SECOND Y[redacted]R HIGHER SECONDARY EXAMINATION [redacted]ARCH 2017

SUBJECT : MATHEMATICS (SCIENCE) CODE. NO: 5018

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
1.	(a)	(b) Symmetric	1	1
	(b)	$fog(x) = f(g(x)) = f(2x-1)$	½	
		$= \|2x-1+1\|$	½	
		$= \|2x\|$		2
		$gof(x) = g(f(x)) = g\|x+1\|$	½	
		$= 2\|x+1\|-1$	½	
	(c)	If (c,d) is the identity element		
		$(a,b) * (c,d) = (a,b)$	½	
		$(a,b) * (c,d) = (a+c, b+d)$	½	
		$(a+c, b+d) = (a,b)$	½	2.
		$(c,d) = (0,0)$	½	
		Not an element of $N \times N$		
		∴ Identity element does not exist		
		Remark.		
		(b) For $fog(x) = f(g(x))$ or $gof(x) = g(f(x))$ only (½)		
		(c) For $a*e = a = e*a$ (½ score)		
		Give 2 score for identity element does not exist		
2.	(a)	d. $2\pi/3$	1	1
	(b)	$\tan^{-1}\left[\dfrac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \pi/4$	1	
		$\tan^{-1}\left[\dfrac{2x^2-4}{-3}\right] = \pi/4$	1	

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		$\frac{2x^2-4}{-3} = 1$	½	3
		$x = \pm \frac{1}{\sqrt{2}}$	½	
		Remark: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ give (1) score		
3.	(a)	a. $\underline{k=0}$	1	1
	(b)	$A^2 = A \cdot A$	½	
		$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$	½	
		$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & \sin\theta\cos\theta + \cos\theta\sin\theta \\ -\sin\theta\cos\theta - \cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$	1	3
		$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$	1	
	(c)	$A' = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$	½	
		$3A' = \begin{bmatrix} 3 & 12 \\ 9 & 3 \end{bmatrix}$	½	2
		$\lvert 3A' \rvert = -99$	1	
		Remark: Since $\lvert A' \rvert = \lvert A \rvert$ for the correct $\lvert 3A \rvert$ give (2) score. For alternate method and correct answer give (2) score.		
4.	a)	(c) 0	1	1
	(b)	$AX = B$ form $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$	½	

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		$\lvert A \rvert = 10$	1	
		$A^{-1} = \frac{Adj\ A}{\lvert A \rvert} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$	2	
		$X = A^{-1}B$		
		$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$	$\frac{1}{2}$	4
		Remark: For only $AX = B$ form give (1) score		
		For $A^{-1} = \frac{Adj\ A}{\lvert A \rvert}$ ($\frac{1}{2}$ score)		
		$X = A^{-1}B$ ($\frac{1}{2}$ score)		
5	(a)	$5a = 1$	1	
		$a = 1/5$	$\frac{1}{2}$	3
		$a = b - \pi/2$	1	
		$b = \frac{1}{5} + \pi/2$	$\frac{1}{2}$	
		Remark: For the idea of continuity give (1) score		
		$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$		
	(b)	$\log\left[(\sin x)^{\cos y}\right] = \log\left[(\cos y)^{\sin x}\right]$	1	
		$\cos y \log \sin x = \sin x \log \cos y$	$\frac{1}{2}$	
		$\cos y \times \frac{1}{\sin x}\cos x + \log \sin x \cdot -\sin y \frac{dy}{dx}$ $= \sin x \frac{1}{\cos y} -\sin y \frac{dy}{dx} + \log \cos y \cdot \cos x$	1	3
		$\frac{dy}{dx} = \frac{\cos x \cdot \log \cos y - \cos y \cot x}{\sin x \tan y - \sin y \cdot \log \sin x}$	$\frac{1}{2}$	

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6	(a)	(d) -1	1	1
	(b)	$f'(x) = 6x^2+18x+12$	1	
		$f'(x) = 0$	1	
		$x = -1$ or -2	½	
		The intervals may be $(-\infty,-2)\,(-2,-1)$ and $(-1,\infty)$	½	
		$f'(x) > 0$ in $(-\infty,-2)$ and $(-1,\infty)$	1	4.
		$\therefore f(x)$ is increasing in $(-\infty,-2)$ and $(-1,\infty)$.		
		Remark: $f(x)$ is increasing when $f'(x)>0$ give (1) score.		
		OR.		
	(a)	(a) 4π	1	1
	(b)	Numbers x and $(16-x)$	1	
		$S = x^3 + (16-x)^3$	½	
		$\frac{ds}{dx} = 3x^2 + 3(16-x)^2 \times -1$	1	
		$\frac{ds}{dx} = 0 \Rightarrow x = 8$	½	
		$\frac{d^2s}{dx^2} = 6x + 6(16-x)$	½	4
		$\frac{d^2s}{dx^2}$ at $x=8$ > 0	½	
		Numbers are 8 and 8		
7	(a)	$\int \frac{x^6}{x^7(x^7+1)}\,dx = \frac{1}{7}\int \frac{dt}{t(t+1)}$ $t = x^7$, $dt = 7x^6dx$	1½	
		$= \frac{1}{7}\left[\int\left(\frac{1}{t} + \frac{-1}{t+1}\right) dt\right.$	1	

