} - \frac{8}{24}\hat{k}$$

$$= \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$

$$18. \vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$$

Shortest distance between lines

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3-6)$$

$$= -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{81 + 9 + 81} = \sqrt{171}$$

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = -27 + 9 + 27 = 9$$

$$\therefore d = \left| \frac{9}{\sqrt{171}} \right|$$

$$= \frac{9}{\sqrt{171}}$$

$$= \frac{9}{3\sqrt{19}}$$

$$= \frac{3}{\sqrt{19}}$$

$$19. i) P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{7} \times \frac{1}{5}$$

$$= \frac{1}{35}$$

$$ii) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{7} + \frac{1}{5} - \frac{1}{35}$$

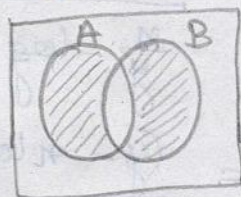
$$= \frac{11}{35}$$

$$iii) P[(A \cap B') \cap (B \cap A')]$$

$A \cap B'$  and  $B \cap A'$

are mutually exclusive events

$$\therefore P[(A \cap B') \cap (B \cap A')] = 0.$$



20.

$$R = \{(a, b) : |a - b| \text{ is even}\}$$

$$A = \{1, 2, 3, 4, 5\}$$

$$|a - a| = 0 \text{ is even.}$$

$\therefore R$  is reflexive.

$$(a, b) \in R \Rightarrow |a - b| \text{ is even}$$

$$\Rightarrow |b - a| \text{ is even}$$

$$\Rightarrow (b, a) \in R$$

$\therefore R$  is symmetric.

$$(a, b) \in R \text{ and } (b, c) \in R$$

$$\Rightarrow |a - b| \text{ is a multiple of } 2 \text{ and}$$

$$|b - c| \text{ is a multiple of } 2$$

$$\Rightarrow |a - c| \text{ is a multiple of } 2$$

$$\Rightarrow (a, c) \in R$$

$\therefore R$  is transitive.

$\therefore R$  is an equivalence relation.

$$21. \quad y^n = n^y$$

taking logarithm on both sides

$$\log y^n = \log n^y$$

$$n \log y = y \log n$$

differentiating w.r.t  $n$

$$n \frac{1}{y} \frac{dy}{dn} + \log y = y \frac{1}{n} + \log n \frac{dy}{dn}$$

$$\frac{n}{y} \frac{dy}{dn} - \log n \frac{dy}{dn} = \frac{y}{n} - \log y$$

$$\left(\frac{n}{y} - \log n\right) \frac{dy}{dn} = \frac{y}{n} - \log y$$

$$\frac{dy}{dn} = \frac{\frac{y}{n} - \log y}{\frac{n}{y} - \log n}$$

$$\frac{dy}{dn} = \frac{(y - n \log y) y}{(n - y \log n) n}$$

22.

$$i) \int n \log n \, dn$$

$$= \log n \int n \, dn - \int \left[ \frac{d}{dn} (\log n) \int n \, dn \right] dn$$

$$= \log n \frac{n^2}{2} - \int \frac{1}{n} \frac{n^2}{2} \, dn$$

$$= \frac{n^2 \log n}{2} - \frac{1}{2} \int n \, dn$$

$$= \frac{n^2 \log n}{2} - \frac{1}{2} \frac{n^2}{2} + C$$

$$= \frac{n^2 \log n}{2} - \frac{n^2}{4} + C$$

$$ii) \int n^2 \sin n \, dn$$

$$= n^2 \int \sin n \, dn - \int \left[ \frac{d}{dn} (n^2) \int \sin n \, dn \right] dn$$

$$= n^2 (-\cos n) - \int 2n (-\cos n) \, dn$$

$$= -n^2 \cos n + 2 \int n \cos n \, dn$$

$$= -n^2 \cos n + 2 \left[ n \int \cos n \, dn \right.$$

$$\left. - \int \left( \frac{d}{dn} (n) \int \cos n \, dn \right) dn \right]$$

$$= -n^2 \cos n + 2 \left[ n \sin n \right.$$

$$\left. - \int \sin n \, dn \right]$$

$$= -n^2 \cos n + 2 \left[ n \sin n - \cos n \right] + C$$

$$= -n^2 \cos n + 2n \sin n + 2 \cos n + C$$

23 to 29 carry 6 scores each

23.

