ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2022.

PART-I/II/III

SUBJECT: Mathematics_ Commerce 80

CODE NO: SY 57 VERSION: S

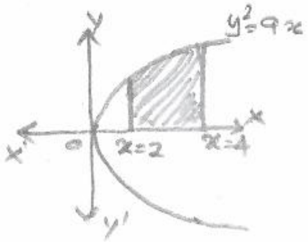
80 SCORES 2½ HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1.		(C) $(8,6) \in R$.	1	1
2.		(B) $\frac{\pi}{3}$	1	1
3.		(C) $m = n$	1	1
4.		$\int_a^6 y\,dx$	1	1
5.		degree = 3	1	1
6.		$\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+1}{3}$	1	1
7.		(C) $k^3 \cdot \lvert A \rvert$	1	1
8.		(B) $a^x \cdot \log a$.	1	1
9		(A) 1, 0, 0	1	1
10		$P(E \cap F) = \frac{1}{6}$	1	1
11		$f \circ g(x) = f(g(x))$ $= f(x^{1/3}) = 8(x^{1/3})^3 = 8x$. $g \circ f(x) = g(f(x))$ $= g(8x^3) = (8x^3)^{1/3} = 2x$.	1 1	2.
12.		$R'(x) = 6x^2 + 36$ Marginal Revenue when $x = 15$ $= 6 \times 15^2 + 36 = 126$	1 1	2

