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MATHEMATICS

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Pariharabodhanam

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പ്രിയ വിദ്യാർത്ഥികളേ,

തിരുവനന്തപുരം നഗരസഭാ പരിധിയിലെ സ്കൂളുകളിൽ പഠിക്കുന്ന വിദ്യാർത്ഥികളുടെ പഠന നിലവാരം വർധിപ്പിക്കുന്നതിനായി നഗരസഭ നടപ്പിലാക്കുന്ന പദ്ധതിയാണ് 'പരിഹാരബോധനം'. മുൻ വർഷങ്ങളിൽ നടത്തിവന്നിരുന്ന പദ്ധതി ഈ വർഷവും വിപുലമായ നിലയിൽ നടപ്പിലാക്കുകയാണ്. പഠനത്തിൽ പിന്നാക്കം നിൽക്കുന്ന വിദ്യാർത്ഥികളെ കൂടുതൽ കരുതൽ നൽകി മുന്നിലേയ്ക്ക് ഉയർത്തുകയെന്നതാണ് നഗരസഭ ഈ പദ്ധതിയിലൂടെ ഉദ്ദേശിക്കുന്നത്. പൊതുവിദ്യാഭ്യാസ രംഗം കൂടുതൽ കരുത്താർജ്ജിച്ച് മുന്നേറുന്ന ഈ കാലഘട്ടത്തിൽ വിദ്യാർത്ഥികൾക്ക് ഗുണമേന്മയുള്ള വിദ്യാഭ്യാസം ഉറപ്പാക്കുന്നതിനും വിവിധ തലങ്ങളിൽ മികവ് തെളിയിക്കാനുള്ള അവസരമൊരുക്കുന്നതിനും സർക്കാരും നഗരസഭയും പ്രതിജ്ഞാബദ്ധമാണ്. അക്കാദമികവും ഭൗതികവുമായ സൗകര്യങ്ങൾ കൂടുതൽ മെച്ചപ്പെട്ട് കേരളത്തിലെ പൊതുവിദ്യാഭ്യാസ രംഗം ശ്രദ്ധേയമായ മാതൃകയായി മാറിയിരിക്കുകയാണ്. ഈ സന്ദർഭത്തിൽ നമ്മുടെ വിദ്യാർത്ഥികൾക്ക് ഉന്നത പഠനത്തിന് ഉപകരിക്കുന്ന തരത്തിൽ പഠന നിലവാരം മെച്ചപ്പെടുത്തുക എന്നതാണ് നാം ലക്ഷ്യമിടുന്നത്. മികച്ച അധ്യാപകരുടെ സഹായത്തോടെ പഠനം അസ്വാദ്യകരമാക്കി മാറ്റിക്കൊണ്ട് കുട്ടികളെ മികച്ച നിലാരത്തിലേയ്ക്ക് ഉയർത്തുകയെന്ന ലക്ഷ്യത്തിന്റെ സാധൂകരണം കൂടിയാണ് പരിഹാരബോധനം എന്ന ബൃഹത് പദ്ധതി. ഈ പദ്ധതിയുടെ ഭാഗമാകുന്ന എല്ലാ പ്രിയപ്പെട്ട വിദ്യാർത്ഥികൾക്കും അഭിനന്ദനങ്ങൾ അറിയിക്കുന്നതോടൊപ്പം മികച്ച വിജയം ആശംസിക്കുന്നു.

സ്നേഹത്തോടെ

ആര്യ രാജേന്ദ്രൻ എസ്.
മേയർ
തിരുവനന്തപുരം നഗരസഭ

പ്രിയപ്പെട്ട കുട്ടികളേ,

തിരുവനന്തപുരം നഗരസഭാ പരിധിയിൽ വരുന്ന ഹൈസ്കൂൾ, ഹയർസെക്കന്ററി വിഭാഗം കുട്ടികളുടെ പഠനനിലവാരം ഉയർത്താനും പൊതുപരീക്ഷയിൽ ഉയർന്ന ഗ്രേഡ് കരസ്ഥമാക്കാനും ലക്ഷ്യമിട്ടുകൊണ്ട് മുൻവർഷങ്ങളെപ്പോലെ പരിഹാരബോധനം പദ്ധതി ഈ വർഷവും നടപ്പിലാക്കിവരുന്നതിൽ അതിയായ സന്തോഷവും അഭിമാനവും ഉണ്ട്. ഈ വർഷത്തെ പൊതുപരീക്ഷയ്ക്ക് നേരത്തെതന്നെ തയ്യാറെടുക്കുന്നതിനും എല്ലാ വിഷയങ്ങളിലെ പാഠഭാഗങ്ങളിലൂടെ ആവർത്തിച്ചുകടന്നുപോകാനും പരിചയപ്പെടാനും സാധിക്കട്ടെ എന്ന് ആശംസിക്കുന്നു.

ഡോ.നീന കെ.എസ്.
ചെയർപേഴ്സൺ
(വിദ്യാഭ്യാസ കായിക സ്റ്റാന്റിംഗ് കമ്മിറ്റി)
തിരുവനന്തപുരം കോർപ്പറേഷൻ

Members participated in the workshop

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2. **Sri. Sureshkumar. T**
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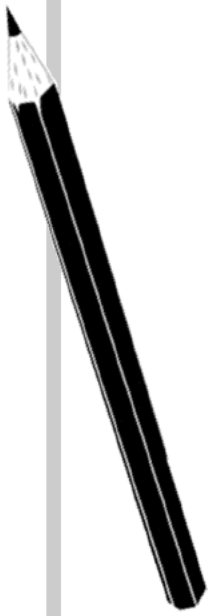

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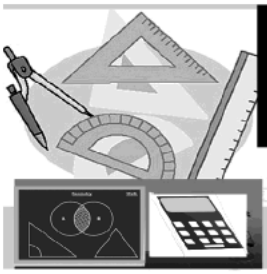
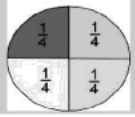
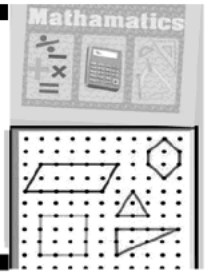


Unit
1

ARITHMETIC SEQUENCES





Points Remember

1. A set of numbers written in order as the first, second, third and so on based on a specific rule is called a number sequence.

   eg: Sequence of squares of natural numbers 1, 4, 9, 16, ...

2. The algebraic form of a sequence is the relation between position and term.

   eg: In the sequence of squares of natural numbers, each term is the square of position. If 'n' is the position,

   $x_n = n^2$, is the algebraic form.

3. A sequence got by starting with any number and adding a fixed number repeatedly is called an arithmetic sequence..

   eg. 1. The sequence of multiples of 3 ie, 3,6,9,12, ...

   2. The sequence of natural numbers leave a remainder 1 on divison by 5 1,6, 11, 16, 21 ...

4. In an arithmetic sequence, if we subtract any term from the term immediately after it, we get the same number. This fixed number is called the common difference.

5. In an arithmetic sequence, the difference between any two terms is the product of the position difference of that terms and the common difference.

   OR

   The difference between any two terms of an arithmetic sequence is a multiple of the common difference.

6. If the terms of an arithmetic sequence are natural numbers, we get the same remainder on division by the common difference.

   Eg. In the arithmetic sequence 4, 7, 10, 13... we get 1 as the remainder on division by 3 for any term.

7. In an arithmetic sequence, term difference divided by position difference, gives the common difference.

   Eg. In the arithmetic sequence 8, 13, 18, 23... $\frac{23-13}{4-2} = \frac{10}{2} = 5$.

8. The difference of any two terms of an arithmetic sequence divided by the common difference gives the position difference and one more than the position difference is the number of terms.

   Eg. In the arithmetic sequence 3, 7, 11, 15, ..., 99.

Number of terms = $\frac{99-3}{4}+1=\frac{96}{4}+1=24+1=25$.

9. Arithmetic Sequences are got by multiplying natural numbers from 1 by a fixed number and then adding a fixed number.

10. If the first term of an arithmetic sequence is 'f' and common difference is 'd' then $x_n = f+(n-1)d$

or

$x_n = dn+(f-d)$

11. Algebraic form of an arithmetic sequence is $x_n = an+b$ where 'a' is the common difference and 'a + b' is the first term.

Eg. In $x_n = 3n+2$. First term is 5 and the common diffence is 3.

12. If the number of terms of an arithmetic sequence is odd, then sum of terms is the product of the number of terms and the middle term.
Eg. Three consecutive terms of an arithmetic sequence are 6, 9, 12, then sum = 3 × 9 = 27

Five consecutive terms of an arithmetic sequence are 8, 14, 20, 26, 32,

then sum = 5 × 20 = 100.

13. In an arithmetic sequence, if the sum of positions of two pairs of terms are equal, then the sums of the pairs of the terms are also equal.
eg: 7 + 12 = 5 + 14 = 3 + 16 = 1 + 18 = ...

So in the arithmetic sequence $x_1, x_2, x_3, x_4, \ldots$, we have

$x_7 + x_{12} = x_5 + x_{14} = x_3 + x_{16} = x_1 + x_{18} = \ldots.$

14. Sum of consecutive natural numbers starting with 1 is half the product of the last number and the next number. Algebraically

$1+2+3+\ldots+n=\frac{1}{2}n(n+1)$

Eg. $1+2+3+\ldots+50=\frac{1}{2}\times 50\times 51=25\times 51=1275$

15. . For the arithmetic sequece $x_n = an+b$, the sum of first 'n' terms is

$S_n = a\times\frac{n(n+1)}{2}+bn$

16. The sum of consecutive terms of an arithmetic sequence is half the product of the number of terms and the sum of the first and last terms.

Algebraically, $x_1+x_2+x_3+\ldots+x_n=\frac{1}{2}n(x_1+x_n)$

17. Sum of consecutive terms of an arithmetic sequence is in the form $s_n = pn^2+qn$, where 2p is the common difference and

p + q is the first term.

