WB-JEE - 2009

PHYSICS & CHEMISTRY QUESTIONS & ANSWERS

1. One Kg of copper is drawn into a wire of 1mm diameter and a wire of 2 mm diameter. The resistance of the two wires will be in the ratio

(A) 2 : 1 (B) 1 : 2 (C) 16 : 1 (D) 4 : 1

Ans : (C)

Hints : $\text{Mass} = \left(\pi r_1^2 \ell_1\right)\sigma$ (Ist wire)

$\text{Mass} = \left(\pi r_1^2 \ell_2\right)\sigma$ (2nd wire)

$\left(\pi r_1^2 \ell_1\right)\sigma = \left(\pi r_2^2 \ell_2\right)\sigma$

$$\frac{\ell_1}{\ell_2} = \left(\frac{r_2}{r_1}\right)^2$$

$$\frac{R_1}{R_2} = \frac{\rho\frac{\ell_1}{A_1}}{\rho\frac{\ell_2}{A_2}} = \frac{\ell_1}{\ell_2} \times \frac{A_2}{A_1} = \frac{\ell_1}{\ell_2} \times \left(\frac{r_2}{r_1}\right)^2$$

$$= \left(\frac{r_2}{r_1}\right)^4$$

$\Rightarrow 16:1$

2. An electrical cable having a resistance of 0.2 Ω delivers 10kw at 200V D.C. to a factory. What is the efficiency of transmission?

(A) 65% (B) 75% (C) 85% (D) 95%

Ans : (D)

Hints : $P = VI \Rightarrow I = \frac{10 \times 10^3}{200} = 50A$, Power loss = $(50)^2 (0.2)$ = 500W

$$\text{Efficiency} = \frac{10000 \times 100}{10000 + 500} = 95.23\%$$

3. A wire of resistance 5 Ω is drawn out so that its new length is 3 times its original length.What is the reistance of the new wire?

(A) 45 Ω (B) 15 Ω (C) 5/3 Ω (D) 5 Ω

Ans : (A)

Hints : $\left(\frac{r_1}{r_2}\right)^2 = \left(\frac{\ell_2}{\ell_1}\right) = \frac{3\ell}{\ell} = 3$

$$\left(\frac{R_2}{R_1}\right) = \frac{\ell_2}{\ell_1} \times \frac{A_1}{A_2} = 3\times\left(\frac{r_1}{r_2}\right)^2 = 3\times 3 \Rightarrow R_2 = 45$$

4. Two identical cells each of emf E and internal resistance r are connected in parallel with an external resistance R. To get maximum power developed across R, the value of R is

(A) R = r/2 (B) R = r (C) R = r/3 (D) R = 2r

Ans : (A)

Hints : $R_{eq} = \frac{r}{2}$

$$I = \frac{2E}{r+2R}$$

For max. power consumption. I should be max. So denominator should be min. for that

$$r+2R = \left(\sqrt{r} \quad \right)$$

5. To write the decimal number 37 in binary, how many binary digits are required?

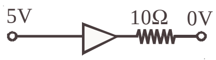
(A) 5 (B) 6 (C) 7 (D) 4

Ans : (B)

Hints :

2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
	1	

6. A junction diode has a resistance of 25 Ω when forward biased and 2500 Ω when reverse biased. The current in the diode, for the arrangement shown will be



(A) $\frac{1}{15}$A (B) $\frac{1}{7}$A (C) $\frac{1}{25}$A (D) $\frac{1}{180}$A

Ans : (B)

Hints : $R_{eq} = 25 + 10 = 35\Omega$

Because diode is forward biased. So $I = \frac{V}{R_{eq}} = \frac{5}{35} = \frac{1}{7} A$

7. If the electron in a hydrogen atom jumps from an orbit with level $n_1 = 2$ to an orbit with level $n_2 = 1$ the emitted radiation has a wavelength given by

(A) $\lambda = 5/3R$ (B) $\lambda = 4/3\,R$ (C) $\lambda = R/4$ (D) $\lambda = 3R/4$

Ans : (B)

Hints : $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$

$\Rightarrow \lambda = \frac{4}{3R}$

8. What is the particle x in the following nuclear reaction :

$^{9}_{4}\mathrm{Be} + ^{4}_{2}\mathrm{He} \longrightarrow ^{12}_{6}\mathrm{C} + \mathrm{x}$

(A) electron (B) proton (C) Photon (D) Neutron

Ans : (D)

Hints : $^{9}_{4}Be + ^{4}_{2}He \longrightarrow ^{12}_{6}C + ^{1}_{0}X$

Hence X represents neutron $\left(^{1}_{0}n\right)$

9. An alternating current of rms value 10 A is passed through a 12 Ω resistor. The maximum potential difference across the resistor is

(A) 20 V (B) 90 V (C) 1969.68 V (D) none

Ans : (C)

Hints : $I_{rms} = 10A$

$I_{rms} = \frac{I_0}{\sqrt{2}} \Rightarrow I_0 = \sqrt{2} \times 10 = 10\sqrt{2}$

Max. P.D. = $\sqrt{2} \times 10 \times 12 = 120 \times 1.414 = 169.68\ V$

10. Which of the following relation represent Biot-Savart's law?

(A) $\mathrm{d}\overline{\mathrm{B}} = \frac{\mu_0}{4\pi}\frac{\overline{\mathrm{dl}} \times \overline{\mathrm{r}}}{\mathrm{r}}$ (B) $\mathrm{d}\overline{\mathrm{B}} = \frac{\mu_0}{4\pi}\frac{\overline{\mathrm{dl}} \times \hat{\mathrm{r}}}{\mathrm{r}^3}$ (C) $\mathrm{d}\overline{\mathrm{B}} = \frac{\mu_0}{4\pi}\frac{\overline{\mathrm{dl}} \times \overline{\mathrm{r}}}{\mathrm{r}^3}$ (D) $\mathrm{d}\overline{\mathrm{B}} = \frac{\mu_0}{4\pi}\frac{\overline{\mathrm{dl}} \times \overline{\mathrm{r}}}{\mathrm{r}^4}$

Ans : (C)

Hints : $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(d\vec{\ell} \times \vec{r}\right)}{r^3}$

Note :– In question paper current (I) is missing

11. $\vec{A}$ and $\vec{B}$ are two vectors given by $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$. The magnitude of the component of $\vec{A}$ along $\vec{B}$ is

(A) $\frac{5}{\sqrt{2}}$ (B) $\frac{3}{\sqrt{2}}$ (C) $\frac{7}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{2}}$

Ans : (A)

Hints : Magnitude of components of $\vec{A}$ along $\vec{B}$ = $\frac{\vec{A}.\vec{B}}{|\vec{B}|} = \frac{\left(2\hat{i} + 3\hat{j}\right).\left(\hat{i} + \hat{j}\right)}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

12. Given $\vec{C} = \vec{A} \times \vec{B}$ and $\vec{D} = \vec{B} \times \vec{A}$. What is the angle between $\vec{C}$ and $\vec{D}$?