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
13.		$u = x^2 \quad du = 2x\,dx$ $\int \frac{2x}{1+x^4}\,dx = \int \frac{1}{1+u^2}\,du$ $= \tan^{-1} u + c$ $= \tan^{-1} x^2 + c.$	 1 1	2
14.		$A(3,8) \quad B(-4,2) \quad C(5,1)$ Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ $= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$ $= \frac{1}{2}[3(2-1) - 8(-4-5) + 1(-4-10)]$ $= \frac{1}{2}[3 + 72 - 14]$ $= \frac{61}{2}$ sq. units	 1 1	2
15		$\vec{r} = \vec{a} + \lambda\vec{b} \quad \vec{a} = -i + 0j + 2k \quad \vec{b} = 3i + 4j + 6k$ $\vec{r} = (-i + 0j + 2k) + \lambda(3i + 4j + 6k)$	1 1	2
16.		$\tan^{-1}[2\cos 2(\sin^{-1} \frac{1}{2})]$ $= \tan^{-1}[2\cos 2 \times \frac{\pi}{6}]$ $= \tan^{-1}[2 \times \cos \pi/3]$ $= \tan^{-1}(2 \times \frac{1}{2})$ $= \tan^{-1}(1) = \frac{\pi}{4}$	1 1	2
17		Let $A = I \cdot A$ $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ $R_2 \to R_2 - R_1 \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} A$	 1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$R_1 \to R_1 - R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} A$ $\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$	1	2
18		$\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx = 2 \cdot \int_0^{\pi/2} \sin^2 x \, dx$ $= 2 \cdot \int_0^{\pi/2} \frac{1 - \cos 2x}{2}$ $= \left[x - \frac{\sin 2x}{2} \right]_0^{\pi/2}$ $= \frac{\pi}{2} - 0 = \frac{\pi}{2}$	1 1	2
19.	a)	$f(x) = 4x+3$ $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3$ $\Rightarrow 4x_1 = 4x_2$ $\Rightarrow x_1 = x_2$ $\therefore f$ is 1-1. Let $y = 4x+3 \Rightarrow 4x = y-3$ $x = \frac{y-3}{4}$ $\forall y \in R$ Let $\therefore f(x)$ is Onto $\therefore f$ is both 1-1 and Onto. Hence f is bijective	1 1	4
	b	Let $g(x) = \frac{x-3}{4}$. $f \circ g(x) = f(g(x)) = 4\left(\frac{x-3}{4}\right) - 3 = x$ $g \circ f(x) = g(f(x)) = \frac{4x+3-3}{4} = \frac{4x}{4} = x$ $\therefore f^{-1} = g(x) = \frac{x-3}{4}$.	1 1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
20	a	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
	b	$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\left(\frac{1/2 + 2/11}{1 - \frac{1}{2}\times\frac{2}{11}}\right)$	1	4.
		$= \tan^{-1}\left(\frac{11+4}{22-2}\right)$	1	
		$= \tan^{-1}\left(\frac{15}{20}\right) = \tan^{-1}\frac{3}{4}$	1	
21.		$f(x) = 2x^3 - 3x^2 - 36x + 7$		
		$f'(x) = 6x^2 - 6x - 36$	1	
		$f'(x) = 0 \Rightarrow 6(x^2 - x - 6) = 0$		
		$\Rightarrow 6(x-3)(x+2) = 0$	1	
		$\Rightarrow x = 3 \quad x = -2$		4
		In $\cancel{(-\infty, 2)}$ $(-\infty, -2)$ $f'(x) > 0$	1	
		In $(-2, 3)$ $f'(x) < 0$		
		In $(3, +\infty)$ $f'(x) > 0$	1	
		Strictly Increasing on $(-\infty, -2) \cup (3, +\infty)$		
		Strictly Decreasing on $(-2, 3)$		
22.	a)	$\int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left\lvert\frac{x-a}{x+a}\right\rvert + c$		
		$\therefore \int \frac{1}{x^2-16}dx = \frac{1}{2\times4}\log\left\lvert\frac{x-4}{x+4}\right\rvert + c$	1	
		$= \frac{1}{8}\cdot\log\left\lvert\frac{x-4}{x+4}\right\rvert + c$	1	
	b)	$\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$		4.
		Put $u = \tan^{-1}x$		
		$du = \frac{1}{1+x^2}dx$	1	
		when $x=0$ $u=0$		
		$x = 1$ $u = \pi/4$.		
		$\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx = \int_0^{\pi/4} u\,du = \left[\frac{u^2}{2}\right]_0^{\pi/4} = \frac{\pi^2}{32}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
23	a	Required Area 		
		$= \int_2^4 y\, dx$	1	
		$= \int_2^4 3\sqrt{x}\, dx$	1	4
		$= 3 \cdot \left[\frac{x^{3/2}}{3/2}\right]_2^4 = 2 \cdot \left[x^{3/2}\right]_2^4$	1	
		$= 2.(8-2\sqrt{2}) = 16-4\sqrt{2}$	1	
24	a)	$a*b = \frac{ab}{4}$ $b*a = \frac{ba}{4} = \frac{ab}{4} = a*b$ $\therefore *$ is commutative	2	
	b)	$(a*b)*c = \left(\frac{ab}{4}\right)*c = \frac{abc}{16}$. $a*(b*c) = a*\left(\frac{bc}{4}\right) = \frac{abc}{16}$ $\therefore (a*b)*c = a*(b*c)$ $*$ is associative	1	4
	c.	Let $a*e = a \Rightarrow \frac{ae}{4} = a$ $\Rightarrow e = 4$. $\therefore$ Identity element $= 4$.	1	
25	~~25~~	Let $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ $R_2 \to R_2 - R_1 \quad R_3 \to R_3 - R_1$ $= \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$	2	
		$= \begin{vmatrix} (b-a) & c(a-b) \\ (c-a) & b(a-c) \end{vmatrix}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$= (a-b)(b-a) \cdot \begin{vmatrix} -1 & c \\ 1 & -b \end{vmatrix}$ $= (a-b)(c-a)(b-c)$	1	4
26		$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ $A \cdot X = B$		
		$\lvert A \rvert = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix}$	1	
		$= 3(2-3) + 2(4+4) + 3(-6-4)$ $= -17 \neq 0.$	1	6
		Now $A^{-1} = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	2	
		$X = A^{-1} \cdot B = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ $= \frac{-1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $x = 1 \quad y = 2 \quad z = 3.$	2	
27		When $x < 2$ $f(x) = x^3 - 3$, a polynomial $\therefore$ Continuous when $x < 2$. when $x > 2$ $f(x) = x^2 + 1$, polynomial $\therefore$ continuous when $x > 2$.	1	
		Now $\lim_{x \to 2^-} f(x) = \lim_{x \to 2} x^3 - 3 = 5$ $\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (x^2 + 1) = 5$ $f(5) = 5$ $\therefore \lim_{x \to 2} f(x) = \cancel{f(2)}\ f(2)$ $\therefore$ Continuous at $x = 2$.	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
	b)	$\frac{d}{dx}(\cos\sqrt{x}) = -\sin\sqrt{x}\cdot\cancel{\frac{1}{2\sqrt{x}}}\times\frac{d}{dx}(\sqrt{x})$		
		$= -\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$	2	
		$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$		6.
	c)	$x^2 + xy + y^2 = 100$		
		Differentiating w.r.to x,		
		$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0.$	1	
		$(x+2y)\frac{dy}{dx} = -(2x+y)$		
		$\frac{dy}{dx} = -\frac{(2x+y)}{(x+2y)}$	1	
28.	a)	$y = mx$ —(1)		
		$\frac{dy}{dx} = m$		
		Using in (1), $y = \frac{dy}{dx}.x$, is the required differential equation.	2	
	b)	$\frac{dy}{dx} = (1+x^2).(1+y^2)$; variable Separable form.		6.
		$\therefore \frac{dy}{1+y^2} = (1+x^2)\,dx.$		
		Integrating,	4	
		$\int\frac{1}{1+y^2}\,dy = \int(1+x^2)\,dx$		
		$\tan^{-1}y = x + \frac{x^3}{3} + c.$		