18. For two arithmetic sequences with same common difference,

- Difference of terms in the same position are equal.
- Difference of the sum of '*n*' terms is *n* × difference of terms in the same position.

Work Sheet - 1

1. 13, 17, 21, ... is an arithemetic sequence.

   a) What is its 11th term?

   b) Write the algebraic form of the sequence

   c) Check whether 249 a term of the sequence

   a) First term of F = ☐

   Common difference of d = ☐

   11th term = dn + ☐ – ☐

   = 4×☐+(☐ – ☐)

   = ☐

   b) Algebraic form is in the form

   xn = ☐n + ☐

   xn = ☐$n + 9$

   c) Each term of the sequence give remainder ☐ when divided by 4 when 249 is divided by 4 the remainder is ☐

   Thus 249 is a term in the sequence Yes ☐ / No☐

Work Sheet - 2

2. The table given below is based on arithmetic sequences. Fill up the empty columns

First term difference	Common Sequence	Arithmetic	10th term	Algebraic form
6	11			
		3, 7, 11, 15,...		
3			39	
7				5n + 2
	7			7n + 3
				6n – 1
	5		50	

Work Sheet- 3

What is the sum of first 20 terms of the arithmetic sequence 5,8,11,.....

First term = ☐ Common difference = ☐

Algebraic form of the sequence

$xn = \square \times n+2$

Algebraic form of sum of first n terms

$Sn = ½ \square\ n(n+1) + \square \times n$

Sum of 20 terms $= \square \times \square \times \square + \square \times \square$

$= \square + \square = \square$

Work Sheet - 4

2. Fill up the following table

Algebraic form (*X*n)	**Common difference**	**First term (f)**	**Arithmetic Sequence**	**10^{th} term (X_{10})**
3n+ 2	3	3+2 = 5	5, 8, 11, ...	X_{10}= 3×10+2 = 30 + 2 = 32
4n + 3				
5n - 4				
3n - 2				
10 n				

Work Sheet - 5

3. Two terms of some arithemeic sequences are given. Fill up the following table

10^{th} term	**15^{th} term**	**d = $\frac{\text{Term difference}}{\text{Position difference}}$**	**First term (f)**	**Algebraic form**
32	47	d = $\frac{47-42}{15-10} = \frac{15}{5}$ = 3	f = $X_1 = X_{10} - 9d$ = 32 – 9 × 3 = 32 – 27 = 5	Xn = dn + f –d = 3n + 5 – 3 = 3n + 2
71	106			
10	30			
50	80			

Work Sheet - 6

Complete the following table.

Sl. No.	Consecutive terms in the Arithmetic sequence	Number of terms	Middle term	Number term ×middle term	Sum of the terms
1	1, 2, 3	3	2	3×2=6	1+2+8=6
2	2, 3, 4				
3	1, 3, 5				
4	5, 8, 11				
5	$x-1, x, x+1$				
6	$x-y, x, x+y$				
7	1, 2, 3, 4, 5				
8	$x-2y, x-y, x, x+y, x+2y$				

Work Sheet - 7

3, 6, 9, 12, 15, 18, 21, 24, 27 are consecutive terms in an arithmetic sequence.

Number of terms = ...

Middle term = ...

If we add the terms which are at same distance from the two ends, then

$x_1 + x_9 = 3 + 27 = 30$

$x_2 + x_8 = ... + ... =$

$x_3 + x_7 = ... + ... =$

$x_4 + x_6 = ... + ... =$

a) Identify the relation among sums of each pair?

b) How the sum is related to the middle term?

Work Sheet - 8

The sum of 4^{th} and 6^{th} terms in an arithmetic sequence is 20

a) Find the sum of its 1^{st} and 9^{th} terms?

b) Write any two pairs of terms having the same sum?

c) Find its 5^{th} term?

d) If 3^{rd} term is 7, then find its 7^{th} term?

Work Sheet - 9

a) $1+2+3+4+....+20 = \frac{20\times 21}{2} = ...$

b) $2+4+6+.....+50 = 25 \times(25+1) = ...$

c) $1+3+5+ ..... + 29 = 15^2 = ...$

Find the following sums

a) $1+2+3+4+ ...+ 50$

b) $2+4+6+8+... + 40$

c) $1+3+5+7+...+19$

Work Sheet - 10

a) $1+2+3+4+...+15 =$ _____

b) $3+6+9+12+.... + 45 = 3\ (1+2+3+....+15)$

$= 3\times ..... = ...$

c) $5+8+11+14+...+47 = \frac{3\times 15\times 16}{2}+15\times 2$

= _____ + _____ =_____

Find the following sums.

a) $1+2+3+4+....+30$

b) $8+16+24+32+...+240$

c) $9+17+25+....+241$

Work Sheet - 11

Complete the following table.

Algebraic form (Xn)	First term (f)	Common difference (d)	Sum of first n terms (S_n)	Sum of first 10 terms (S_{10})
3n + 2	3+2 = 5	3	$Sn = \frac{3n(n+1)}{2} + 2n$ $= \frac{3n^2+3n}{2} + 2n$ $= \frac{3n^2}{2} + \frac{3n}{2} + 2n$ $= \frac{3n^2}{2} + \frac{7n}{2}$	$S_{10} = \frac{3\times10\times11}{2} + 2\times10$ $= 3\times5\times11+20$ = 165 + 20 = 185
6n +4				
10n – 3				
7n + 1				

Work Sheet - 12

Complete the following table.

Arithmetic Sequence	First term (f)	Common difference (d)	Sum of first 10 terms (Sn)
5, 8, 11,.....	5	3	$S_n = \frac{n}{2}[2f + (n-1)d]$ $S_{10} = \frac{10}{2}[2\times5 + (10-1)3]$ = 5 (10 + 9 ×3) = 5 (10 + 27) = 5 ×37= 185
12, 23, 34, ...			
15, 22, 29, ...			
10, 16, 22, ...			

Work Sheet - 13

Complete the following table

Algebraic form of the sum of arithmetic sequence	**First term (f)**	**Common difference (d)**	**Sum of first 10 terms (S_{10})** **10th term X_{10} = f+ 9d**
$3n^2+2n$	3+2 = 5	2×3 = 6	$S_{10}=3\times10^2+2\times10$ $X_{10}=5+9\times6$ = 3×100+20 = 5 + 54 = 300 + 20 = 59 = 320
$2n^2+5n$			
n^2+n			
$5n^2+4n$			

Work Sheet - 14

Complete the following table.

Arithmetic Sequences	**Difference of first terms**	**Number of terms**	**Difference of sums**
4, 7, 10, ...	15 – 4 = 11	20	20 × 11 = 220
15, 18, 21, ...			
1, 6, 11, ...			
7, 12, 17, ...			
5, 13, 21, ...			
12, 20, 28, ...			
21, 27, 33, ...			
11, 17, 23, ...			

Work Sheet - 17

Look at the following Pattern

1

2 3

4 5 6

---

a) Next two lines are

7 ☐ ☐ ☐

☐ ☐ ☐ ☐ ☐

b) Last number in the 4^{th} line = ☐ = 1 + 2+ 3 + ☐

c) Last number in the 9^{th} line = 1 + 2+3+ ..... + ☐ = ☐

d) First number in the 10^{th} line = ☐

e) Last number in the 10^{th} line = ☐

f) How many numbers in the 10^{th} line?

g) Sum of the numbers in the 10^{th} line = $\frac{10}{2}$[___+___] = ☐

Practice Problems

1. a. Write an arithmetic sequence with common difference 4.
   b. What is the 15^{th} term of this sequence.
   c. Is 145, the difference of any two terms of this sequence? Why?
2. a. Write the algebraic form of the arithmetic sequence 5,8,11...
   b. Write the 8^{th} and 24^{th} term of this sequence.
3. The 8^{th} term of an arithmetic sequence is 25 and 16^{th} term is 49.
   a. What is the common difference?
   b. What is its first term?
   c. Write the algebraic form of this arithmetic sequence.
4. The algebraic form of an arithmetic sequence is $x_n = 7n + 3$.
   a. Write the first 3 terms.
   b. What is the remainder on dividing its terms by 7?
   c. Is 500, a term of this sequence? Justify.

5. Consider arithmetic sequence 6, 11, 16, ...
   a. What is the common difference?
   b. Write the algebraic from of the sequence.
   c. Find the sum of first 20 terms?
6. The sum of 2^{nd} and 30^{th} terms of an arithmetic sequence is 50.
   a. What is the sum of 1^{st} and 31^{st} terms?
   b. What is the sum of 15^{th} and 17^{th} terms?
   c. What is the 16^{th} term?
   d. What is the sum of first 31 terms?
7. Answer the following questions based on the given pattern.

$$1 = 1$$
$$1 + 3 = 4$$
$$1 + 3 + 5 = 9$$

   .....

   .....

   .....

   a. Write the next two lines?
   b. Find the sum of first 15 odd numbers?
   c. Find $\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \ldots + \frac{29}{2}$?
8. The sum of first 9 terms of an arithmetic sequence is 90.
   a. What is its 5^{th} term?
   b. Write three arithmetic sequences with 90 as the sum of first 9 terms.
9. a. Write the sequence of natural numbers between 100 and 300 which leave a remainder 2 on division by 3.
   b. How many terms are there in this sequence?
   c. Find the sum of terms of this sequence.
10. The algebraic form of the sum of first 'n' terms of an arithmetic sequence is $S_n = 2n^2 + 3n$
    a. Write the first term and common difference.
    b. Write the algebraic form of this sequence.
    c. Find the sum of first 25 terms?
11. The sum of natural numbers from 1 to 100 is 5050. Using this, find the sum of the terms of the arithmetic sequence 3, 6, 9, ... 300.
12. The algebraic form of two arithmetic sequences are $x_n = 6n + 2$ and $x_n = 6n - 2$. What is the difference of the sums of first 25 terms of these sequences.