$$A = \frac{1}{2} (A + A^T) + \frac{1}{2} (A - A^T)$$

$$A^T = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{2} (A + A^T) + \frac{1}{2} (A - A^T)$$

$$= \frac{1}{2} \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= A //$$

24.  $Ax = B$

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix}$$

$$= -17 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$x = A^{-1}B = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{-1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x = 1, y = 2, z = 3$$

25. i)  $f(x) = 3 - 4x$

Let  $f(x_1) = 3 - 4x_1$

$f(x_2) = 3 - 4x_2$

$f(x_1) = f(x_2) \Rightarrow 3 - 4x_1 = 3 - 4x_2$

$$4x_1 = 4x_2$$

$$x_1 = x_2$$

$\therefore f(x)$  is one-one.

Let  $y \in \mathbb{R}$ , and  $f(x) = y$

i.e.  $3 - 4x = y$

$$\Rightarrow 4x = 3 - y$$

$$x = \frac{3-y}{4} \in \mathbb{R}$$

Every  $y$  has a pre-image under  $f$ .  $f$  is onto.

$\therefore f$  is bijective

ii)  $f(x) = 2x+1, g(x) = x^2$

$$g \circ f(x) = g(f(x))$$

$$= g(2x+1)$$

$$= \underline{(2x+1)^2}$$

$$f \circ g(x) = f(g(x))$$

$$= f(x^2)$$

$$= \underline{2x^2+1}$$

26. curve is  $y = x^2 - 2x + 7$

slope of tangent,  $\frac{dy}{dx} = 2x - 2$

i) slope of the lines are same since they are parallel

$$2x - y + 9 = 0$$

$$y = 2x + 9$$

$$\text{slop} = 2.$$

$$\therefore 2n - 2 = 2$$

$$2n = 4$$

$$n = 2$$

$$\begin{aligned} \text{But } y &= n^2 - 2n + 7 \\ &= 4 - 4 + 7 \\ &= 7. \end{aligned}$$

Point is  $(2, 7)$ .

$\therefore$  Equation of the tangent line

$$\text{is } y - y_0 = m(n - n_0)$$

$$y - 7 = 2(n - 2)$$

$$y - 7 = 2n - 4$$

$$2n - y + 3 = 0$$

ii) Lines are  $\perp$  then  $m_1 m_2 = -1$ .

$$5y - 15n = 13$$

$$5y = 15n + 13$$

$$y = 3n + \frac{13}{5}$$

$\therefore$  slope = 3.

$$\therefore (2n - 2) \cdot 3 = -1$$

$$6n - 6 = -1$$

$$6n = 5$$

$$n = \frac{5}{6}$$

$$\begin{aligned} \therefore y &= n^2 - 2n + 7 \\ &= \frac{25}{36} - \frac{10}{6} + 7 \\ &= \frac{217}{36} \end{aligned}$$

$\therefore$  Point is  $(\frac{5}{6}, \frac{217}{36})$ .

Equation of tangent line is

$$y - \frac{217}{36} = 3(n - \frac{5}{6})$$

$$n + 3y - \frac{227}{12} = 0$$

$$12n + 36y - 227 = 0.$$

27.

$$i) \int \frac{1}{1+n^2} dn = \tan^{-1} n + C$$

$$ii) \int \frac{dn}{n^2 - 6n + 13}$$

$$n^2 - 6n + 13 = n^2 - 6n + 9 + 13 - 9$$

$$= (n - 3)^2 + 4$$

$$= (n - 3)^2 + 2^2$$

$$\therefore \int \frac{dn}{n^2 - 6n + 13} = \int \frac{dn}{(n - 3)^2 + 2^2}$$

$$\text{put } t = n - 3$$

$$dt = dn$$

$$= \int \frac{dt}{t^2 + 2^2}$$

$$= \frac{1}{2} \tan^{-1} \left( \frac{t}{2} \right) + C$$

$$= \frac{1}{2} \tan^{-1} \left( \frac{n - 3}{2} \right) + C$$

$$iii) \int \frac{\tan^{-1} n}{1+n^2} dn$$

$$\text{put } \tan^{-1} n = t$$

$$dt = \frac{1}{1+n^2} dn$$

$$\therefore \int \frac{\tan^{-1} n}{1+n^2} dn = \int t dt$$

$$= \frac{t^2}{2} + C$$

$$= \frac{(\tan^{-1} n)^2}{2}$$

$$\int_0^1 \frac{\tan^{-1} n}{1+n^2} dn = \left[ \frac{(\tan^{-1} n)^2}{2} \right]_0^1$$