(A) 30° (B) 60° (C) 90° (D) 180°

Ans : (D)

Hints : $\vec{C}$ and $\vec{D}$ are antiparellel since $\vec{A} \times \vec{B} = -\left(\vec{B} \times \vec{A}\right)$

13. The acceleration 'a' (in ms^{-2}) of a body, starting from rest varies with time t (in s) following the equation a = 3t + 4
The velocity of the body at time t = 2s will be

(A) 10 ms^{-1} (B) 18 ms^{-1} (C) 14 ms^{-1} (D) 26 ms^{-1}

Ans : (C)

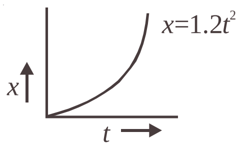
Hints : a = 3t + 4

$$\frac{dV}{dt} = 3t + 4$$

$$\int_0^V dV = \int_0^t (3t+4)dt$$

$$V = \frac{3t^2}{2} + 4t = \frac{12}{2} + 8 = 14 \quad m/s$$

14. Figure below shows the distance-time graph of the motion of a car. If follows from the graph that the car is



(A) at rest (B) in uniform motion

(C) in non-uniform acceleration (D) uniformly accelerated

Ans : (D)

Hints : Slope is increasing with constant rate. i.e motion is uniformaly accelerated

$x = 1.2t^2 \Rightarrow v = 2.4t \Rightarrow a = 2.4\ m/s^2$

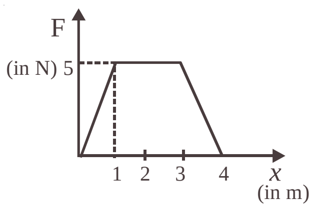
15. Two particles have masses m & 4m and their kinetic energies are in the ratio 2: 1. What is the ratio of their linear momenta ?

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{16}$

Ans : (A)

Hints : $\frac{KE_1}{KE_2} = \frac{\frac{p_1^2}{2m}}{\frac{p_2^2}{2\times 4m}} = \frac{2}{1} \Rightarrow \frac{p_1}{p_2} = \frac{1}{\sqrt{2}}$

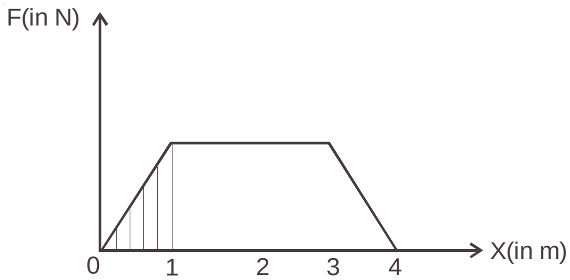
16. The force F acting on a particle moving in a straight line is shown below. What is the work done by the force on the particle in the 1st meter of the trajectory ?



(A) 5 J (B) 10 J (C) 15 J (D) 2.5 J

Ans : (D)

Hints : Work done in 1 meter = area of shaded curve = 1/2 × 1×5 =2.5 J



17. If the kinetic energy of a body changes by 20% then its momentum would change by –

(A) 20% (B) 24% (C) 40% (D) 44%

Ans : (No answer matching)

Hints : $\dfrac{\dfrac{p_f^2}{2m} - \dfrac{p_i^2}{2m}}{\dfrac{p_i^2}{2m}} \times 100 = 20$

$\Rightarrow \dfrac{p_f}{p_i} = \sqrt{1.2} = 1.095 \Rightarrow \dfrac{p_f - p_i}{p_i} = 0.095$

Therefore % increase = 9.5%

18. A bullet is fired with a velocity u making an angle of 60° with the horizontal plane. The horizontal component o the velocity of the bullet when it reaches the maximum height is

(A) u (B) 0 (C) $\dfrac{\sqrt{3}u}{2}$ (D) $\dfrac{u}{2}$

Ans : (D)

Hints : Horizontal velocity would be constant so the value of velocity at the highest point will be u/2

19. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is

(A) K (B) zero (C) $\dfrac{K}{4}$ (D) $\dfrac{K}{2}$

Ans : (C)

Hints : At highest point kinetic energy = 1/2m (v cos 60°)^2 = 1/4 × 1/2m v^2 = K/4

20. The poisson's ratio of a material is 0.5. If a force is applied to a wire of this material, there is a decrease in the cross-sectional area by 4%. The percentage increase in the length is :

(A) 1 % (B) 2% (C) 2.5% (D) 4%

Ans : (D)

Hints : Poisson ratio = 0.5

Therefore density is constant hence change in volume is zero we have

V = A × ℓ = constant

log V = log A + log ℓ or $\dfrac{dA}{A} + \dfrac{d\ell}{\ell} = 0 \Rightarrow \dfrac{d\ell}{\ell} = -\dfrac{dA}{A}$

That is 4%

21. Two spheres of equal masses but radii r_1 and r_2 are allowed to fall in a liquid of infinite column. The ratio of their terminal velocities is

(A) 1 (B) $r_1 : r_2$ (C) $r_2 : r_1$ (D) $\sqrt{r_1} : \sqrt{r_2}$

Ans : (Data incomplete)

Hints : We have $v_T = \dfrac{2r^2(\sigma - \rho)g}{9\eta}$

$$\frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{(\sigma_1 - \rho)}{(\sigma_2 - \rho)} \text{ ; given } m_1 = m_2 \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{\sigma_2}{\sigma_1}$$

22. Two massless springs of force constants K_1 and K_2 are joined end to end. The resultant force constant K of the system is