13. Find the following sums:
    a. 1 + 2 + 3 + ... + 40
    b. 5 + 6 + 7 + ... + 44
    c. 6 +12 + 18 + ... + 240
    d. –1 + 5 + 11 + ... + 233
14. The 7^{th} term of an arithmetic sequence is 17 and 17^{th} term is 7.
    a. What is the common difference?
    b. Find the 24^{th} term?
    c. Find the sum of first 47 terms?
15. The 5^{th} term of an arithmetic sequence is 40 and 31^{st} term is 160.
    a. What is the sum of 17^{th} and 19^{th} terms?
    b. Find the 18^{th} term?
    c. Find the sum of first 35 terms?
    d. What is the sum of first 35 terms of the arithmetic sequence with 5^{th} term 43 and 31^{st} term 163?
16. The 6^{th} term of an Arithmetic sequence is 28 and 11^{th} is 63\
    a) Find the first term
    b) What is its common difference
    c) Write the algebraic form of the sequence.

ANSWERS

Work Sheet - 1

a) $f = 13$

$d = 17 - 13$

$= 4$

$x11 = dn + f - d$

$= 4 \times 11 + (13 - 4)$

$= 44 + 9$

$= 53$

b) $xn = an + b$

$xn = 4n + 9$

c) remainder = 1

249 is divided by 4, remainder is 1

$\therefore$ 249 a term of this sequence.

Work Sheet - 2

f	d	Arithmetic sequence	10^{th}term	Algebraic form
6	11	17,28,39,.....	115	11n+5
3	4	3,7,11,.....	39	4n–1
3	4	3,7,11,.....	39	4n–1
7	5	7,12,17,.....	52	5n+2
10	7	10,17,24,.....	73	7n+3
5	6	5,11,17.....	59	6n–1
5	5	5,10,15,.....	50	5n

Work Sheet - 3

First term = 5, Common difference - 3

$xn = 3n + 2$

$Sn = \frac{1}{2} \times 3n(n+1) + 2n$

$S20 = \frac{1}{2} \times 3 \times 20 \times 21 + 2 \times 20$

$= 600 + 70$

$= 670$

Work Sheet - 4

Algebraic form (Xn)	Common difference (d)	First term (f)	Arithematic Sequence	Algebraic form
3n+2	3	3+2+ = 5	5, 8, 11, ...	X_{10} = 3×10+2 = 30 + 2 = 32
4n + 3	4	4+3 = 7	7, 11, 15, ...	X_{10} = 4×10+3 = 40 + 3 = 43
5n – 4	5	5 – 4 = 1	1, 6, 11, ...	X_{10} = 5×10–4 = 50 – 4 = 46
3n – 2	3	3 – 2 = 1	1, 4, 7, ...	X_{10} = 3×10 – 2 = 30 – 2 = 28
10n	10	10 × 1 = 10	10, 20, 30,...	X_{10} =10×10 = 100

Work Sheet - 5

10^{th} term	15^{th} term	d = $\frac{\text{Term difference}}{\text{Position difference}}$	First term (f)	Algebraic form
32	47	$d = \frac{47-32}{15-10} = \frac{15}{5} = 3$	f= X_1 = X_{10} – 9d = 32 – 9 × 3 = 32 – 27 = 32 – 27 = 5	Xn = dn + f – d = 3n + 5 – 3 = 3n + 2
71	106	$d = \frac{106-71}{15-10} = \frac{35}{5} = 7$	f= X_1 = X_{10} – 9d = 71 – 9 ×7 = 71 – 63 = 8	Xn = dn + f – d = 7n + 8 –7 = 7n + 1

10	30	$d=\frac{30-10}{15-10}=\frac{20}{5}=4$	f= X_1 = X_{10} – 9d = 10 – 9× 4 = 10 – 36 = – 26	Xn = dn + f – d = 4n + (–26) – 4 = 4n – 30
50	80	$d=\frac{80-50}{15-10}=\frac{30}{5}=6$	f= X_1 = X_{10} – 9d = 50 – 9 × 6 = 50 – 54 = – 4	Xn = dn + f – d = 6n + (–4) – 6 = 6n – 10

Work Sheet - 6

Sl. No.	Consecutive terms in the arithemetic sequence	Number of terms	Middle term	Number of terms × middletrem	Sum of the terms
1.	1, 2, 3	3	2	3 × 2 = 6	1+2+3=6
2	2, 3, 4	3	3	3×3 = 9	2+3+4 = 9
3	1, 3, 5	3	3	3×3 = 9	1 + 3 + 5 = 9
4	5, 8, 11	3	8	3×8 = 24	5+8+11= 24
5	x–1, x, x+1	3	x	3×x = 3x	(x–1)+x+(x+1)=3x
6	$x-y$, x , $x+y$	3	x	3×x= 3x	(x–y)+x+(x+y)= 3x
7	1, 2, 3, 4, 5	5	3	5×3 = 15	1+2+3+4+5=15
8	x–2y, x–y, x, x+y, x+2y	5	x	5×x = 5x	(x–2y)+(x–y) +x+ (x+y) + (x+2y) = 5x

Work Sheet - 7

3, 6, 9, 12, 15, 18, 21, 24, 27

Number of terms = 9

Middle term = 15

$x_1 + x_9 = 3 + 27 = 30$

$x_2 + x_8 = 6 + 24 = 30$

$x_3 + x_7 = 9 + 21 = 30$

$x_4 + x_6 = 12 + 18 = 30$

a) In an arithmetic sequence if the sum of position of two pairs of terms are equal, then the sums of the pairs of the terms are also equal.

b) Sums are two times the middle term.

Work Sheet - 8

a) 20

b) x_2+x_8, x_3+x_7

c) 10

d) $x_3+ x_7 = 20$

$x_7 = 20 - x_3 = 20 - 7 = 13$

Work Sheet - 9

a) $1 + 2+ 3+ ...+ 20 = \frac{20\ (20+1)}{2} = \frac{20\times21}{2} = 210$

b) $2 + 4+ 6+ ... + 50 = 25\ (25+1) = 25\times26 = 650$

c) $1 + 3+ 5 + ...+ 29 = 15^2 = 225$

Sum

a) $1 + 2+3+ 4+ ...+ 50 = \frac{50\ (50+1)}{2} = \frac{50\times51}{2} = 1275$

b) $2+4+6+8+...+40) = 20\times21 = 420$

c) $1+3+5\times7+...+19 = 10^2 = 100$

Work Sheet - 10

a) $1+ 2+ 3+4+ ... +15 = \frac{15\ \times16}{2} = 15\times8 = 120$

b) $3+6+9+12+...+45 = 3\ (1+2+3+....+15) = 3\times120 = 360$

c) $5+8+11+14+....+47 = \frac{3\ \times15\times16}{2} +15\times2 = 360+30 = 390$

Sum

a) $1+2+3+4+....+30 = \frac{30\ \times31}{2} = 15\times31 = 465$

b) $8+16+24+32+...+240 = 8(1+2+3+4+.....+30) = 8\ \times\ 465 = 3720$

c) $9+17+25+...+241 = 3720 + 30\times1 = 3720+30 = 3750$

Work Sheet - 11

Algebraic form	First term	Common difference	Sum of first n terms	Sum of first 10 terms
3n + 2	3+2 = 5	3	$S_n = \frac{3n(n+1)}{2} + 2n$ $= \frac{3n^2+3n}{2} + 2$ $= \frac{3n^2}{2} + \frac{3n}{2} + 2n$ $= \frac{3}{2}n^2 + \frac{7}{2}n$	$S_{10} = \frac{3\times10\times11}{2} + 2\times10$ $= 3\times5\times11+20$ $= 165 + 20$ $= 185$
6n + 4	6+4 = 10	6	$S_n = \frac{6n(n+1)}{2} + 4n$ $= 3n\ (n+1) + 4n$ $= 3n^2 + 3n + 4n$ $= 3n^2 + 7n$	$S_{10} = \frac{6\times10\times11}{2} + 4\times10$ $= 3\times10\times11+40$ $= 330 + 40$ $= 370$
10n –3	10–3=7	10	$S_n = \frac{10n(n+1)}{2} + (-3)n$ $= 5n(n+1) - 3n$ $= 5n^2+5n-3n$ $= 5n^2 + 2n$	$S_{10} = \frac{10\times10\times11}{2} + (-3)\times10$ $= 5\times10\times11-30$ $= 550 - 30$ $= 520$
7n+1	7+1=8	7	$S_n = \frac{7n(n+1)}{2} + 1n$ $= \frac{7n^2+7n}{2} + 1n$ $= \frac{7n^2}{2} + \frac{7n}{2} + n$ $= \frac{7n^2}{2} + \frac{9n}{2}$	$S_{10} = \frac{7\times10\times11}{2} + 1\times10$ $= 7\times5\times11+10$ $= 385 + 10$ $= 395$

Work Sheet - 15

Algebraic form of the Sum of arithmatic sequence	First term (f)	Common difference (d)	Sum of first 10 term (S_{10})	10th term X_{10} = f+9d
$3n^2+2n$	3+2 = 5	2×3=6	$S_{10} = 3\times10^2 + 2\times10$ = 3×100+20 = 300 + 20 = 320	$X_{10} = 5+9\times6$ = 5+54 = 59
$2n^2+5n$	2+5 =7	2×2 = 4	$S_{10} = 2\times10^2 + 5\times10$ = 2 ×100+50 = 200+50 = 250	$X_{10} = 7+9\times4$ = 7+36 = 43
n^2+n	1+1= 2	2×1=2	$S_{10} = 10^2 + 10$ = 100 + 10 = 110	$X_{10} = 2+9\times2$ = 2 + 18 = 20
$5n^2+4n$	5+4= 9	2×5=10	$S_{10} = 5\times10^2 + 4\times10$ = 5 ×100+40 = 500 + 40 = 540	$X_{10} = 9+9\times10$ = 9+90 = 99