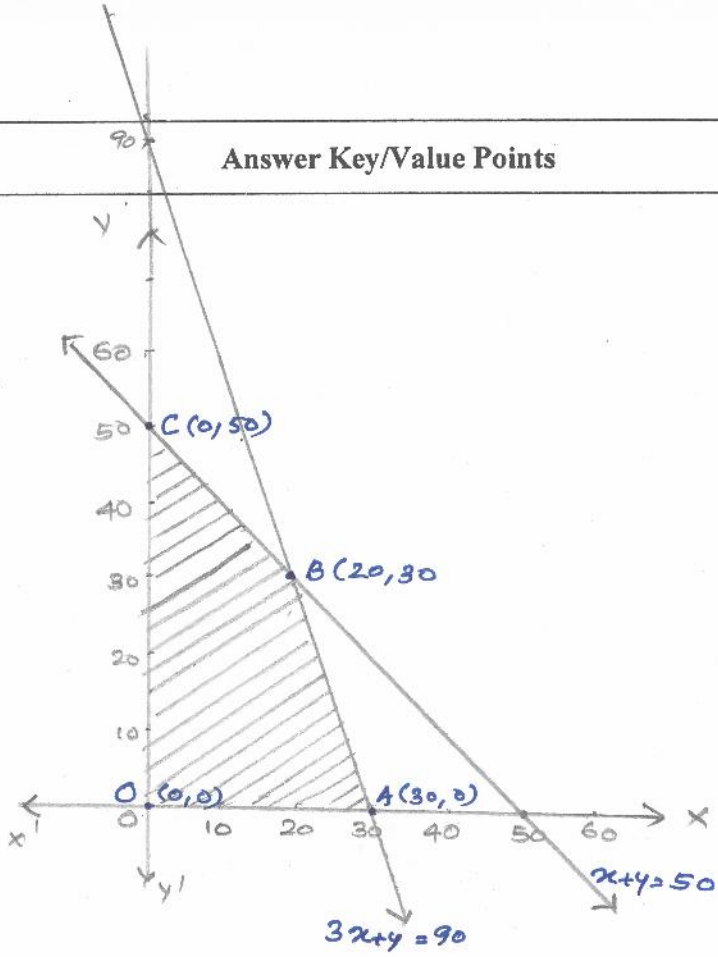
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
29	a)	$\vec{a}+\vec{b} = 2\hat{i}+3\hat{j}+4\hat{k}$ $\vec{a}-\vec{b} = 0\hat{i}-\hat{j}-2\hat{k}$	2	
	b)	$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 2\times0 + 3\times\bar{1} + 4\times\bar{2}$ $= 0-3-8 = -11.$	2	6.
	c)	$(\bar{a}+\bar{b})\times(\bar{a}-\bar{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$ $= (-6+4)\hat{i} - (-4-0)\hat{j} + (-2-0)\hat{k}.$ $= -2i+4j-2k.$	2	
30.	a)	$x^y = y^x$ Taking log on both sides. $y\log x = x\log y$ Differentiating w r to x. $y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx} = x\cdot\frac{1}{y}\frac{dy}{dx} + \log y\cdot 1$ $\left[\log x - \frac{x}{y}\right]\frac{dy}{dx} = \log y - \frac{y}{x}$ $\frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$ $= \frac{y\,[x\log y - y]}{x\,[y\log x - x]}$	3	
	b)	$y = \sin^{-1} x$ $\therefore$ ~~$\sin y$~~ ~~$= x$~~ Differentiating w r to x		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$. $\sqrt{1-x^2} \cdot \frac{dy}{dx} = 1$ D. w r to x again. $\sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot -2x = 0$. $(1-x^2) \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \times -x = 0$. $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.	3	6.
31.	a)	Let $y = \sqrt{x}$ and let $x = 25$ and $\Delta x = 0.3$. Then $\Delta y = \sqrt{x+\Delta x} - \sqrt{x}$ $= \sqrt{25.3} - \sqrt{25} = \sqrt{25.3} - 5$ $\therefore 5 + \Delta y = \sqrt{25.3}$. Now $\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$ or $\Delta y \approx \left(\frac{dy}{dx}\right) \cdot \Delta x$ $\therefore \sqrt{25.3} \approx 5 + \frac{dy}{dx} \cdot \Delta x$ $\approx 5 + \frac{1}{2\sqrt{x}} \cdot \Delta x$ $\approx 5 + \frac{1}{2\sqrt{25}} \cdot 0.3$ $\approx 5 + 0.03$ ≈ 5.03	3	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
	b).	$P(x) = 41 + 72x - 18x^2$ $p'(x) = +72 - 36x$ $p''(x) = -36 < 0$. Now $p'(x) = 0 \Rightarrow 36x = +72$ $x = +2$. at $x = +2$ $p''(x) < 0$ $\therefore P(x)$ is Maximum when $x = -2$ $\therefore$ Maximum profit $= P(+2)$ $= 41 + 72 \times +2 - 18 \times (2)^2$ $= 113$ units	3	6.
32.	a)	$\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{\lvert\vec{b_1}\rvert \cdot \lvert\vec{b_2}\rvert}$ $\vec{b_1} = i - j + k$. $\vec{b_2} = 2i + j + 2k$. $\vec{a_1} = \hat{i} + 2\hat{j} + k$ $\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$ $= \frac{2 - 1 + 2}{\sqrt{1+1+1}\ \sqrt{4+1+4}}$ $= \frac{3}{\sqrt{3} \cdot \sqrt{9}} = \frac{1}{\sqrt{3}}$ $\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.	2	6.
	b)	$SD = \left\lvert \frac{(\vec{a_2} - \vec{a_1}) \cdot (\bar{b_1} \times \bar{b_2})}{\lvert \bar{B_1} \times \bar{b_2} \rvert} \right\rvert$ $\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 0\hat{j} + 3\hat{k}$ $SD = \left\lvert \frac{(i - 3j - 2k) \cdot (-3i + 0j + 3k)}{\sqrt{9+0+9}} \right\rvert$ $= \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$.	4	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
33.	a)	$A = \begin{bmatrix}1 & 0\\ 3 & 2\end{bmatrix}$ $\quad a_{ij} = 2i-j$	2	
	b)	$A = \begin{bmatrix}6 & -2 & 2\\ -2 & 3 & 4\\ -2 & -1 & 3\end{bmatrix}$ $\quad A' = \begin{bmatrix}6 & -2 & -2\\ -2 & 3 & -1\\ 2 & 4 & 3\end{bmatrix}$ Let $P = \frac{1}{2}.(A+A')$ $= \frac{1}{2}.\begin{bmatrix}12 & -4 & 0\\ -4 & 6 & 3\\ 0 & 3 & 6\end{bmatrix}$ Let $Q = \frac{1}{2}(A-A')$ $= \frac{1}{2}\begin{bmatrix}0 & 0 & 4\\ 0 & 0 & 5\\ -4 & -5 & 0\end{bmatrix}$ Now $P' = P$ ~~and~~ $\therefore P$ is Symmetric $Q' = -Q$ $\quad \therefore Q$ is Skew symmetric Also $P+Q = \frac{1}{2}\begin{bmatrix}12 & -4 & 0\\ -4 & 6 & 3\\ 0 & 3 & 6\end{bmatrix} + \frac{1}{2}\begin{bmatrix}0 & 0 & 4\\ 0 & 0 & 5\\ -4 & -5 & 0\end{bmatrix}$ $= \begin{bmatrix}6 & -2 & 2\\ -2 & 3 & 4\\ -2 & -1 & 3\end{bmatrix}$ $= A.$ $\therefore A$ is the Sum of P and Q where P is Symmetric and Q is skew Symmetric.	6	8