(A) $K = \dfrac{K_1 + K_2}{K_1K_2}$ (B) $K = \dfrac{K_1 - K_2}{K_1K_2}$ (C) $K = \dfrac{K_1K_2}{K_1 + K_2}$ (D) $K = \dfrac{K_1K_2}{K_1 - K_2}$

Ans : (C)

Hints : In series $K_{eff} = \dfrac{K_1K_2}{K_1 + K_2}$

23. A spring of force constant k is cut into two equal halves. The force constant of each half is

(A) $\dfrac{k}{\sqrt{2}}$ (B) k (C) $\dfrac{k}{2}$ (D) 2k

Ans : (D)

Hints : As $K\ell$ = constant

$$K' = 2K$$

24. Two rods of equal length and diameter have thermal conductivities 3 and 4 units respectively. If they are joined in series, the thermal conductivity of the combination would be

(A) 3.43 (B) 3.5 (C) 3.4 (D) 3.34

Ans : (A)

Hints : In series $R = R_1 + R_2$

$$\frac{2\ell}{K_{eff}A} = \frac{\ell}{K_1A} + \frac{\ell}{K_2A}$$

$$K_{eff} = \frac{24}{7} = 3.43$$

25. 19 g of water at 30° C and 5 g of ice at – 20° C are mixed together in a calorimeter. What is the final temperature of the mixture? Given specific heat of ice = 0.5 cal $g^{-1}(°C)^{-1}$ and latent heat of fusion of ice = 80 cal g^{-1}

(A) 0° C (B) – 5° C (C) 5° C (D) 10° C

Ans : (C)

Hints : $5 \times .5 \times 20 + 5 \times 80 + 5t = 19 \times 1 \times (30 - t)$

$t = 5°C$

26. It is difficult to cook rice in an open vessel by boiling it at high altitudes because of

(A) low boiling point and high pressure (B) high boiling point and low pressure

(C) low boiling point and low pressure (D) high boiling point and high pressure

Ans : (C)

Hints : At high altitude pressure is low and boiling point also low

27. The height of a waterfall is 50 m. If g = 9.8 ms^{-2} the difference between the temperature at the top and the bottom of the waterfall is:

(A) 1.17 °C (B) 2.17° C (C) 0.117° C (D) 1.43° C

Ans : (C)

Hints : $\frac{mgh}{J} = ms\Delta t \Rightarrow \Delta t = 0.117°C$

28. The distance between an object and a divergent lens is m times the focal length of the lens. The linear magnification produced by the lens is

(A) m (B) $\frac{1}{m}$ (C) m + 1 (D) $\frac{1}{m+1}$

Ans : (D)

Hints : u = – mf

$$\frac{1}{v} - \frac{1}{(-mf)} = -\frac{1}{f} \Rightarrow \quad \frac{1}{v} = -\frac{1}{f}\left(1+\frac{1}{m}\right) \Rightarrow -\frac{v}{u} = \left(\frac{1}{1+m}\right)$$

29. A 2.0 cm object is placed 15 cm in front of a concave mirror of focal length 10 cm. What is the size and nature of the image?

(A) 4 cm. real (B) 4 cm, virtual (C) 1.0 cm, real (D) None

Ans : (A)

Hints : $\frac{1}{v} - \frac{1}{15} = \frac{1}{-10} \Rightarrow v = -30 \ \text{cm}$

$m = \frac{-30}{-15} = 2$, image size = 4 cm

30. A beam of monochromatic blue light of wavelength 4200 Å in air travels in water of refractive index 4/3. Its wavelength in water will be:

(A) 4200 Å (B) 5800 Å (C) 4150 Å (D) 3150 Å

Ans : (D)

Hints : In water $\lambda = \frac{4200}{\frac{4}{3}} = 3150 \overset{0}{A}$

31. Two identical light waves, propagating in the same direction, have a phase difference δ. After they superpose the intensity of the resulting wave will be proportional to

(A) cos δ (B) cos (δ/2) (C) $\cos^2$ (δ/2) (D) $\cos^2\delta$

Ans : (C)

Hints : $I = 4I_0 \cos^2\left(\frac{\delta}{2}\right) \Rightarrow I \propto \cos^2\left(\frac{\delta}{2}\right)$

32. The equation of state for n moles of an ideal gas is PV = nRT, where R is a constant. The SI unit for R is

(A) JK^{-1} per molecule (B) $JK^{-1}\ mol^{-1}$ (C) $J\ Kg^{-1}\ K^{-1}$ (D) $JK^{-1}\ g^{-1}$

Ans : (B)

Hints : $JK^{-1}\ mol^{-1}$

33. At a certain place, the horizontal component of earth's magnetic field is $\sqrt{3}$ times the vertical component. The angle of dip at that place is

(A) 30° (B) 60° (C) 45° (D) 90°

Ans : (A)

Hints : $\tan\theta = \frac{V}{H} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

34. The number of electron in 2 coulomb of charge is

(A) 5×10^{29} (B) 12.5×10^{18} (C) 1.6×10^{19} (D) 9×10^{11}

Ans : (B)

Hints : $n = \frac{2}{1.6\times10^{-19}} = 12.5\times10^{18}$

35. The current flowing through a wire depends on time as $I = 3t^2 + 2t + 5$. The charge flowing through the cross section of the wire in time from t = 0 to t = 2 sec. is

(A) 22 C (B) 20 C (C) 18 C (D) 5 C

Ans : (A)

Hints : $Q = \int_0^2 \left(3t^2 + 2t + 5\right) dt = 22\,C$

36. If the charge on a capacitor is increased by 2 coulomb, the energy stored in it increases by 21%. The original charge on the capacitor is

(A) 10 C (B) 20 C (C) 30 C (D) 40 C

Ans : (B)

Hints : $\frac{\frac{q_f^2}{2C} - \frac{q_i^2}{2C}}{\frac{q_i^2}{2C}} \times 100 = 21 \quad and \quad q_f - q_i = 2$

solving we get $q_i = 20$ coulomb

37. The work done in carrying a charge Q once around a circle of radius r about a charge q at the centre is

(A) $\frac{qQ}{4\pi\varepsilon_0 r}$ (B) $\frac{qQ}{4\pi\varepsilon_0}\frac{1}{\pi r}$ (C) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{2\pi r}\right)$ (D) 0

Ans : (D)