Work Sheet - 13

Arithmetic Sequence	Difference of First Term	Number of Terms	Difference of Sum
4, 7, 10, ...	15 – 4 = 11	20	20×11=220
15, 18, 21, ...			
1, 6, 11, ...			
7, 12, 17, ...	7 – 1 = 6	25	25×6 = 150
5, 13, 21, ...			
12, 20, 28, ...	12 – 5 = 7	30	30×7 = 210
21, 27, 33,...			
11, 17, 23, ...	21 – 11 = 10	25	25×10 = 250

Work Sheet - 14

a) 7, 8, 9, 10

11, 12, 13, 14, 15

b) 10 = 1+2+3+4

c) $1+2+3+\ldots+9 = \frac{9\times10}{2} = 45$

d) 46

e) 1 + 2+ 3+ ...+10 = 55

f) 10

g) $\frac{10}{2}(46+55) = 5\times101 = 505$

Answers

1. a. 1, 5, 9, ...; 4, 8, 12, ...; 6, 10,14 ...; are examples.
   b. $x_{15} = f + 14d$. Or $x_{15} = d\times15 + (f - d)$
   c. The difference between any two terms of this sequence is a multiple of 4. Since 145 is not a multiple of 4, it is not the difference between any two terms of this sequence.
2. a. $x_n = 3n + 2$
   b. $x_8 = 3 \times 8 + 2 = 24 + 2 = 26$
   $x_{24} = 3 \times 24 + 2 = 72 + 2 = 74$
3. a. $x_8 + 8d = x_{16}$
   $25 + 8d = 49$ OR $x_{16} - x_8 = 49 - 25$
   $8d = 49 - 25 = 24$ $8d = 24$
   $d = \frac{24}{8} = 3$ $d = \frac{24}{8} = 3$
   b. First term $x_1 = x_8 - 7d = 25 - 3 \times 7 = 25 - 21 = 4$
   c. $x_n = 3n + 1$
4. a. $x_1 = 7 \times 1 + 3 = 10$
   $x_2 = 7 \times 2 + 3 = 17$
   $x_3 = 7 \times 3 + 3 = 24$
   b. The terms of this sequence leaves a remainder 3 on division by 7.
   c. 500 leave a remainder 3 on division by 7. So 500 is a term.
5. a. Common difference = 5
   b. Algebraic from, $x_n = dn + (f - d) = 5n + 1$

c. $S_{20} = 5 \times \frac{20 \times 21}{2} + 20 \times 1$

$= 1050 + 20 = 1070$

6. a. $x_1 + x_{31} = 50$

b. Since $x_2 + x_{30} = 50$

$x_{15} + x_{17}$ is also 50.

c. Middle term is half the sum of the pair

$x_{16} = \frac{50}{2} = 25$

d. Sum of 31 terms $= 31 \times x_{16}$

$= 31 \times 25$

$= 775$

7. a. $1 + 3 + 5 + 7 = 16$

$1 + 3 + 5 + 7 + 9 = 25$

b. Sum of first 'n' consecutive odd numbers is n^2.

$\therefore$ Sum of first 15 odd numbers $= 15^2 = 225$

c. $\frac{1}{2}(1 + 3 + 5 + \ldots + 15) = \frac{1}{2} \times 225 = \frac{225}{2}$

8. a. $9 \times x_5 = 90$

$x_5 = \frac{90}{9} = 10$

b. 2, 4, 6, 8, 10, 12, 14, 16, 18

–10, –5, 0, 5, 10, 15, 20, 25, 30

–2, 1, 4, 7, 10, 13, 16, 19, 22

That is, in any arithmetic sequence with middle term 10 and any number as common difference the sum of 9 terms is always 90.

9. a. $x_1 = 101$

$d = 3$

Sequence 101, 104, 107, 110, ... 229

b. Number of terms $= \frac{299 - 101}{3} + 1$

$= \frac{198}{3} + 1 = 66 + 1 = 67$

c. $s_{67} = \frac{1}{2}n(x_1 + x_{67})$

$= \frac{1}{2} \times 67 \times (101 + 299)$

$= \frac{1}{2} \times 67 \times 400$

$= 13400$

10. a. In the algebraic form of the sum $S_n = pn^2 + qn$ of the arithmetic sequence 2p is the common difference.

$s_n = 2n^2 + 3n$

$\therefore d = 2 \times 2 = 4$

First term $x_1 = 2 + 3 = 5$

b. Algebraic form of the arithmetic sequence is $x_n = 4n + 1$

c. $s_{25} = 2 \times 25^2 + 3 \times 25$

$= 2 \times 625 + 3 \times 25$

$= 1250 + 75$

$= 1325$

11. $1 + 2 + 3 + 4 \ldots + 100 = 5050$

$3 + 6 + 9 + \ldots + 300 = 3(1 + 2 + 3 + \ldots + 100)$

$= 3 \times 5050$

$= 15150$

12. Algebraic form of first arithmetic sequence, $x_n = 6n + 2$

Algebraic form of second arithmetic sequence, $x_n = 6n - 2$

First term of the first sequence $= 8$

First term of the second sequence $= 4$

In two arithmetic sequences with same common difference, the difference between the first terms is also the difference between any terms at the same position.

Difference between first terms $= 8 - 4 = 4$

$\therefore$ Difference of sums $= 25 \times 4 = 100$

13. (a) $1 + 2 + 3 + \ldots\ldots + 40 = \frac{40 \times 41}{2} = 820$

(b) $5 + 6 + 7 + \ldots\ldots + 44 = 820 + 40 \times 4 = 980$

(c) $6 + 12 + 18 + \ldots\ldots + 240 = 6\,(1 + 2 + 3 + \ldots\ldots + 40)$

$= 6 \times 820 = 4920$

(d) $-1 + 5 + 11 + \ldots\ldots + 233 = 4920 - 40 \times 7$

$= 4640$

14. (a) Common difference $= \frac{\text{Term difference}}{\text{Position difference}}$

$= \frac{7-17}{17-7}$

$= -1$

(b) 24^{th} term $= x_{17} + 7d$

$= 7 + 7 \times (-1)$

$= 0$

(c) Sum of first 47 terms $= 47 \times 24^{th}$ term

$= 47 \times 0$

$= 0$

15. $x_5 = 40,\ x_{31} = 160$

(a) $x_{17} + x_{19} = x_5 + x_{31}$

$= 40 + 160$

$= 200$

(b) $x_{18} = \frac{200}{2} = 100$

(c) $S_{35} = 35 \times 100$

$= 3500$

(d) $x_5 = 43,\ x_{31} = 163$

Sum of 35- terms $= 3500 = 35 \times 3$

$= 3605$

16. $x_6 = 28,\ x_{11} = 63$

(a) $x_1 = x_6 - 5d$

$= 28 - 35$

$= -7$

(b) Common difference $= \frac{35}{5} = 7$

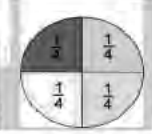
(c) Algebraic form, $x_n = dn + (f - d)$

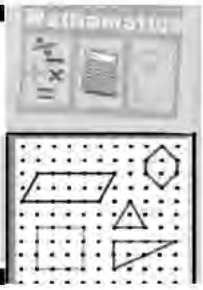
$= 7n + (-7 - 7)$

$= 7n - 14$

Unit 2

CIRCLES

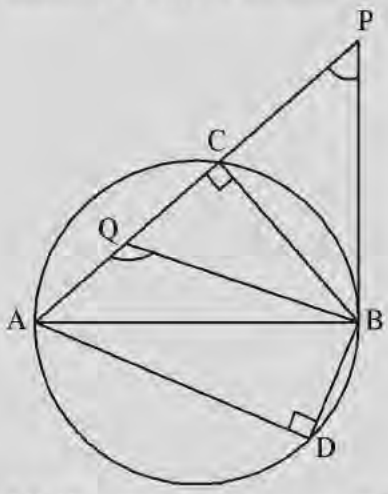




Points to Remember

- If we join the ends of a diameter of a circle to any point on the circle, we get a right angle.

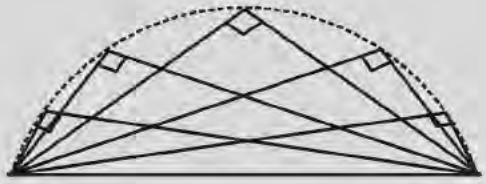
  If we join the end points of a diameter of a circle to a point inside the circle, we get an angle more than a right angle, and if the point is outside, the angle is less than a right angle.



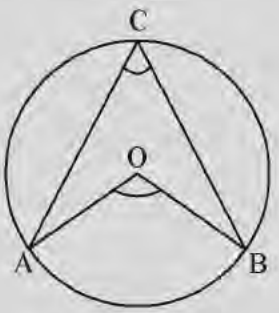
$\angle ACB = 90^\circ$, $\angle ADB = 90^\circ$

$\angle APB < 90^\circ$, $\angle AQB > 90^\circ$

- All the mutually perpendicular lines drawn from the two end points of a line will meet at the circle with the line as diameter.

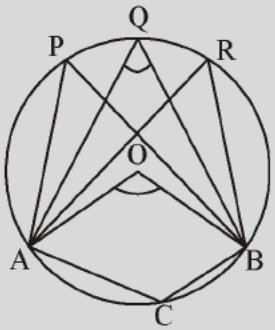


- The angle made by any arc of a circle on the alternate arc is half the angle made at the centre.



$$\angle ACB = \frac{1}{2}\angle AOB$$

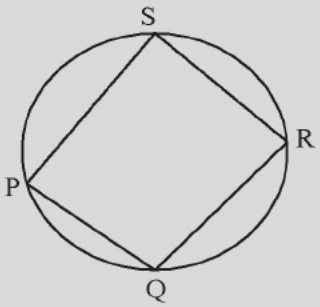
- All the angles made by an arc on the alternate arc are equal and a pair of angles on alternate arcs are supplementary.



