SECOND YEAR HIGHER SECONDARY SAMPLE QUESTION PAPER 2023
MATHEMATICS
PART-III

Time: 2 Hours
Cool off time: 15 minutes

Maximum: 60 Scores

Answer any 6 questions from 1 to 8. Each question carries 3 marks.

1. (i) The relation R={(1,1), (2,2), (3,3), (1,2),(2,3)} in the set A={1,2,3} is
a) reflexive b)symmetric c) transitive d) reflexive and symmetric. (1)
(ii) Show that the relation R in the set Z of integers given by R={(a,b):2 divides a-b} is an equivalence relation. (2)

2.If A= $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

3.(i)If A is a matrix of order 3 ,then $|2A| = ...|A|$. (1)
(ii)Evaluate the determinant $\begin{vmatrix} cosx & -sinx \\ sinx & cosx \end{vmatrix}$. (2)

4. (i) Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular if $\vec{a}.\vec{b} = ...$ (1)
(ii)Find the area of the parallelogram with adjacent sides $\vec{a} = 3i + j + 4k$ and $\vec{b} = i - j + k$. (2)

5. Find the angle between the following pair of lines:
$\vec{r} = 2i - 5j + k$ λ($3i + 2j + 6k$) and
$\vec{r} = 7i - 6k$ +μ($i + 2j + 2k$).

6. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that exactly one of them solves the problem.

7. Find aii points of discontinuity of the function defined by f(x)= $\begin{cases} x+2 & \text{if } x<1 \\ 0 & \text{if } x=1 \\ x-2 & \text{if } x>1. \end{cases}$

8. (i) The derivative of $\tan(2x)$ =............
a) $sec^2 2x$ b) $2sec^2 x$ c) $2sec^2 2x$ d) $sec^2 x$. (1)
(ii) Find $\frac{dy}{dx}$ if $x = acost$ and $y = a(1 - sint)$. (2)

Answer any 6 questions from 9 to 16. Each question carries 4 marks.

9. (i) Show that $f : R \rightarrow R$ given by $f(x) = 4x - 3$ is a bijection. (2)
(ii)Also find the inverse of f. (2)

10. (i) The principal value of $\tan^{-1}(-1) = ...$ (1)
(ii) Show that $\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{2}{11}) = \tan^{-1}(\frac{3}{4})$. (3)

11. Express the matrix $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ as the sum of a symmetric and a skew symmetric matrices.

12. (i) The area of the region bounded by the curve $y=f(x)$, x axis and the lines $x=a$ and $x=b$ is

a) $\int_a^b y\,dx$ b) $\int_b^a y\,dx$ c) $\int_b^a x\,dy$ d) $\int_a^b x\,dy$. (1)

(ii)Find the area of the circle $x^2+y^2=9$ using integration. (3)

13. (i) Write the order of the differential equation: $x^2\frac{d^2y}{dx^2}=1+\left(\frac{dy}{dx}\right)^3$. (1)

(ii) Solve: $x\frac{dy}{dx}+2y=x^2$. (3)

14. (i) Find the unit vector in the direction of $\vec{a}+\vec{b}$ where $\vec{a}=2i+2j-5k, \vec{b}=-i+7k$. (2)

(ii) Find the projection of the vector $2i+3j+2k$ on the vector $i+j+k$. (2)

15. Find the shortest distance between the lines:

$\vec{r}=i+2j+3k$ +λ($i-3j+2k$) and

$\vec{r}=4i+5j+6k$ μ($2i+3j+k$).

16. Bag I contains 3 red and 4 black balls while BagII contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from BagII.

Answer any 3 questions from 17 to 20. Each question carries 6 marks.

17. Solve the system of linear equations by matrix method:

$$x-y+2z=7$$
$$3x+4y-5z=-5$$
$$2x-y+3z=12 \quad .$$

18. Find the following integrals:

(a) $\int \frac{\tan^{-1}x}{1+x^2}dx$ (2)

(b) $\int \frac{dx}{(x-1)(x-2)}$ (2)

(c) $\int_0^{\pi/2} \frac{\sqrt{cosx}}{\sqrt{sinx}+\sqrt{cosx}}dx$. (2)

19. (a) An edge of a variable cube is increasing at a rate of 3cm/s. How fast is the volume of the cube increasing when the edge is 10cm long. (2)

(b) Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is (i) strictly increasing (ii) strictly decreasing. (2)

(c) Find the absolute maximum value and absolute minimum value of the function $f(x)=sinx+cosx$, $x\in[0,\pi]$. (2)

20. Solve the following linear programming problem graphically: Maximise $Z=3x+4y$ subject to $x+2y\leq10; 3x+y\leq15; x,y\geq0$.