Hints : Work done by conservative force in a round trip is zero

38. Four capacitors of equal capacitance have an equivalent capacitance C_1 when connected in series and an equivalent capacitance C_2 when connected in parallel. The ratio $\frac{C_1}{C_2}$ is:

(A) 1/4 (B) 1/16 (C) 1/8 (D) 1/12

Ans : (B)

Hints : $C_1 = \frac{C}{4} \quad and \quad C_2 = 4C \Rightarrow \frac{C_1}{C_2} = \frac{1}{16}$

39. Magnetic field intensity H at the centre of a circular loop of radius r carrying current I e m.u is

(A) r/I oersted (B) 2πI/r oersted (C) I/2πr oersted (D) 2πr/I oersted

Ans : (B)

Hints : $H = \frac{\mu_0 I}{2r} = \frac{\mu_0}{4\pi} \times \frac{2\pi I}{r}$

In e m.u system $\frac{\mu_0}{4\pi} = 1$. So $H = \frac{2\pi I}{r}$

40. Which of the following materials is the best conductor of electricity?

(A) Platinum (B) Gold (C) Silicon (D) Copper

Ans : (D)

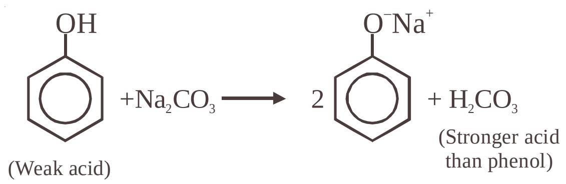
41. Which statement is incorrect

(A) Phenol is a weak acid (B) Phenol is an aromatic compound

(C) Phenol liberates CO_2 from Na_2CO_3 soln (D) Phenol is soluble in NaOH

Ans : (C)

Hints : Phenol does not liberate CO_2 from Na_2CO_3 solution



Note : Strong acid is not formed by weak acid

42. In which of the following reactions new carbon-carbon bond is not formed :

(A) Cannizaro reaction (B) Wurtz reaction (C) Aldol condensation (D) Friedel-Craft reaction

Ans : (A)

Hints : In cannizaro's reaction no new C–C bond is formed

e.g. $\text{H-}\overset{\overset{O}{\|}}{C}\text{-H} + \text{H-}\overset{\overset{O}{\|}}{C}\text{-H} \xrightarrow{50\%NaOH} CH_3OH + HCOO^-Na^+$

43. A compound is formed by substitution of two chlorine for two hydrogens in propane. The number of possible isomeric compounds is

(A) 4 (B) 3 (C) 5 (D) 2

Ans : (C)

Hints : $C_3H_8 \xrightarrow[+2Cl]{-2H} C_3H_6Cl_2$, following isomers of $C_3H_6Cl_2$ is possible

(I) (II) (III) (IV)

Due to presence of chiral carbon compound (IV) is optically active and forms an enantiomer. So total no of isomers =5

44. Which one of the following is called a carbylamine?

(A) R CN (B) R $CONH_2$ (C) R–CH=NH (D) R NC

Ans : (D)

45. For making distinction between 2-pentanone and 3-pentanone the reagent to be employed is
(A) $K_2Cr_2O_7/H_2SO_4$ (B) Zn-Hg/HCl (C) SeO_2 (D) Iodine/NaOH

Hints : In 2-pentanone *ie.*, $CH_3\text{–}\overset{\overset{\Large O}{||}}{C}\text{–}CH_2CH_2CH_3$, $CH_3\text{–}\overset{\overset{\Large O}{||}}{C}\text{–}$ group is present due to which it can show iodoform test. *i.e.*,

$$CH_3\text{–}\overset{\overset{O}{||}}{C}\text{–}CH_2\text{–}CH_2\text{–}CH_3 \xrightarrow{I_2/NaOH} \underset{\text{(Yellow ppt.)}}{CHI_3\downarrow} + CH_3CH_2\text{–}CH_2\text{–}\overset{\overset{O}{||}}{C} - O^- Na^+$$

46. Which one of the following formulae does not represent an organic compound?
(A) $C_4H_{10}O_4$ (B) $C_4H_8O_4$ (C) $C_4H_7ClO_4$ (D) $C_4H_9O_4$
Ans : (D)
Hints : Unsaturation factor = 0, 1, 1, 0.5 Hence (D)

47. The catalyst used for olefin polymerization is
(A) Ziegler-Natta Catalyst (B) Wilkinson Catalyst (C) Raney nickel catalyst (D) Merrifield resin
Ans : (A)
Hints : $TiCl_3 + (C_2H_5)_3Al$

48. The oxidant which is used as an antiseptic is :
(A) $KBrO_3$ (B) $KMnO_4$ (C) CrO_3 (D) KNO_3
Ans : (B)

49. Which of the following contributes to the double helical structure of DNA
(A) hydrogen bond (B) covalent bond (C) disulphide bond (D) van-der Waal's force
Ans : (A)

50. The monomer used to produce orlon is
(A) CH_2=CHF (B) CH_2=C Cl_2 (C) CH_2=CH Cl (D) CH_2=CH–CN
Ans : (D)
Hints : Orlon or PAN
Monomer $\Rightarrow$ CH_2=CH–CN

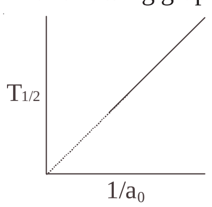
51. 1 mole of photon, each of frequency 2500 S^{-1}, would have approximately a total energy of :
(A) 1 erg (B) 1 Joule (C) 1 eV (D) 1 MeV
Ans : (A)
Hints : Total Energy = Nhv = $6.022 \times 10^{23} \times 6.626 \times 10^{-34}$ J.S. $\times$ 2500 s^{-1} = 9.9 erg $\approx$ 10 erg
In (A) option, it should be 10 erg instead of 1 erg.

52. If n_t number of radioatoms are present at time t, the following expression will be a constant :
(A) n_t/t (B) In n_t/t (C) d In n_t/dt (D) t n_t
Ans : (C)

Hints : $-\dfrac{dN}{dt} = \lambda N \Rightarrow -\dfrac{d\ln N}{dt} = \lambda$

Hence (C)

53. The following graph shows how $T_{1/2}$ (half-life) of a reactant R changes with the initial reactant concentration a_0.