$$= \frac{(\tan^{-1} 1)^2}{2}$$

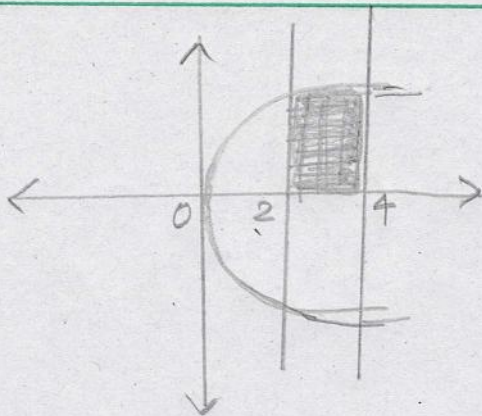
$$= \frac{\left(\frac{\pi}{4}\right)^2}{2}$$

$$= \frac{\pi^2}{32}$$



28.

i)



The required area

$$= \int_2^4 y \, dx$$

$$= \int_2^4 3\sqrt{x} \, dx$$

$$= 3 \int_2^4 x^{1/2} \, dx$$

$$= 3 \left[ \frac{x^{3/2}}{3/2} \right]_2^4$$

$$= 2 \left[ 4^{3/2} - 2^{3/2} \right]$$

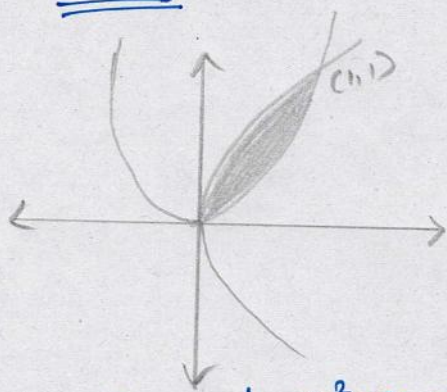
$$= 2 \left[ 2^3 - 2\sqrt{2} \right]$$

$$= \underline{\underline{16 - 4\sqrt{2} \text{ sq. units}}}$$

$$y^2 = 9x$$

$$y = 3\sqrt{x}$$

ii)



$y = x^2$  and  $y^2 = x$ .  
we get (1,1).

Required Area

$$= \int_0^1 (f(x) - g(x)) \, dx$$

$$= \int_0^1 (\sqrt{x} - x^2) \, dx$$

$$= \int_0^1 (x^{1/2} - x^2) \, dx$$

$$= \left[ \frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right]_0^1$$

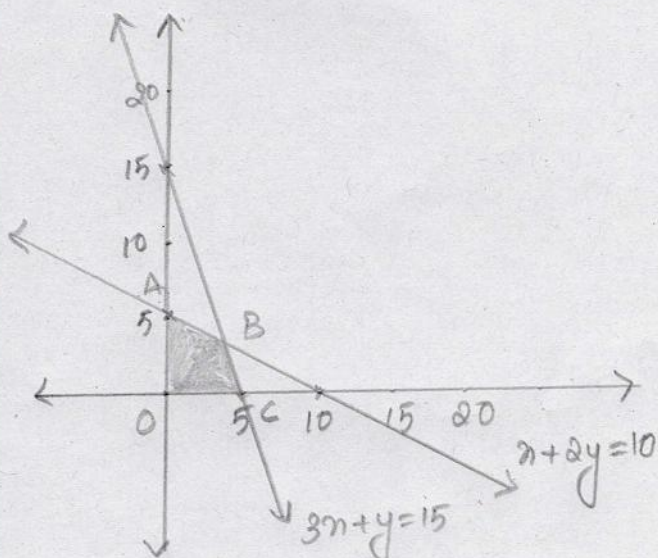
$$= \frac{2}{3} - \frac{1}{3} = \underline{\underline{\frac{1}{3} \text{ sq. units}}}$$

29.  $x + 2y = 10$ 

x	y
0	5
10	0

 $3x + y = 15$ 

x	y
0	15
5	0



Point	function, $3x + 2y$
O(0,0)	0
A(0,5)	10
B(4,3)	<u>18</u> - Maximum.
C(5,0)	15

Maximum value is 18 at (4,3)