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		$= \frac{1}{7}\left[\log x^7 - \log(1+x^7)\right] + c$	½	3
		<u>Remark</u>: Put $u = x^7+1$ or $u = x^7$ } give (1) score.		
	(b)	$\int_1^4 \lvert x-2\rvert dx = \int_1^2 (2-x)dx + \int_2^4 (x-2)dx$	1	
		$= \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^4$	1	3
		$= \left(0 - \frac{-1}{2}\right) + \left(\frac{4}{2} + 0\right)$	½	
		$= \frac{5}{2}$	½	
		<u>Remark</u>: $\int_1^4 (x-2)dx = \left[\frac{(x-2)^2}{2}\right]_1^4$ give (1) score. For concept of $\lvert x\rvert$ (½) property (½) score.		
		Alternative method:		
		$\int_1^2 (2-x)dx + \int_2^4 (x-2)dx$		
		$= \left(2x - \frac{x^2}{2}\right)_1^2 + \left(\frac{x^2}{2} - 2x\right)_2^4$		
		$= 5/2$.		
8		$I = \int_0^{\pi/2} \log \sin x \, dx$		
		$= \int_0^{\pi/2} \log \sin(\pi/2 - x)dx$	½	
		$I = \int_0^{\pi/2} \log \cos x \, dx$	½	
		$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x)\, dx$	1	4
		$= \int_0^{\pi/2} \log \sin 2x - \int_0^{\pi/2} \log 2 \, dx$	1	
		$= I - \pi/2 \log 2$	½	
		$I = -\pi/2 \log 2$	½	

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8		OR. $\int_0^4 x^2 dx = \lim_{h\to 0} h\left[h^2+(2h)^2+\cdots+((n-1)h)^2\right]$	1	
		$= \lim_{h\to 0} h^3\left[1^2+2^2+\cdots+(n-1)^2\right]$	1	
		$= \lim_{h\to 0} h^3 \frac{n(n-1)(2n-1)}{6}$	1	4
		$= \lim_{h\to 0} h^3n^3 \cdot \frac{(1-1/n)(2-1/n)}{6}$	½	
		$= \frac{4\times 4\times 8}{6} = \frac{64}{3}$	½	
		Remark: $\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+\cdots + f(a+(n-1)h)\right]$ (1) score. For $(b-a)$, h, $f(a)$, $f(a+h)$ give ½ score each. For direct integration and correct answer (1) score.		
9.	(a)	(c) 1	1	1
	(b)		½	
		Point of intersection $(4a, 4a)$	1	
		Area $= \int_0^{4a} \sqrt{4ax}\, dx - \int_0^{4a} \frac{x^2}{4a} dx$	2	5
		$= 2\sqrt{a}\left(\frac{x^{3/2}}{3/2}\right)_0^{4a} - \frac{1}{4a}\left(\frac{x^3}{3}\right)_0^{4a}$	1	
		$= \frac{16}{3}a^2$	½	

Remark: $A = \int_a^b y\,dx$ (½) score.
Figure only (1) score. Figure is not necessary

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10	(a)	(d) 2	1	1
	(b)	Solving second order differential equation is out of syllabus. Remark: Attempting question 10 give 5 score.		5
11.	(a)	b $7/\sqrt{3}$	1	1
	(b)	$A = \lvert\bar{a}\times\bar{b}\rvert$	½	
		$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$	½	2
		$= i + j - 3k$	½	
		$\lvert\bar{a}\times\bar{b}\rvert = \sqrt{11}$	½	
12	(a)	a 60° $(\pi/3)$	1	1
	(b)	$\lvert\bar{a}+\bar{b}+\bar{c}\rvert = 0$	1	
		$(\bar{a}+\bar{b}+\bar{c})^2 = 0$	1	
		$\lvert\bar{a}\rvert^2 + \lvert\bar{b}\rvert^2 + \lvert\bar{c}\rvert^2 + 2\bar{a}\cdot\bar{b} + 2\bar{a}\cdot\bar{c} + 2\bar{b}\cdot\bar{c} = 0$	1	4
		$3 + 2(\bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c} + \bar{b}\cdot\bar{c}) = 0$	1	
		$\bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c} + \bar{b}\cdot\bar{c} = -3/2$		
		Remark: For alternate method and correct answer (4) $\lvert\bar{a}\rvert = \lvert\bar{b}\rvert = \lvert\bar{c}\rvert = 1$ (1 score)		
13.	(a)	(a) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-3}$	1	1
	(b)	$\bar{a}_1 = \bar{i} + 2\bar{j} + 3\bar{k}$ $\bar{a}_2 = \bar{i} + \bar{j} + \bar{k}$	1	
		$\bar{b}_1 = \bar{i} + \bar{j} + \bar{k}$ $\bar{b}_2 = \bar{i} + \bar{j} + \bar{k}$.		
		$\bar{a}_2 - \bar{a}_1 = -\bar{j} - 2\bar{k}$	½	