<table>
<tr><th>Qn. No</th><th>Sub Qns</th><th>Answer Key/Value Points</th><th>Score</th><th>Total Score</th></tr>
<tr><td>34.</td><td></td><td>



Shaded region is the feasible region

<table>
<tr><th>Corner Points</th><th>Value of Z.</th></tr>
<tr><td>(0,0)</td><td>Z = 0</td></tr>
<tr><td>(30,0)</td><td>Z = 120</td></tr>
<tr><td>(20,30)</td><td>Z = 110</td></tr>
<tr><td>(0, 50)</td><td>Z = 50</td></tr>
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Z is Maximum when $x = 30, y = 0$.

and the maximum value of Z is 120.

</td><td>8</td><td>8</td></tr>
<tr><td>35</td><td>a)</td><td>

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{7}{13} + \frac{9}{13} - \frac{4}{13}$$
$$= \frac{12}{13}$$

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Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$ $= \dfrac{4/13}{9/13} = \dfrac{4}{9}$	3	
	b).	$\sum_{i=1}^{n} P(k) = 1 \Rightarrow 0.1 + k + 2k + 2k + k = 1$ $\Rightarrow k = 0.15$ $P(X < 3) = P(X=0) + P(X=1) + P(X=2)$ $= 0.1 + k + 2k.$ $= 0.1 + 3k$ $= 0.1 + 3 \times 0.15$ $= 0.55$ $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$ $= 2k + 2k + k$ $= 5k$ $= 5 \times 0.15$ $= 0.75$	5	8