The order of the reaction will be :
(A) 0 (B) 1 (C) 2 (D) 3

Ans : (C)

Hints : $t_{1/2} \propto \frac{1}{a^{n-1}}$

Hence (C)

54. The second law of thermodynamics says that in a cyclic process :

(A) work cannot be converted into heat
(B) heat cannot be converted into work
(C) work cannot be completely converted into heat
(D) heat cannot be completely converted into work

Ans : (D)

Hints : Because 0 K temperature is unattainable.

55. The equilibrium constant (K) of a reaction may be written as :

(A) $K = e^{-\Delta G/RT}$
(B) $K = e^{-\Delta G^0/RT}$
(C) $K = e^{-\Delta H/RT}$
(D) $K = e^{-\Delta H^0/RT}$

Ans : (B)

Hints : $\Delta G^o = -RT \ln K$

$$\Rightarrow \frac{\Delta G^o}{-RT} = \ln K$$

$$\therefore K = e^{-\Delta G^o/RT}$$

56. For the reaction $SO_2 + \frac{1}{2}O_2 = SO_3$, if we write $K_p = K_c(RT)^x$, then x becomes

(A) –1
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 1

Ans : (B)

Hints : $K_P = K_C(RT)^x$

$x = (\Sigma n_{(g)})_P - (\Sigma n_{(g)})_R$

$$= 1 - \frac{3}{2} = -\frac{1}{2}$$

57. If it is assumed that ${}^{235}_{92}U$ decays only by emitting α and β particles, the possible product of the decay is :

(A) ${}^{225}_{89}Ac$
(B) ${}^{227}_{89}Ac$
(C) ${}^{230}_{89}Ac$
(D) ${}^{231}_{89}Ac$

Ans : (B)

Hints : New mass no. = 235 – 2 × 4 = 227

New at. no. = 92 – 2 × 2 + 1 = 92 – 4 + 1 = 89

58. The time taken for 10% completion of a first order reactin is 20 mins. Then, for 19% completion, the reaction will take

(A) 40 mins
(B) 60 mins
(C) 30 mins
(D) 50 mins

Ans : (A)

Hints : $t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$

$$20 = \frac{2.303}{\lambda}\log\frac{100}{90} \quad \text{....... (i)}$$

$$t = \frac{2.303}{\lambda}\log\frac{100}{81} \quad \text{.......(ii)}$$

equation (i) / (ii)

$\therefore t = 40$ min.

59. Which of the following will decrease the pH of a 50 ml solution of 0.01 M HCl?

(A) addition of 5 ml of 1 M HCl
(B) addition of 50 ml of 0.01 M HCl
(C) addition of 50 ml of 0.002 M HCl
(D) addition of Mg

Ans : (A)

Hints : $50\text{ ml } 0.01\text{ M} \equiv 50 \times 0.01 = 0.5$ millimole

$5\text{ ml } 1\text{ (M)} \equiv 5 \times 1 = 5$ millimole

Total millimoles $= 5.5$ millimole

Total volume $= 55$ ml.

$$\text{Molarity} = \frac{5.5}{55} = 0.1(\text{M}) = 10^{-1}\ (\text{M})$$

pH = 1

60. Equal volumes of molar hydrochloric acid and sulphuric acid are neutralised by dilute NaOH solution and x kcal and y kcal of heat are liberated respectively. Which of the following is true?

(A) x=y
(B) $x = \dfrac{y}{2}$
(C) x=2y
(D) none of the above

Ans : (B)

Hints : Enthalpy of 1 g equivalent of strong acid and 1 g equivalent strong base = 13.7 kcal

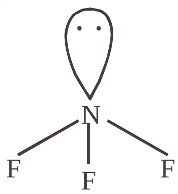
Equal volume contains double eq. of H_2SO_4 than HCl

61. Hybridisation of central atom in NF_3 is

(A) sp^3
(B) sp
(C) sp^2
(D) dsp^2

Ans : (A)

Hints :



3σ & 1 lone pair
Hyb. = sp^3

62. Of the following compounds the most acidic is

(A) As_2O_3
(B) P_2O_5
(C) Sb_2O_3
(D) Bi_2O_3

Ans : (B)

Hints : In a group as we go downwards, the oxide basic character increases hence maximum acidic oxide is P_2O_5

63. The half-life of a radioactive element is 10 hours. How much will be left after 4 hours in 1 g atom sample?

(A) 45.6×10^{23} atoms
(B) 4.56×10^{23} atoms
(C) 4.56×10^{21} atoms
(D) 4.56×10^{20} atoms

Ans : (B)

Hints : $t_{1/2} = 10$ hr. $\quad \text{K} = \dfrac{0.693}{10}$

$$4 = \frac{2.303 \times 10}{0.693} \log \frac{1}{\text{N}}$$

$$\log \frac{1}{\text{N}} = \frac{4 \times 0.693}{2.303 \times 10} = 0.12036$$

$$\log \text{N} = -0.12036 = \bar{1}.87964$$

$\text{N} = 7.575 \times 10^{-1}$ g atoms

$\therefore$ No. of atoms $= 7.575 \times 10^{-1} \times 6.023 \times 10^{23}$ atoms $= 4.56 \times 10^{23}$ atoms

64. For the Paschen series the values of n_1 and n_2 in the expression $\Delta E = Rhc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ are

(A) $n_1=1$, $n_2=2, 3, 4$........ (B) $n_1=2$, $n_2=3, 4, 5$........ (C) $n_1=3$, $n_2=4, 5, 6$........ (D) $n_1=4$, $n_2=5, 6, 7$........

Ans : (C)

Hints : In Paschen series electron shifting to third shell i.e., $n_1 = 3$ to $n_2 = 4, 5, 6,$ ......

65. Under which of the following condition is the relation $\Delta H = \Delta E + P\Delta V$ valid for a closed system?

(A) Constant Pressure (B) Constant temperature

(C) Constant temperature and pressure (D) Constant temperature, pressure and composition

Ans : (A)

Hints : This is applicable when pressure remains constant.