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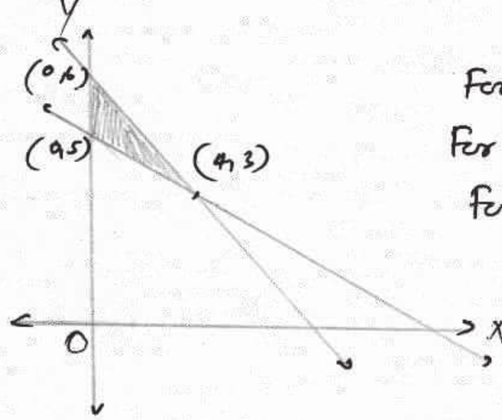
		$S.D = \left\lvert \frac{\bar{b}\times(\bar{a}_2-\bar{a}_1)}{\lvert b\rvert} \right\rvert$	$\frac{1}{2}$	3
		$= \sqrt{2}$	1	
		Remark: $SD = \left\lvert \frac{(\bar{a}_2-\bar{a}_1)\cdot(\bar{b}_1\times\bar{b}_2)}{\lvert b_1\times b_2\rvert} \right\rvert$ give ($\frac{1}{2}$ score)		
		For $SD = 0$ give (3 score)		
		For $\lvert b\rvert = \sqrt{3}$ give ($\frac{1}{2}$ score)		
14.	(a)	b $2/\sqrt{3}$	1	1
	(b)	$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$	1	
		$(2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z-5\lambda-1=0$	$\frac{1}{2}$	
		$(2\lambda+1)\cdot 1+(3\lambda+1)\times{-1}+(4\lambda+1)\times 1=0$	$\frac{1}{2}$	
		$\lambda = -\frac{1}{3}$	$\frac{1}{2}$	3
		$(x+y+z-1)-\frac{1}{3}(2x+3y+4z-5)=0$	$\frac{1}{2}$	
		$x-z+2=0$		
		Remark: Concept of $\perp^r$ planes give (1) score.		
15.	(a)	 For I Quadrant	$\frac{1}{2}$	
		For two lines	2	3
		For feasible region	$\frac{1}{2}$	

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	(b)	Corner points (0,5) (0,6) (4,3)	2	2
	(c)	at (0,5) z = 200 at (0,6) z = 240 at (4,3) z = 320 ←	1	1
		z is max^m at (4,3) and it is 320		
		Remarks : For any two correct corner points (2) score : max^m according to the obtained corner points give (1) score		
16.	(a)	(a) $\frac{P(A)}{P(B)}$	1	1
	(b)(i)	$P(E_1) = \quad P(E_2) = \frac{1}{2}$	½	
		$P(A/E_1) = 5/9 \quad P(A/E_2) = 3/7$	½	2
		P (the ball drawn is red)		
		$= \frac{1}{2} \times \frac{5}{9} + \frac{1}{2} \times \frac{3}{7}$	½	
		$= \frac{62}{126} = \frac{31}{63}$	½	
	(ii)	P (the ball was drawn from bag I)		
		$= \dfrac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{5}{9} + \frac{1}{2} \times \frac{3}{7}}$	1	2
		$= \frac{27}{62}$	1	
		Remark : $P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)$ (½) score.		

$P(E_1/A) = \dfrac{P(E_1) \times P(A/E_1)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)}$ (½) score.

(10/10)

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		<u>OR.</u>		
16.	(i)	$\sum P(x) = 1$	½.	1
		$k = \frac{7}{16}$	½.	
	(ii)	$E(x) = \sum x\, P(x)$	1	
		$= 0 + \frac{2}{16} + \frac{14}{16} + \frac{15}{16} + \frac{4}{16}$	½	
		$= \frac{35}{16}$	½	
		$E(x^2) = \sum x^2\, P(x)$	½	
		$= 0 + \frac{2}{16} + \frac{28}{16} + \frac{45}{16} + \frac{16}{16}$		
		$= \frac{91}{16}$	½	
		Variance $= \sum x^2 P(x) - \left(\sum x\, P(x)\right)^2$	½	
		$= \frac{91}{16} - \left(\frac{35}{16}\right)^2$		4.
		$= \frac{231}{256}$	½	
		$= \underline{0.902}$		