$\angle P = \angle Q = \angle R$

$\angle P + \angle C = 180^o$, $\angle Q + \angle C = 180^o$, $\angle R + \angle C = 180^o$

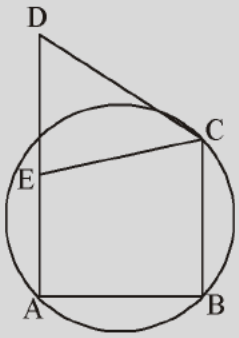
- If all the four vertices of a quadrilateral are on a circle, then its opposite angles are supplementary.



$\angle P + \angle R = 180^o$

$\angle Q + \angle S = 180^o$

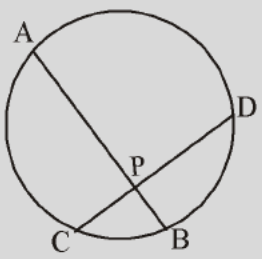
- If one vertex of a quadrilateral is outside the circle drawn through the other three vertices, then the sum of the angles at this vertex and the opposite vertex is less than 180°, if it is inside the circle, the sum is more than 180°.



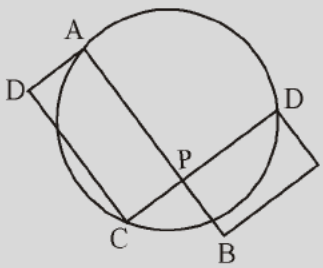
$\angle ABC + \angle D < 180^o$

$\angle ABC + \angle AEC > 180^o$

- If the opposite angles of a quadrilateral are supplementary, we can draw a circle passing through all four of its vertices.
- If two chords of a circle intersect within the circle, then the product of the parts of the two chords are equal. That is the rectangle formed by the parts of the same chord have equal area.

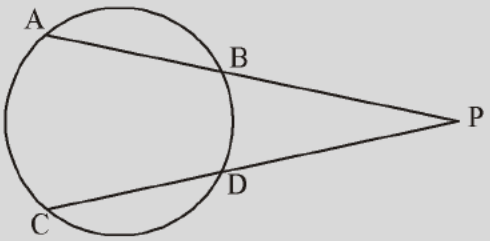


PA × PB = PC × PD



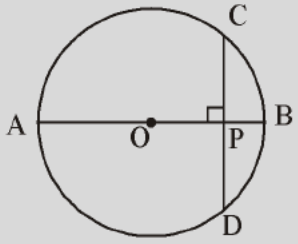
The area of two rectangels are equal

◆ If two chords of a circle intersect externally, the product of the distances from the external point to the ends of the chords are equal.

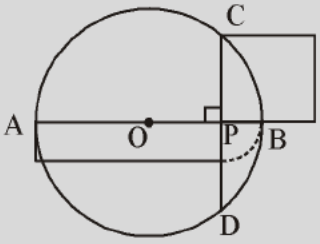


PA × PB = PC × PD

◆ The product of the parts into which a diameter of a circle is cut by a perpendicular chord is equal to the square of half the chord. That is the area of the rectangle formed of parts into which a diameter of a circle is cut by a perpendicular chord is equal to the area of the square formed by half the chord.

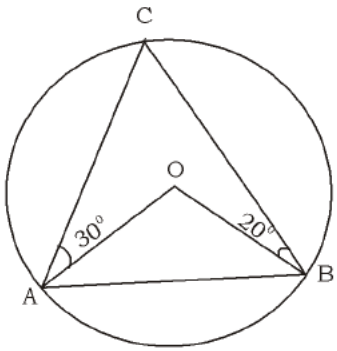


PA × PB = PC^2



Area of the square is equal to area of the rectangle.

Work Sheet - 1



In the figure O is the centre of the circle and A,B,C are points on the circle. $\angle OAC = 30^0$, $\angle OBC = 20^0$. Find all angles of ΔABC and ΔOAB

In the figure join OC ΔOAC is an isosceles triangle. (☐ = ☐)

$\angle OCA = \angle$☐ = ☐

ΔOBC is an isosceles triangle (☐ = ☐)

$\angle OCB = \angle$☐ = ☐

$\angle ACB = \angle$☐ + $\angle$☐ = ☐ + ☐ = ☐

$\angle AOB = 2 \times \angle$☐ = 2 × ☐ = ☐

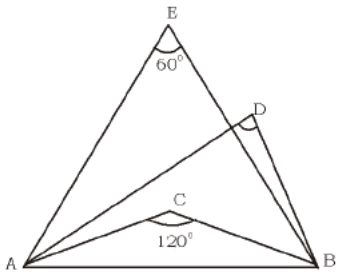
ΔOAB is an isosceles triangle ($\square$ = $\square$)

$$\angle OAB = \angle OBA = \frac{180^0 - \angle\square}{2} = \frac{180^0 - \square}{2} = \square$$

Angles of ΔOAB

$\angle OAB = \square$, $\angle OBA = \square$, $\angle AOB = \square$

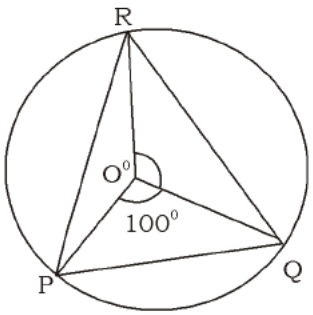
Work Sheet - 2



If a circle is drawn with AB as diameter, find the position of the points C,D,E related to the circle (Inside, Outside or on the circle)

Worksheet - 3

In the figure O is the centre of the circle. Find all angles of ΔPQR



$\angle POQ$ = $\square$

$\angle PRQ$ = ½×$\square$ = $\square$

$\angle QOR$ = $\square$

$\angle QPR$ = ½× $\square$ = $\square$

$\angle PQR$ = 180°–($\square$+$\square$)

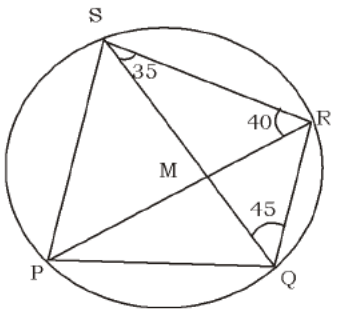
= 180 – $\square$ = $\square$

Angles of ΔPQR

$\angle P$ = $\square$, $\angle Q$ = $\square$, $\angle R$ = $\square$

Worksheet - 4

In the figure P,Q,R,S are points on the circle. Find the angles of the quadrilateral ABCD and the angles between the diagonals.



$\angle SQR = 45^0$ $\angle SPR =$ ☐ (Reason)

$\angle PRS = 40^0$ $\angle PQS =$ ☐ (Reason)

$\angle RSQ = 35^0$ $\angle RPQ =$ ☐ (Reason)

$\angle QPS =$ ☐ + ☐ = ☐

$\angle QRS = 180^0 - \angle$☐ $= 180^0 -$ ☐ = ☐

$\angle PQR =$ ☐ + ☐ = ☐

$\angle PSR = 180^0 - \angle$☐ $= 180^0 -$ (☐ + ☐)

ΔPMQ ൽ , $\angle PMQ = 180^0 -$ (☐ + ☐) = ☐

$\angle RMS =$ ☐ (Reason)

$\angle QMR = 180^0 -$ ☐ = ☐

$\angle PMS =$ ☐ (Reason)

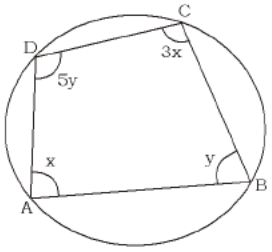
Worksheet - 5

In the figure ABCD is a cyclic quadrilateral

$\angle A = x$, $\angle B = y$, $\angle C = 3x$, $\angle D = 5y$

a) Find x andy

b) Find all angles of quadrilateral ABCD



a) $\angle A + \angle C =$ ☐ (Reason)

□ + □ = □

□ = □

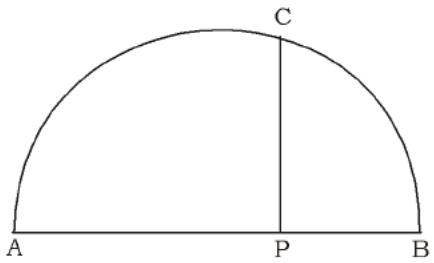
$\angle B + \angle D =$ □ (Reason)

□ + □ = □

□ = □

b) $\angle A =$ □ $\angle B =$ □ $\angle C =$ □ $\angle D =$ □

Worksheet - 6



In the figure AB is the diameter of the semicircle. Also PC $\perp$ AB PA= 8cm PC= 4cm

a) Find PB

b) Find the radius of the semicircle

a) PA × PB = □²

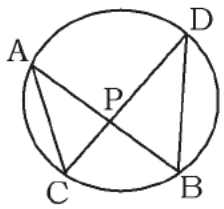
□ × PB = □

PB = □

b) Diameter of the semicircle AB = □ + □ = □

Radius = $\frac{\square}{\square}$ = □

Worksheet - 7



In the figure chords AB and CD intersect at P

a) Which angle is equal to $\angle ACD$?

a) Which angle is equal to $\angle ACD$?

c) Which angle is equal to $\angle APC$?