66. An organic compound made of C, H and N contains 20% nitrogen. Its molecular weight is :

(A) 70 (B) 140 (C) 100 (D) 65

Ans : (A)

Hints : Nitrogen at. wt. = 14 in a molecule minimum one atom of N is present

i.e., $20\% \equiv 14$ Molecular weight = 70

$100\% \equiv 14 \times 5 = 70$

67. In Cu-ammonia complex, the state of hybridization of Cu^{+2} is

(A) sp^3 (B) d^3s (C) sp^2f (D) dsp^2

Ans : (D)

Hints : In $[Cu(NH_3)_4]^+$

Cu^{+2} is in a state of dsp^2 hybridization and shape of the complex is square planar. (One e^- is excited from 3*d* to 4*p* during complex formation)

68. The reaction that takes place when Cl_2 gas is passed through conc. NaOH solution is :

(A) Oxidation (B) Reduction (C) Displacement (D) Disproportionation

Ans : (D)

Oxidation

Hints : $\overset{0}{Cl_2}$ + NaOH (conc. & hot) $\overset{-1}{NaCl} + NaCl\overset{+5}{O_3} + H_2O$

Reduction

Hence the reaction is disproportionation

69. "Electron" is an alloy of

(A) Mg and Zn (B) Fe and Mg (C) Ni and Zn (D) Al and Zn

Ans : (A)

Hints : Electron is an alloy of Mg(95%) + Zn(4.5%) and Cu(0.5%)

70. Blackened oil painting can be restored into original form by the action of :

(A) Chlorine (B) BaO_2 (C) H_2O_2 (D) MnO_2

Ans : (C)

Hints : Blackening of oil painting is due to PbS which is oxidised by H_2O_2 to form white $PbSO_4$

$\underset{\text{(Black)}}{PbS} + H_2O_2 \rightarrow \underset{\text{(white)}}{PbSO_4} + H_2O$

71. Of the following acids the one which has the capability to form complex compound and also possesses oxidizing and reducing properties is :

(A) HNO_3 (B) HNO_2 (C) HCOOH (D) HCN

Ans : (B) $H\overset{+3}{N}O_2$

Hints : Here oxidation state of N lies between –3 to +5

72. Atoms in a P_4 molecule of white phosphorus are arranged regularly in the following way :

(A) at the corners of a cube

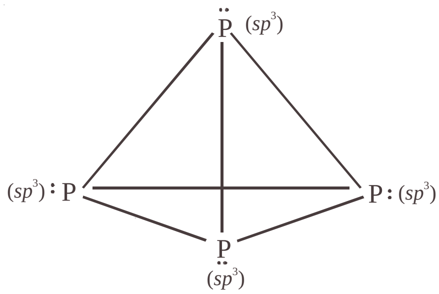
(B) at the corners of a octahedron

(C) at the corners of a tetrahedron

(D) at the centre and corners of a tetrahedron

Ans : (C)

Hints :



73. Which of the following statements is not correct

(A) Silicon is extensively used as a semiconductor

(B) Carborundum is SiC

(C) Silicon occurs in free state in nature

(D) Mica contains the element silicon

Ans : (C)

Hints : Silicon exist in nature in combined state as SiO_2

74. In aluminium extraction by the Bayer process, alumina is extracted from bauxite by sodium hydroxide at high temperature and pressures :

$$Al_2O_3\,(s) + 2OH^-(aq) \longrightarrow 2Al_2O_2^-(aq) + H_2O\,(1)$$

Solid impurities such as Fe_2O_3 and SiO_2 are removed and then $Al(OH)_4^-$ is reprecipitated :

$2Al(OH)_4^- \longrightarrow Al_2O_3.3H_2O\,(s) + 2OH^-\,(aq)$. In the industrial world :

(A) Carbon dioxide is added to precipitate the alumina

(B) Temperature and pressure are dropped and the supersaturated solution seeded

(C) Both (A) and (B) are practised

(D) The water is evaporated

Ans : (B)

75. The addition of HBr to 2-pentene gives

(A) 2-bromopentane only

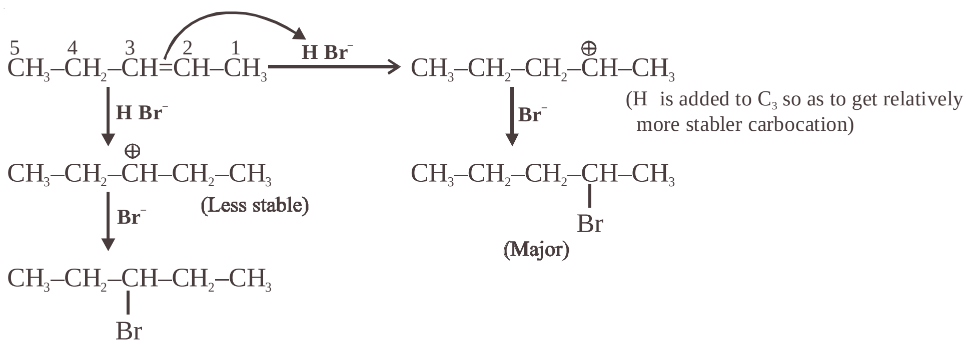
(B) 3-bromopentane only

(C) 2-bromopentane and 3-bromopentane

(D) 1-bromopentane and 3-bromopentane

Ans : (C)

Hints :



76. Ethelene can be separated from acetylene by passing the mixture through :

(A) fuming H_2SO_4 (B) pyrogallol (C) ammoniacal Cu_2Cl_2 (D) Charcoal powder

Ans : (C)

Hints : $H\text{–}C\equiv C\text{–}H + Cu_2Cl_2 \rightarrow \underset{\textbf{Red ppt.}}{Cu^+C^- \equiv C^-Cu^+\downarrow}$

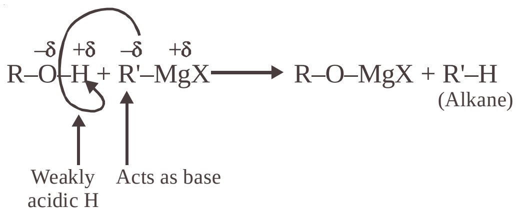
$H_2C{=}CH_2 + Cu_2Cl_2 \rightarrow$ No. ppt

77. Reaction of R OH with R′ MgX produces :

(A) RH (B) R′ H (C) R - R (D) R′ - R′

Ans : (B)

Hints :



