+1 MATHEMATICS 2022

MODEL EXAMINATION ANSWER KEY

AMC

1, (a) 4	1
(b) $\phi, \{1\}, \{2\}, \{1,2\}$	1
(c) $(6,12] = \{x : x \in R, 6 < x \leq 12\}$	1
2, (a) 45°	1
(b) $\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = -\frac{4}{5}$	1
$\tan x = \frac{\sin x}{\cos x} = \frac{3/5}{-4/5} = \frac{-3}{4}$	1
3, (a) $a_n = 5n+1$	
$a_1 = 6, \; a_2 = 11, \; a_3 = 16, \; a_4 = 21$	
$6, 11, 16, 21$	1
(b) $a = 6, \; d = 11-6 = 5$	
$S_n = \frac{n}{2}[2a+(n-1)d]$	1
$= \frac{n}{2}[12+(n-1)5] = \frac{n}{2}[5n+7]$	1
4, (a) Slope $= \frac{y_2-y_1}{x_2-x_1} = \frac{5-1}{4-2} = \frac{4}{2} = 2$	1
(b) Since the Points are collinear	
$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$	1
$x(-4)+2(6)+4(-2)=0$	
$-4x+12-8=0$	
$-4x=-4$	
$x=1$	1

5, $r = 5$

Since centre lie on x-axis, let it be $(h, 0)$

Equation of the circle is

$$(x-h)^2 + (y-0)^2 = 5^2 \quad \text{—— (1)}$$

Since $(2,3)$ is a point on the circle

$$(2-h)^2 + 9 = 25$$

$$(2-h)^2 = 16$$

$$2-h = \pm 4$$

$$h = 2 \pm 4 = 6 \text{ or } -2$$

Equation of the circle is

$$(x-6)^2 + y^2 = 25 \text{ or } (x+2)^2 + y^2 = 25$$

6, (a) 8

(b) distance $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$$= \sqrt{2^2 + (-6)^2 + 8^2}$$

$$= \sqrt{4+36+64} = \sqrt{104}$$

7, (a) $\lim\limits_{x \to 4} \dfrac{4x+3}{x-2} = \dfrac{16+3}{4-2} = \dfrac{19}{2}$

(b) $\lim\limits_{x \to 1} \dfrac{x^3-1}{x^2-1} = \lim\limits_{x \to 1} \dfrac{x^3-1^3}{x^2-1^2} = \lim\limits_{x \to 1} \dfrac{\dfrac{x^3-1^3}{x-1}}{\dfrac{x^2-1^2}{x-1}}$

$$= \frac{3 \cdot 1^2}{2 \cdot 1} = 3/2$$

8, (a) It is false that $\sqrt{7}$ is rational

(b) Contrapositive: If a number n^2 is not even then n is not even.

Converse: If a number n^2 is even then n is even.

9, (a) $A \cup B = B$ 1

(b) (i) $A' = \{1,3,5,7,9\}$

$B' = \{1,4,6,8,9\}$ 1

(ii) $A \cup B = \{2,3,4,5,6,7,8\}$ 1

(iii) $LHS = (A \cup B)' = \{1,9\}$

$RHS = A' \cap B' = \{1,9\}$ 1

$\therefore (A \cup B)' = A' \cap B'$

10, (a) $R = \{(1,3),(2,6),(3,9),(4,12)\}$ 1

domain $= \{1,2,3,4\}$, Range $= \{3,6,9,12\}$ 2

(b) $f(x) = 2x - 5$

$f(0) = -5$ 1

11; (a) $LHS = 1$

$RHS = \frac{3-1}{2} = 1$ 1

$\therefore P(1)$ is true

(b) Assume that $P(K)$ is true

$P(K): 1+3+3^2+ \cdots +3^{K-1} = \frac{3^K - 1}{2}$ 1

Now $P(K+1): 1+3+3^2+ \cdots +3^{K-1} + 3^K$

$= \frac{3^K - 1}{2} + 3^K$

$= \frac{3^K - 1 + 2 \cdot 3^K}{2}$ 1

$= \frac{3 \cdot 3^K - 1}{2} = \frac{3^{K+1} - 1}{2}$ 1

$\therefore P(K+1)$ is true.

Hence by the P.M.I, the statement is true for all natural numbers.