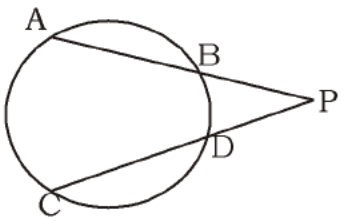
d) $\frac{PA}{\square} = \frac{\square}{PB}$ (Reason)

e) PA×PB= ☐×☐

f) Fill in the blanks

PA	PB	PC	PD
8	4	—	16
—	3	2	9
12	—	4	6
8	5	4	—

വർക്ക് ഷീറ്റ് - 8



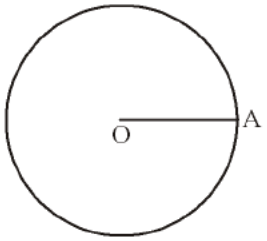
Chords AB and CD intersect at a point P outside the circle. AB=10cm, PB=2cm, PD=3cm. Find the length of chord CD

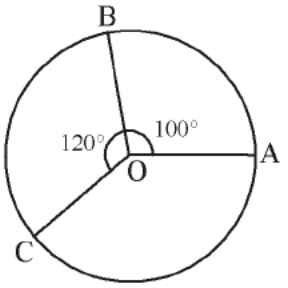
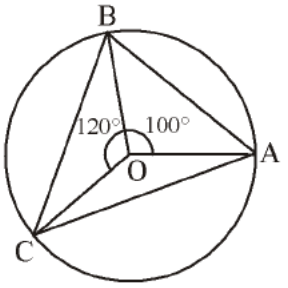
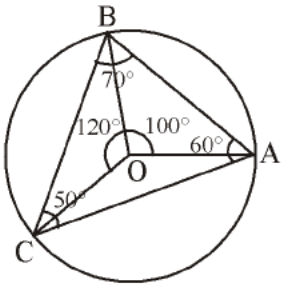
PA = ☐ + ☐ = ☐

PA × ☐ PC× ☐

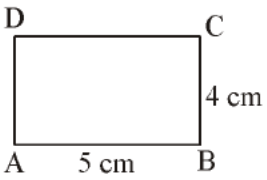
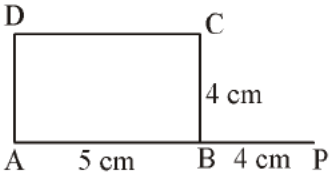
☐ × ☐ = PC × ☐

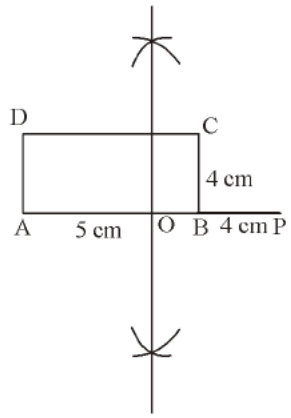
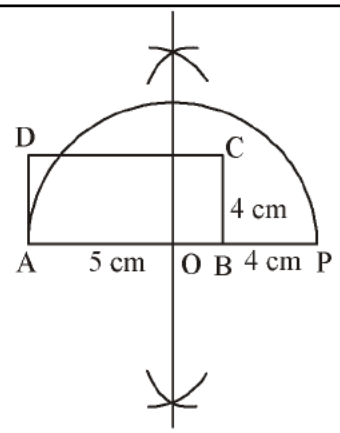
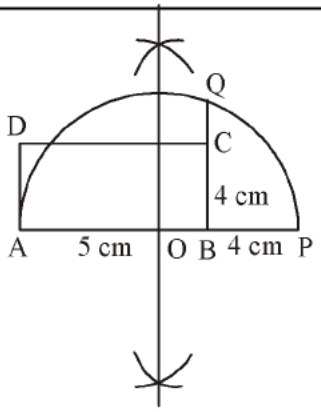
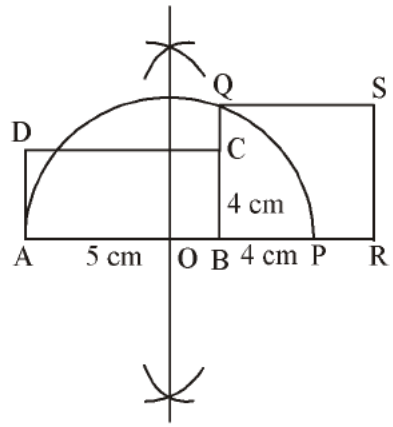
Work Sheet - 9

1. Find the third angle of the triangle	180 – (60 +70) = 180 – 130 = 50^0
2. Calculate the double of the angles of triangle	120^0, 140^0, 100^0
3. Draw a circle of radius 3cm and draw one radius	

4. Draw two central angles with measures 100^0 and 120^0	
5. Join AB, BC, AC	 ΔABC is the required triangle
6. Angles of ΔABC are 50^0, 60^0 and 70^0	

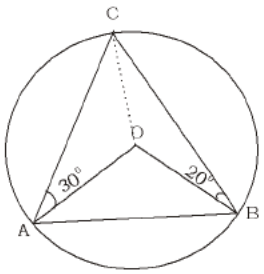
Work sheet- 11

Draw a rectangle ABCD with sides AB = 5cm BC = 4cm	
Extend AB to P with BP = 4cm	

Draw perpendicular bisector of AP	
Draw semicircle with centre O and OA as radius	
Extended BC to Q $\left[AB \times BP = BQ^2\right]$	
Draw square BRSQ of length of sides equal to BQ	

Answerkey

Worksheet 1



In the figure join OC. ΔOAC is an isosceles triangle (OA=OC)

$\angle OCA = \angle OAC = 30^0$

ΔOBC is an isosceles triangle (OB=OC)

$\angle OCB = \angle OBC = 20^0$

$\angle ACB = \angle OCA + \angle OCB = 30^0 + 20^0 = 50^0$

$\angle AOB = 2 \times \angle ACB = 2 \times 50^0 = 100^0$

ΔOAB is an isosceles triangle. (OA=OB)

$$\angle OAB = \angle OBA = \frac{180^0 - \angle AOB}{2} = \frac{180^0 - 100^0}{2} = 40^0$$

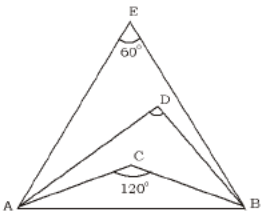
Angles of ΔABC

$\angle A = 30^0 + 40^0 = 70^0$, $\angle B = 20^0 + 40^0 = 60^0$, $\angle C = 50^0$

Angles of ΔOAB

$\angle OAB = 40^0$, $\angle OBA = 40^0$, $\angle AOB = 100^0$

Worksheet 2

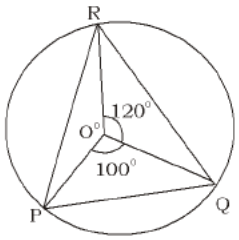


$\angle C = 120^0 > 90^0$, C is inside the circle

$\angle D = 90^0$ D is a point of the circle

$\angle E = 60^0 < 90^0$ E is outside the circle

Worksheet 3



$\angle POQ = 100^0$

$\angle PRQ = \frac{1}{2} \times 100 = 50^0$

$\angle QOR = 120^0$

$\angle QPR = \frac{1}{2} \times 120^0 = 60^0$

$\angle PQR = 180^0 - \left(50^0 + 60^0\right)$

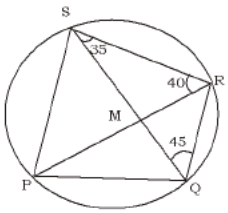
$= 180^0 - 110^0$

$= 70^0$

Angles of ΔPQR

$\angle P = 60^0, \angle Q = 70^0, \angle R = 50^0$

Worksheet - 4



$\angle SQR = 45^0$ $\angle SPR = 45^0$ (All angles made by an arc on the alternate are equal)

$\angle PRS = 40^0$ $\angle PQS = 40^0$ (All angles made by an arc on the alternate are equal)

$\angle RSQ = 35^0$ $\angle RPQ = 35^0$ (All angles made by an arc on the alternate are equal)

$\angle QPS = \angle SPR + \angle RPQ = 45^0 + 35^0 = 80^0$

$\angle QRS = 180 - \angle QPS = 180^0 - 80^0 = 100^0$

$\angle PQR = \angle PQS + \angle SQR = 40^0 + 45^0 = 85^0$

$\angle PSR = 180^0 - \angle PQR = 180^0 - 85^0 = 95^0$

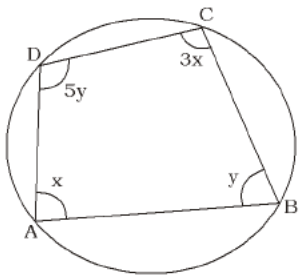
ΔPMQ ൽ $\angle PMQ = 180^0 - (35^0 + 40^0) = 180^0 - 75^0 = 105^0$

$\angle RMS = \angle PMQ = 105^0$ (Opposite angles)

$\angle QMR = 180^0 - 105^0 = 75^0$

$\angle PMS = \angle QMR = 75^0$ (Opposite anngles)

Worksheet - 5



a) $\angle A + \angle C = 180^0$ (Opposite angles of a cyclic quadrilateral are supplymentary)

$x + 3x = 180^0$

$4x = 180^0$

$x = \frac{180^0}{4} = 45^0$

$\angle B + \angle D = 180^0$ (Opposite angles of a cyclic quadrilateral are supplymentary)

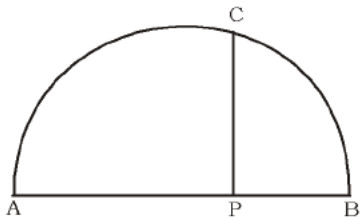
$y + 5y = 180^0$

$6y = 180^0$

$y = \frac{180^0}{6} = 30^0$

b) $\angle A = x = 45^0$ $\angle B = y = 30^0$ $\angle C = 135^0$ $\angle D = 150^0$

Worksheet - 6



a) $PA \times PB = PC^2$

$8 \times PB = 4^2$

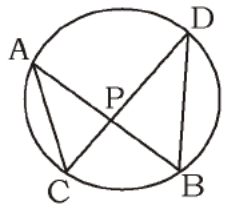
$$PB = \frac{16}{8} = 2$$

b) Diameter of the semicircle AB=PA+PB

$= 8 + 2 = 10cm$

$$\text{Radius} = \frac{10}{2} = 5cm$$

Worksheet - 7



a) $\angle BAC = \angle BDC$

b) $\angle ACD = \angle ABD$

c) $\angle APC = \angle BPD$ (Opposite angles)