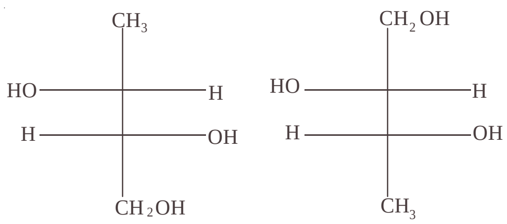
78. In the compound $HC \equiv C - CH = CH_2$ the hybridization of C–2 and C–3 carbons are respectively :

(A) sp^3 & sp^3 (B) sp^2 & sp^3 (C) sp^2 & sp (D) sp^3 & sp

Ans : (C)

Hints : $\overset{4}{H\text{–}C}\equiv\underset{sp}{\overset{3}{C}}\text{–}\underset{sp^2}{\overset{2}{C}}H{=}\overset{1}{C}H_2$ (Double bond is preferred)

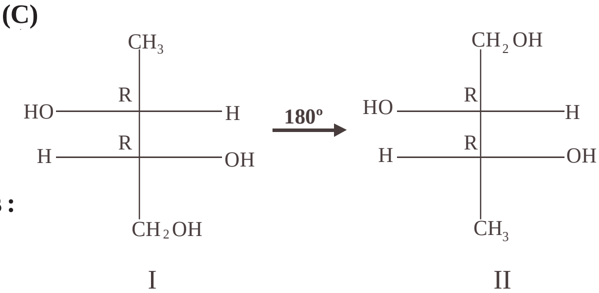
79. The two structures written below represent



(A) pair of diastereomers (B) pair of enantiomers (C) same molecule (D) both are optically inactive

Ans : (C)

Hints :



I & II are same Fischer projection because 180° rotation doesn't change configuration

80. Which of the following carbocations will be most stable ?

(A) $Ph_3\overset{+}{C}$ (B) $CH_3-\overset{+}{C}H_3$ (C) $(CH_3)_2\overset{+}{C}H$ (D) $CH_2=CH-\overset{+}{C}H_2$

Ans : (A)

Hints : $Ph\text{–}\overset{\oplus}{\underset{\underset{Ph}{|}}{C}}\text{–}Ph$ (Highly resonance stabilized)

PHYSICS

SECTION-II

1 The displacement x of a particle at time t moving under a constant force is t = $\sqrt{x}+3$, x in meters, t in seconds. Find the work done by the force in the interval from t = 0 to t = 6 second.

A. $t=\sqrt{x}+3 \Rightarrow x=(t-3)^2 \Rightarrow \mathrm{v}=2(t-3)$

v at t = 0, – 6 m/s

v at t = 6 sec., 6 m/s

change in KE is zero $\Rightarrow$ work done = 0

2 Calculate the distance above and below the surface of the earth at which the acceleration due to gravity is the same

A. $$\frac{GM}{(R+h)^2}=\frac{GM(R-h)}{R^3}$$

on solving we get

$-Rh + R^2 - h^2 = 0$

$$h=\frac{-R+\sqrt{R^2+4R^2}}{2}=\frac{(\sqrt{5}-1)R}{2}$$

3 A ray of light travelling inside a rectangular glass block of refractive index $\sqrt{2}$ is incident on the glass-air surface at an angle of incidence of 45°. Show that the ray will emerge into the air at an angle of refraction equal to 90°

A. Given C = 45°

$$\sin c=\frac{1}{\mu}=\frac{1}{\sqrt{2}}=\sin 45°$$

So the ray will graze the interface after refraction at an angle of 90°

4 Two cells each of same e.m.f 'e' but of internal resistances r_1 and r_2 are connected in series through an external resistance R. If the potential difference between the ends of the first cell is zero, what will be the value of R in terms r_1 and r_2 ?

A. $I=\dfrac{2e}{r_1+r_2+R}$; now $e - Ir_1 = 0$

$\Rightarrow r_2 - r_1 + R = 0,\ R = (r_1 - r_2)$

5 At time t = 0, a radioactive sample has a mass of 10 gm. Calculate the expected mass of radioactive sample after two successive mean lives.

A. Two successive mean lives = $\dfrac{2}{\lambda}$

No. of nuclei after two mean lives = $N_0 e^{-(\lambda)\left(\frac{2}{\lambda}\right)}=\dfrac{N_0}{e^2}$

Therefore mass = $\dfrac{10}{e^2}\ gm$

CHEMISTRY

SECTION-II

6 Calculate the number of H^+ ion present in 1 ml of a solution whose pH is 10.

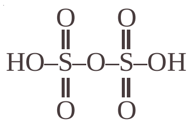
A. pH = 10

$[H^+] = 10^{-10}$ M

In 1000 ml solution there are 6.023×10^{13} H^+ ions

In 1 ml solution there are 6.023×10^{10} H^+ ions

7 Give the structure of pyro-sulfuric acid. How would you prepare it? What would you observe when colourless HI is added to pyro-sulfuric acid?

A. 

($H_2S_2O_7$)

(Pyro-sulfuric acid)

(Oleum)

Preparation of $H_2S_2O_7$: $\underset{\textbf{(98\%)}}{H_2SO_4} + SO_3 \longrightarrow \underset{\textbf{(Oleum)}}{H_2S_2O_7}$

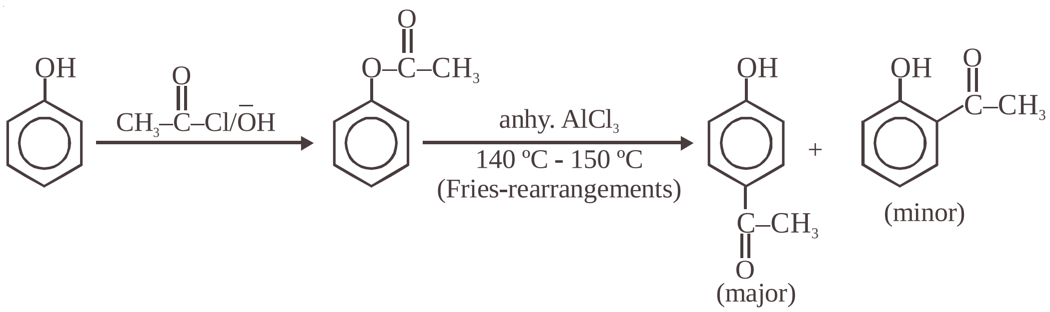
$\underset{\textbf{(Colourless)}}{H_2SO_4 + 2HI} \longrightarrow 2H_2O + SO_2 + \underset{\textbf{(Violet colour)}}{I_2}$

8 Write with a balanced chemical equation how gypsum is used for the conversion of ammonia into ammonium sulfate without using H_2SO_4.