12, (a) $\frac{7!}{5!} = \frac{7\times6\times5!}{5!} = 7\times6 = 42$ 1

(b) Number of 4 digit numbers $= {}^{9}P_{4}$ 1

$= 3024$ 1

(c) $^{17}C_{17} = 1$ 1

13, (a) 5 1

(b) $\left(x^2+\frac{3}{x}\right)^4 = {}^4C_0(x^2)^4 + {}^4C_1(x^2)^3\cdot\left(\frac{3}{x}\right)$

$+ {}^4C_2(x^2)^2\left(\frac{3}{x}\right)^2 + {}^4C_3(x^2)\left(\frac{3}{x}\right)^3 + {}^4C_4\left(\frac{3}{x}\right)^4$ 2

$= x^8 + 12x^5 + 54x^2 + \frac{108}{x} + \frac{81}{x^4}$ 1

14, Let $\frac{a}{r}$, a, ar be the terms

Given $\frac{a}{r}\cdot a\cdot ar = 1$

$a^3 = 1 \Longrightarrow a = 1$ 1

Given $\frac{a}{r} + a + ar = \frac{39}{10}$

$\frac{1}{r} + 1 + r = \frac{39}{10}$

xig by $10r$, $10 + 10r + 10r^2 = 39r$ 1

$10r^2 - 29r + 10 = 0$

$r = \frac{29 \pm \sqrt{841 - 4\times100}}{2\times10}$

$= \frac{29\pm21}{20} = \frac{50}{20}$ or $\frac{8}{20}$

$r = 5/2$ or $2/5$ 1

Terms are $\frac{2}{5}, 1, 5/2$ or $5/2, 1, 2/5$ 1

15, (a) $y = 0$ | 1

(b) $3x + 2y - 12 = 0$

$\text{slope} = \frac{-a}{b} = -\frac{3}{2}$ | 1

$3x + 2y = 12$

$\frac{3x}{12} + \frac{2y}{12} = 1$

$\frac{x}{4} + \frac{y}{6} = 1$ | 1

x-intercept $= 4$, y-intercept $= 6$ | 1

16, $a^2 = 16$ $\quad$ $b^2 = 9$

$a = 4$ $\quad$ $b = 3$

$c^2 = a^2 - b^2 = 16 - 9 = 7$ | 1

$c = \sqrt{7}$

$\text{foci} = (\pm c, 0) = (\pm\sqrt{7}, 0)$ | 1

$\text{eccentricity} = \frac{c}{a} = \frac{\sqrt{7}}{4}$ | 1

$LLR = \frac{2b^2}{a} = \frac{18}{4} = \frac{9}{2}$ | 1

17, $n(S) = {}^{52}C_1 = 52$ | 1

(i) $n(A) = {}^{13}C_1 = 13$

$P(\text{a diamond}) = P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$ | 1

(ii) $n(B) = {}^{48}C_1 = 48$

$P(\text{not an ace}) = P(B) = \frac{n(B)}{n(S)} = \frac{48}{52} = \frac{12}{13}$ | 1

(iii) $n(C) = {}^{26}C_1 = 26$

$P(\text{a black card}) = P(C) = \frac{n(C)}{n(S)}$ | 1

$= \frac{26}{52} = \frac{1}{2}$

18, (a) $LHS = \dfrac{\tan(\pi/4 + x)}{\tan(\pi/4 - x)}$

$$= \dfrac{\dfrac{\tan\pi/4 + \tan x}{1 - \tan\pi/4 \cdot \tan x}}{\dfrac{\tan\pi/4 - \tan x}{1 + \tan\pi/4 \cdot \tan x}}$$ 1

$$= \dfrac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}} = \left(\dfrac{1+\tan x}{1-\tan x}\right)^2 = RHS$$ 2

(b) $\cos 3x + \cos x - \cos 2x = 0$

$2\cos 2x \cdot \cos x - \cos 2x = 0$

$\cos 2x(2\cos x - 1) = 0$ 1

$\cos 2x = 0$ or $2\cos x - 1 = 0$

$2x = (2n+1)\pi/2$ $\quad$ $2\cos x = 1$ 1

$\cos x = 1/2$

$x = (2n+1)\pi/4, n \in Z$ $\quad$ $\cos x = \cos \pi/3$ 1

$x = 2n\pi \pm \pi/3, n \in Z$

19, (a) $i^4 = 1$ 1

(b) Let $z = 1 - i$

$\bar{z} = 1 + i$, $|z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$ 1

$z^{-1} = \dfrac{\bar{z}}{|z|^2} = \dfrac{1+i}{2} = \dfrac{1}{2} + i\dfrac{1}{2}$ 1