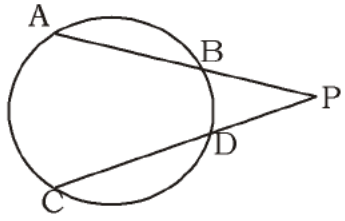
d) $\frac{PA}{PD} = \frac{PC}{PB}$ (ΔPAC and ΔPBD are similar triangles)

e) PA × PB = PC × PD

f)

PA	PB	PC	PD
8	4	2	16
6	3	2	9
12	2	4	6
8	5	4	10

Worksheet - 8



PA = PB + AB = 2 + 10 = 12

PA × PB = PC × PD

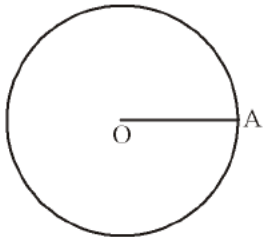
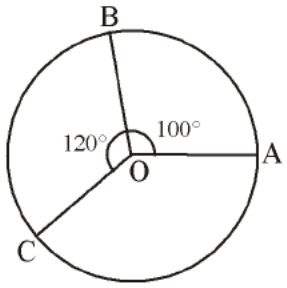
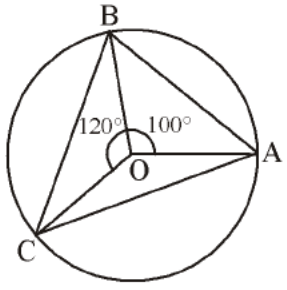
$12 \times 2 = PC \times 3$

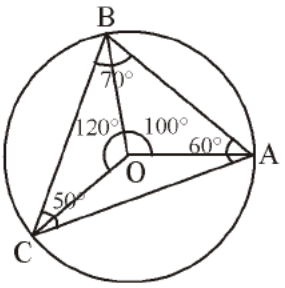
$$PC = \frac{24}{3} = 8 \text{ cm}$$

CD = PC - PD

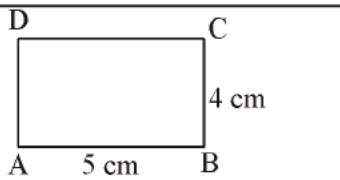
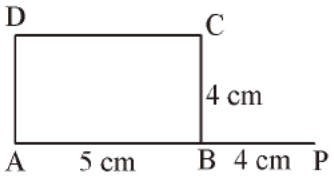
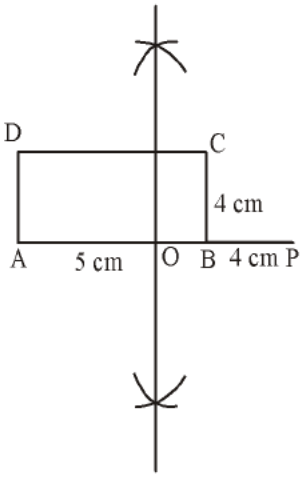
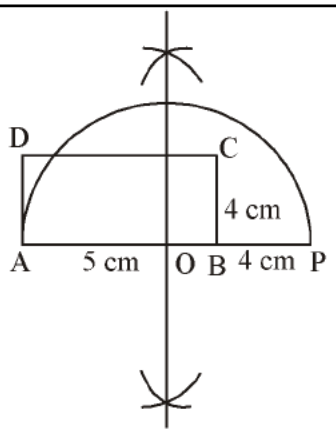
= 8 – 3 = 5cm

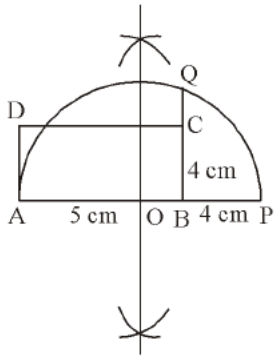
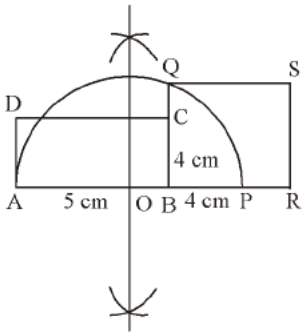
Work Sheet - 9

1. Find the third angle of the triangle	180 – (60 +70) = 180 – 130 = 50^0
2. Calculate the double of the angles of triangle	120^0, 140^0, 100^0
3. Draw a circle of radius 3cm and draw one radius	
4. Draw two central angles with measures 100^0 and 120^0	
5. Join AB, BC, AC	 ΔABC is the required triangle

6. Angles of ΔABC are 50^0, 60^0 and 70^0	

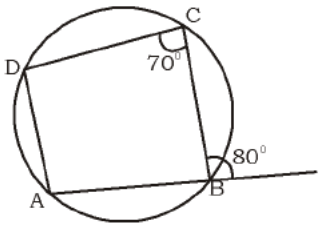
Work sheet- 10

Draw a rectangle ABCD with sides AB = 5cm BC = 4cm Extend AB to P with BP = 4cm	 
Draw perpendicular bisector of AP	
Draw semicircle with centre O and OA as radius	

Extended BC to Q $\left[AB \times BP = BQ^2\right]$	
Draw square BRSQ of length of sides equal to BQ	

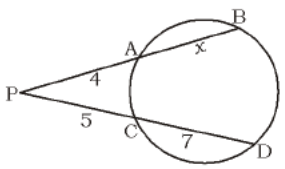
Additional Questions for pratice

1. Draw a triangle with two angles 35^0 and 65^0 and has circumradius 4 cm.

2. 

In the figure ABCD is a cyclic quadrilateral $\angle CBE = 80^0$, $\angle BCD = 70^0$. Find the measure of all angles of quadrilateral ABCD.

3. Draw the rectangle of sides 6cm and 3cm. Draw a square of the same area.

4. 

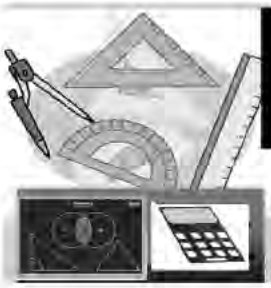
4. In the figure AB and CD meet at a point P Outside the circle PA=4cm, PC=5cm and CD=7cm

   a) Find PD

   b) Let AB = $x$cm. Find the length of PB

   c) Write the relation between PA, PB, PC, PD

   d) Find the length of AB



Unit 3

MATHEMATICS OF CHANCE

Points to Remember

The probability of occurring an event is the number, which indicates how many part of the favourable outcomes of it to the total outcomes.

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total Number of outcomes}}$$

$$\text{Geometric probability} = \frac{\text{Area of required region}}{\text{Total area}}$$

$$\text{Probability of pairs} = \frac{\text{Number of favourable pairs}}{\text{Total Number of pairs}}$$

Worksheet - 1

A box contains 18 black balls and 7 white balls. A ball is taken from the box. What is the probability that it is black?

What is the probability that it is white?

Total number of balls in the box =_____

Total number of black balls =_____

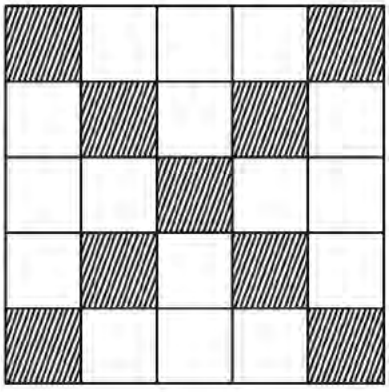
Total number of white balls =_____

Probability of getting a black ball =_____

Probability of getting a white ball =_____

Worksheet- 2

In the figure a large square is cut in to smaller squares of equal size. Without looking put a dot inside the larger square. What is the probability of it being in the shaded region?



Total number of smaller squares =_____

Total number of smaller squares shaded =_____

Probability that the dot is in the shaded region =_____

Worksheet - 3

Two dice are rolled together and two digit numbers are formed with the

numbers turned up.

a. What is the probability of both digits being the same ?

b. What is the probability of both digits even?

c. What is the probability of both digits odd?

d. What is the probability of one digit being even and other odd?

Complete the table in this way.

	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21					
3	31					
4	41					
5	51					
6	61					

Total number of two digits numbers =_____

a. Total number of two digit numbers with both digits same =_____

Probability of both digits being same =_____

b. Total number of two digit numbers with both digits even

Probability of getting both digits even =_____

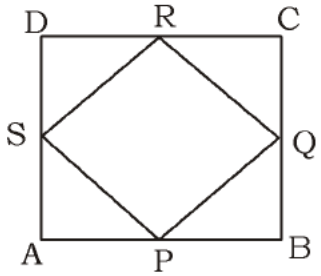
c. Total number of two digit numbers with both digits odd =_____

Probability of getting both digits odd =_____

d. Total number of two digit numbers with one digit even and other odd =_____

Probability of getting one digit even and other odd =_____

Worksheet - 4



In the figure ABCD is a square in which P,Q,R,S are the mid points of the sides AB,BC,CD and AD respectively without looking put a dot inside the larger square.

a) What is the probability that the dot being inside the small square?

b) Let x be the side of large square

Side of small square = ...

Area of small square = ...

Area of large square = ...

Probability of getting the dot inside the small square $\frac{\square}{\square}$

Worksheet - 5

There are 45 students in class 10 A out of which 25 are girls and 40 students in class 10 B out of which 20 are girls.

If one from each class is selected.

a. What is the probability of both being girls?

b. What is the probability of both being boys?

