SECOND YEAR HIGHER SECONDARY SAMPLE QUESTION PAPER 2023
MATHEMATICS

Maximum : 60 scores | Time : 2 Hours

Cool-off Time : 15 minutes

General instructions to Candidates:

- There is a 'Cool-off Time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off Time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculators, figures and graphs should be shown in the answer sheet itself.
- Give equations wherever necessary.
- Electronic devices except non-programmed calculators are not allowed in the examination hall.

Answer any six questions from 1 to 8. Each carries 3 scores

1. Consider the set of all lines in a plane. Show that parallelism is an equivalence relation whereas perpendicularity is not. (3)

2. Let $A = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

   Show that $A^2 = \begin{bmatrix} \sin 2\theta & \cos 2\theta \\ \cos 2\theta & -\sin 2\theta \end{bmatrix}$ (3)

3. Find the area of triangle formed by $(1, -1)$, $(2, 1)$ and $(3, 5)$ (3)

4. Find the relation between a and b if, $f(x) = \begin{cases} ax + b & x \leq 2 \\ a - bx & x > 2 \end{cases}$

   is continuous at $x = 2$ (3)

5. If $x^y = y^x$. Find $\frac{dy}{dx}$. (3)

6. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{b} = 5\hat{i} + 2\hat{j} - 3\hat{k}$. Find the projection of $\vec{a}$ along $\vec{b}$. (3)

7. Find the vector equation of the line passing through $(2, 1, -3)$ in the direction of the vector $3\hat{i} - 2\hat{j} + 5\hat{k}$. Write also the cartesian equation. (3)

8. If $P(A) = 0.7$, $P(B) = 0.5$ and $P(A \cup B) = 0.9$. Find $P(A|B)$ and $P(B|A)$. (3)

Answer any six questions from 9 to 16. Each carries 4 scores

9. (a) Let $A = \{1, 2, 3\}$, $B = \{2, 4, 5, 6\}$ and $f = \{(1, 4), (2, 2), (3, 5)\}$. Check whether f is one-one and onto. (2)

   (b) Show that $f : R \longrightarrow R$ defined by $f(x) = 2x + 1$ is a bijective function. (2)

10. (a) Find the principle value of $\cos^{-1}(-\frac{1}{2})$

    (i) $\frac{\pi}{2}$ (ii) $\frac{2\pi}{3}$ (iii) $\frac{\pi}{6}$ (iv) $\frac{5\pi}{6}$ (1)

    (b) Prove that $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $x\epsilon[-\frac{1}{2}, \frac{1}{2}]$. (3)

11. Express $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & 1 & 7 \\ -5 & 6 & 4 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices. (4)

12. Find the area of the region formed by $y^2 = 3x$, x axis and the ordinates at $x = 1$ and $x = 3$. (4)

13. (a) The order and degree of the differential equation $3\left(\frac{dy}{dx}\right)^2 + \frac{d^2y}{dx^2} + 3y - 7 = 0$

    (i) 1, 1
    (ii) 2, 1
    (iii) 1, 2
    (iv) 2, 2 (1)

    (b) Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = 3$ (3)

14. (a) Find the unit vector perpendicular to both $\vec{a} = 3\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 7\hat{k}$. (2)

    (b) Prove that $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$ (2)

15. Find the shortest distance between the skew lines $\vec{r} = \hat{i} - 2\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = \hat{j} + 5\hat{k} + \kappa(\hat{i} - \hat{j} + \hat{k})$. (4)

16. Bag I contains 2 red and 4 black balls, bag II contains 5 red and 3 black balls. A ball is taken from one of the bags and it is found to be red. Find the probability that it is from bag II. (4)

Answer any three questions from 17 to 20. Each carries 6 scores

17. Solve the system of equation using matrix method

$$\begin{aligned} x + y + z &= 2 \\ 2x - y + z &= 7 \\ x + 3y + 2z &= 1 \end{aligned}$$

(6)

18. Maximize $Z = 3x + 4y$
Subject to constraints

$$\begin{aligned} x + 2y &\leq 4 \\ 2x + y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

(6)

19. Evaluate

(a) $\int_0^{\frac{\pi}{2}} \frac{\sin^{2023} x}{\cos^{2023} x + \sin^{2023} x} dx$ (2)

(b) $\int \frac{x}{(x-1)(x+2)} dx$ (2)

(c) $\int \frac{e^{\tan^{-1} x}}{(1+x^2)} dx$ (2)

20. (a) Find the intervals in which the function given by $f(x) = x^3 - 3x^2 + 7x + 5$ is increasing or decreasing. (2)

(b) Show that the right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base. (4)

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