(c) Let $z = 1 - i$

$a = 1, b = -1, r = \sqrt{a^2+b^2} = \sqrt{2}$ 1

$\tan\theta = \dfrac{b}{a} = -1 \implies \theta = -\pi/4$ 1

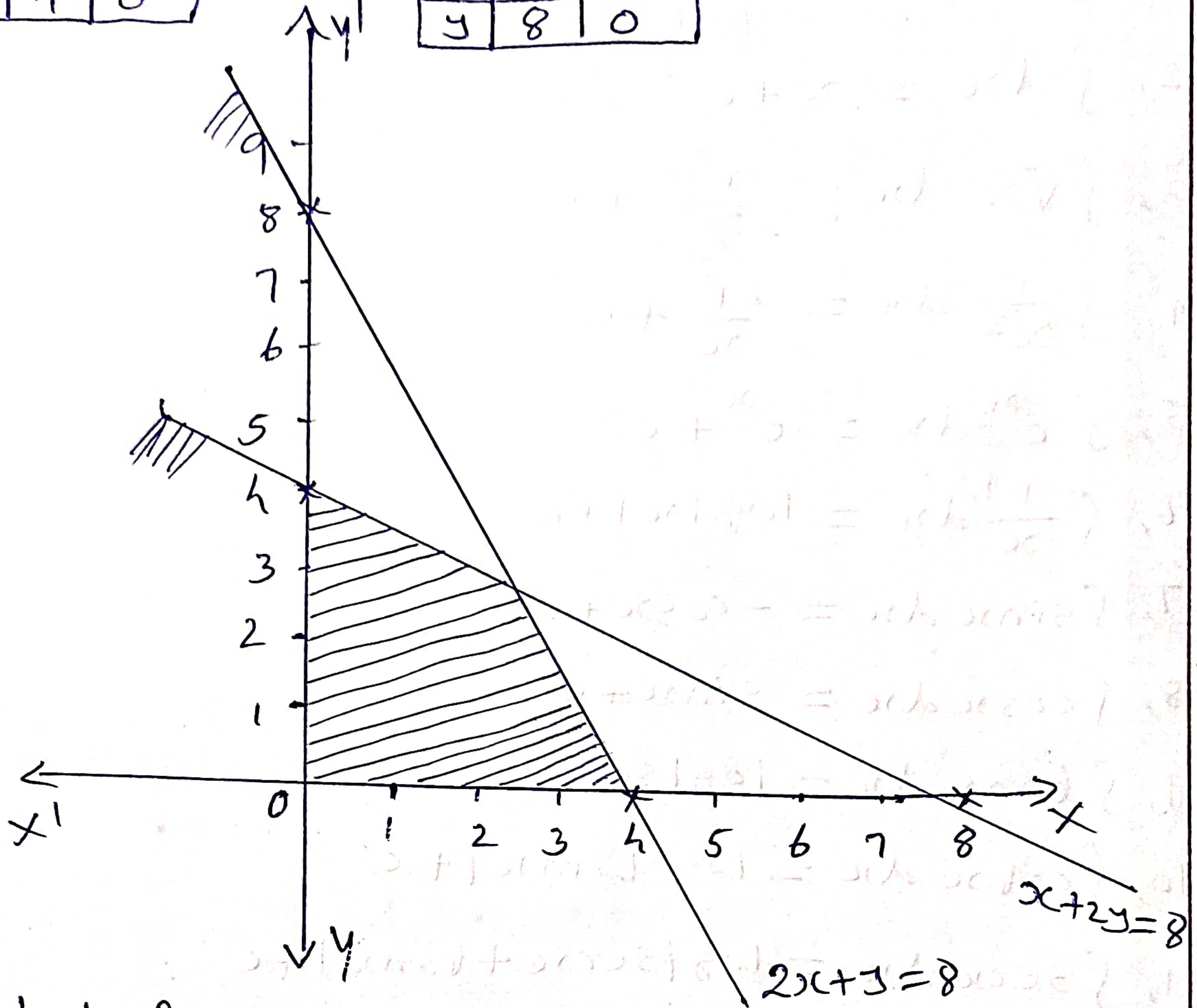
$z = r(\cos\theta + i\sin\theta) = \sqrt{2}\left[\cos(-\pi/4) + i\sin(-\pi/4)\right]$ 1

20, $x+2y=8$

x	0	8
y	4	0

$2x+y=8$

x	0	4
y	8	0



Shaded portion is the solution region. 2

4

21, (a) $f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ 1

$$= \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$ 1

$$= \lim_{h\to 0} \cos\left(\frac{2x+h}{2}\right) \cdot \lim_{h\to 0} \frac{\sin(h/2)}{h/2}$$

$$= \cos\left(\frac{2x}{2}\right) \cdot 1 = \cos x$$ 1

(b) $\frac{d}{dx}(5\sin x - 6\cos x + 7)$

$= \frac{d}{dx} 5\sin x - \frac{d}{dx} 6\cos x + \frac{d}{dx} 7$ 1

$= 5\frac{d}{dx}\sin x - 6\frac{d}{dx}\cos x + 0$ 1

$= 5\cos x + 6\sin x$ 1

22)

class	f	x	fx	fx^2
0-10	5	5	25	125
10-20	8	15	120	1800
20-30	15	25	375	9375
30-40	16	35	560	19600
40-50	6	45	270	12150
	50		1350	43050

$N = \Sigma f = 50$, $\Sigma fx = 1350$, $\Sigma fx^2 = 43050$

(i) $\bar{x} = \text{mean} = \frac{\Sigma fx}{N} = \frac{1350}{50} = 27$ 2

(ii) Variance $(\sigma^2) = \frac{\Sigma fx^2}{N} - (\bar{x})^2$

$= \frac{43050}{50} - (27)^2$ 3

$= 132$

Standard deviation $= \sqrt{\text{variance}}$

$= \sqrt{132} = 11.49$ 1