Total number of students in 10 A =_____

Total number of girls in 10 A =_____

Total number of boys in 10 A =_____

Total number of Students in 10 B =_____

Total number of girls in 10 B =_____

Total number of boys in 10 B =_____

Total number of pairs = Total number of Students in 10A × Total number of students in 10 B

= ____ × ____ = ____

Total number of pairs both being girls = ____ × ____ = ____

Total number of pairs both being boys = ____ × ____ = ____

Total number of pairs of one boy and other girl

= Total number of boys in 10 A × Total number of girls in 10 B + Total number of girls in 10 A × Total number of boys in 10 B

= ____ × ____ + ____ × ____ = ____ × ____ = ____

Total number of pairs of getting atleast one girl = Total number of pairs – Total number of pairs of both boys.

i. Probability of both being girls = $\frac{\text{Total number of pairs both being girls}}{\text{Total number of pairs}}$

$= \frac{\square}{\square}$

ii. Probability of both being boys = $\frac{\square}{\square}$

iii. Probability of one being a boy and other a girl = $\frac{\square}{\square}$

iv. Probability of atleast one girl = $\frac{\square}{\square}$

ANSWERS

Worksheet : 1

Total number of balls = 18 + 7 = 25

Number of black balls = 13

Number of white balls = 7

Probability of getting a black ball = $\frac{18}{25}$

Probability of getting a white ball = $\frac{7}{25}$

Worksheet : 2

Total numbers of small squares = 25

Total numbers of small shaded squares = 9

Probability of putting a dot on the shaded part = $\frac{9}{25}$

Worksheet : 3

	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Total numbers of two digit numbers = 36

a. Total number of two digit numbers in which both digits are same = 6 [11, 22, 33, 44, 55, 66]

Probability of getting both digits same = $\frac{6}{36} = \frac{1}{6}$

b. Total number of two digit numbers with both digits even = 9

Probability of getting both digits even = $\frac{9}{36} = \frac{1}{4}$

c. Total number of two digit numbers with both digits odd = 9

Probability of getting both digits odd= $\frac{9}{36}=\frac{1}{4}$

d. Total number of two digit numbers with one digit even and other odd = 18

Probability of getting one digit even and other odd = $\frac{18}{36}=\frac{1}{2}$

Worksheet : 4

Let x be the side of the large square

Side of small square = $\frac{\left(x\times\sqrt{2}\right)}{2}=\frac{x}{\sqrt{2}}$

Area of the small square = $\left(\frac{x}{\sqrt{2}}\right)^2=\frac{x^2}{2}$

Area of the large square = x^2

Probability of getting the dot inside the small square = $\frac{\frac{x^2}{2}}{x^2}=\frac{1}{2}$

Worksheet : 5

Total number of students in 10 A = 45

Total number of girls in 10 A = 25

Total number of boys in 10 A = 20

Total number of students in 10 B = 40

Total number of girls in 10 B = 20

Total number of boys in 10 B = 20

Total number of pairs = 45 × 40 = 1800

Total number of pairs with both being girls = 25 × 20 = 500

Total number of pairs with both being boys = 20 × 20 = 400

i. Probability of both being girls = $\frac{500}{1800}=\frac{5}{18}$

ii. Probability of both being boys = $\frac{400}{1800}=\frac{4}{18}=\frac{2}{9}$

PRACTICE PROBLEMS

1. One is asked to say a three digit number. What is the probability of all the digits being the same?
2. There are 4 black beads and 5 white beads in a box. 6 black beads and 5 white beads is another box.
   a. A bead is taken out from the first box without looking. What is the probability of it being black ? What about white?
   b. Which box is preferable to get a white bead?
   c. If all the beads are put in a single box and take a bead without looking, what is the probability of it being black?
3. There are white and black beads in a box, with total 16. The probability of getting a black bead from this box is ½ . Then,
   a. How many black beads are there?
   b. How many white beads are there?
   c. What is the probability of getting a white bead?
   d. If one more black bead is put into this box, what is the probability of getting a black bead ?
4. Slips numbered 1 to 100 are put in a box. One is taken out from it without looking.
   a. What is the probability of it being a multiple of 4 ?
   b. What is the probability of it being a multiple of 5 ?
5. Slips numbered odd numbers less than 10 are put in a box and slips numbered natural numbers from 1 to 5 in another box. One slip is taken from each box without looking.
   a. What is the probability of both being odd?
   b. What is the probability of both being even?

Answers

1. $\frac{1}{100}$
2. a) $\frac{4}{9}$
   b) $\frac{5}{9}$
3. a) 8
   b) 8
   c) ½

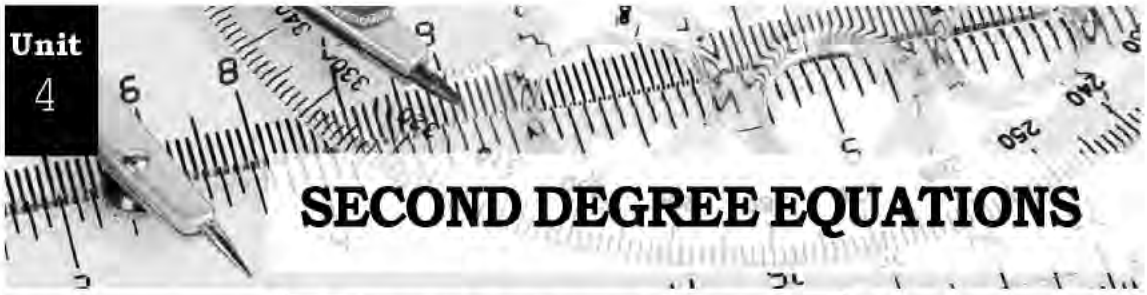
4. a) $\frac{1}{4}$

   b) $\frac{1}{5}$

5. a) $\frac{2}{5}$

   b) $\frac{3}{5}$

Unit 4



SECOND DEGREE EQUATIONS

Points to Remember

Algebraic form

- Two consecutive natural numbers : $x, x + 1$
- Two consecutive odd numbers : $x, x + 2$
- Two consecutive even numbers : $x, x + 2$
- Two consecutive terms of an arithmetic sequence with common difference 'd' : $x, x + d$
- Two numbers with sum 12 : $x, 12 - x$
- Two numbers with difference 12 : $x, x + 12$ or $x, x - 12$

- $ax^2 + bx + c = 0$, $a \neq 0$ is the general form of a second degree equation and its solution is.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- A second degree equation can have at most 2 solutions

Activity-1

When each side of a square is increased by 3 cm, the area will become 100 cm^2. What is the length of the side of the original square?

One side of the original square = x

One side of the new square = $x +$ ☐

Area of the new square = $(x +$ ☐$)^2$

$(x +$ ☐$)^2$ = 100

$x +$ ☐ $= \sqrt{100}$

$x +$ ☐ $=$ ☐

$x =$ ☐ - ☐

$=$ ☐

∴ One side of the original square = ☐ cm

Activity-2

The length of a rectangle is 6 cm more than its breadth and its area is 91 cm^2. What are the lengths of the sides of the rectangle?

Breadth = x

Length = $x +$ ☐

Area = $x(x +$ ☐$)$

$x(x +$ ☐$) = 91$

The number to be added to change it into a perfect square = ☐

$x^2 + 6x +$ ☐ $= 91 +$ ☐

$(x +$ ☐$)^2 =$ ☐

$x +$ ☐ $=$ ☐

$x =$ ☐ - ☐

$=$ ☐

∴ Breadth of the rectangle = ☐ cm

Length of the rectangle = ☐ $+ 6$

$=$ ☐ cm

Activity -3

Length of a rectangle is 2cm less than twice its breadth. The length of its diagonal is 5cm. Find the length and breadth of the rectangle.

Let breadth = x,

Then length = ☐

$x^2 + (2x - 2)^2 =$ ☐

$5x^2 -$ ☐ $-$ ☐ $= 0$

a = 5, b = ☐ , c = ☐

$b^2 - 4ac =$ ☐

= ☐

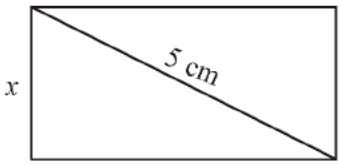
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{8 \pm \sqrt{\square}}{10}$$

= ☐, ☐

Breadth = ☐

Length = ☐



PRACTICE PROBLEMS

1. The square of the difference of 3 times a natural number and 1 is 121. What is the number ?
2. One side of rectangle is 2cm longer than the other side and its diagonal is 10cm. What are the lengths of its sides?
3. What should be the lengths of the sides of a rectangle having perimeter 28 cm and area 24cm^2.
4. In the figure, the chord AB extended and the tangent at the point ?

4. The sum of the squares of two consecutive odd numbers is 130. What are the numbers?
5. The sum of the perpendicular sides of a right angled triangle is 14 cm and its area is 24 cm^2. Find the length of its sides?

Answers

1. Number = 3
2. breadth = 6cm
   length = 8cm
3. Length = 12 cm
   breadth = 2cm
4. Consicutive odd number = 7, 9
5. Sides of a right triangle 6cm, 8cm, 10cm

Answers

Worksheet - 1

1. One side of the original square = x
   Side of the new square = $x+3$
   Area of the new square = $(x+3)^2$
   $(x+3)^2 = 100$
   $x+3 = \sqrt{100}$
   $x+3=100,\ x+3=10-3$
   $x=y$
   One side of original square = 7cm

Worksheet - 2

Breadth = x
Length = $x+6$
Area = $x(x+6)$
$x(x+6) = 91$
$x^2+6x+9=91+9$
$(x+3)^2 = 100$
$x+3 = \sqrt{100}$

$x+3 = 10$

$x = 10-3$

$x = 7$

Breadth of rectangle = 7cm

Length of rectangle = 7+6=13cm

Worksheet - 3

Breadth - x

Then length = $2x-2$

$x^2+(2x-2)^2=25$

$5x^2 - 8x+4-25=0$

$5x^2-8x-21=0$

$a=5 \ b=-8 \ c=-21$

$b^2=4ac=(-8)^2-4\times5\times-21$

$= 64+420$

$= 484$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{484}}{2 \times 5}$$

$$= \frac{8 \pm 22}{10}$$

$$x = \frac{8+22}{10}, x = \frac{8-22}{10}$$

$$x = \frac{30}{10}, x = \frac{-14}{10}$$

$$x = 3, x = \frac{-14}{10}$$

Breadth = 3cm

The length = 2×5

= 4cm