A. Balanced reaction is

$2NH_3 + CaSO_4 + CO_2 + H_2O = (NH_4)_2SO_4 + CaCO_3$

9 Convert phenol to p-hydroxy acetophenone in not more than 2 steps.

A. 

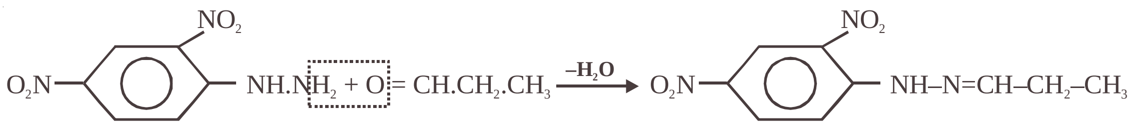
10 An organic compound 'A' on treatment with ammoniacal silver nitrate gives metallic silver and produces a yellow crystalline precipitate of molecular formula $C_9H_{10}N_4O_4$, on treatment with Brady's reagent. Give the structure of the organic compound 'A'.

A. Compound (A) is an aldehyde. It should be propanal CH_3CH_2CHO

Reactions :

(i) $CH_3CH_2CHO \xrightarrow[AgNO_3]{Ammoniacal} Ag\downarrow$

(Tollen's reagent)

(ii) 

(2, 4-Dinitro phenyl hydrazine)

(Brady's reagent)

(Yellow ppt. with mol. formula $C_9H_{10}N_4O_4$)

WB-JEE - 2009

BIOLOGY
QUESTIONS & ANSWERS

1. The length of DNA hgaving 23 base pair is

(A) 78 Å (B) 78.4 Å (C) 74.8 Å (D) 78.2 Å

Ans : (D)

Hints : Distance between adjacent base pairs = 3.4Å

2. Which I_g is produced in primary immune response?

(A) I_gA (B) I_gE (C) I_gG (D) I_gM

Ans : (D)

Hints : IgM is produced in primary response to the given antigen

3. The average diameter of Red Blood Corpuscles of man is

(A) 7.2 μm (B) 8.1 μm (C) 9.2 μm (D) 10.3 μm

Ans : (A)

Hints : The average diameter of RBC of man is 7.2 μm

4. FAD is electron acceptor during oxidation of which of the following?

(A) α -ketoglutarate $\longrightarrow$ Succinyl CoA (B) Succinic acid $\longrightarrow$ Fumaric acid

(C) Succinyl CoA $\longrightarrow$ Succinic acid (D) Fumaric acid $\longrightarrow$ Malic acid

Ans : (B)

Hints : FAD is electron acceptor during oxidation of succinic acid to fumaric acid

5. The chemical nature of hormones secreted by α & δ cells of pancreas is –

(A) Glycolipid (B) Glycoprotein (C) Steroid (D) Polypeptide

Ans : (D)

Hints : Hormones produced by α cells (glucagon) and β cells (somatostatin) are polypeptide

6. The genetic material of Rabies virus is

(A) Double stranded RNA (B) Single stranded RNA (C) Double stranded DNA (D) ssDNA

Ans : (B)

Hints : The genetic material of Rabies virus is ss RNA

7. T-lymphocyte is produced in

(A) Bone marrow (B) Spleen (C) Pancreas (D) Thymus

Ans : (A)

Hints : T-lymphocyte are produced in bone marrow but mature in thymus

8. How many ATP molecules are obtained from fermentation of 1 molecule of glucose?
(A) 2 (B) 4 (C) 3 (D) 5
Ans : (A)
Hints : Two molecules of ATP are produced by fermentation of one molecule of glucose

9. Number of nitrogenous bases in a Codon is
(A) 3 (B) 2 (C) 1 (D) 5
Ans : (A)
Hints : Three nitrogenous bases are found in a codon.

10. A character which is expressed in a hybrid is called
(A) Dominant (B) Recessive (C) Co-dominant (D) Epistatic
Ans : (A)
Hints : Dominant gene is expressed in a hybrid

11. In which stage of cell division chromosomes are most condensed?
(A) Prophase (B) Metaphase (C) Anaphase (D) Telophase
Ans : (B)
Hints : Chromosome is most condensed in metaphase

12. Which of the following is correct
(A) Haemophilic-Y chromosome (B) Down's syndrome - 21st chromosome
(C) Sickle cell anaemia-X chromosome (D) Parkinson's disease-X and Y chromosome
Ans : (B)
Hints : Down's syndrome is trisomy of 21st chromosome

13. Genetically engineered bacteria are being employed for production of
(A) Thyroxine (B) Human insulin (C) Cortisol (D) Epinephrine
Ans : (B)
Hints : Human insulin is now being produced by genetically engineered bacteria (E.coli). This insulin is called Humulin

14. Scientific name of sunflower is
(A) Hibiscus rosa-sinensis (B) Solanum nigram (C) Oryza sativa (D) Helianthus annus
Ans : (D)
Hints : Helianthus annuus is sunflower

15. By which of the following methods, new and better varieties of plants can be formed?
(A) Selection (B) Grafting
(C) Hybridization (D) Hybridization followed by selection
Ans : (D)
Hints : Better variety of plant can be formed by hybridisation followed by selection.

16. Which one is product of aerobic respiration?
(A) Malic acid (B) Ethyl alcohol (C) Lactic acid (D) Pyruvic acid
Ans : (A)
Hints : Malic acid is product of aerobic respiration

17. CO_2 acceptor in C_3 cycle is
(A) OAA (B) RUBP (C) PEP (D) Malic acid
Ans : (B)
Hints : RUBP (Ribulose 1.5. biphosphate) is CO_2 acceptor in C_3 plant

18. Virus was discovered by whom?
(A) Stanley (B) Ivanowsky (C) Herelle (D) Beijerinek
Ans : (B)
Hints : Ivanowsky discovered virus

19. Electron microscope is based on principle of
(A) Electromagnetic theory (B) Resolution of glass lenses (C) Magnification of glass lenses (D) Refraction of light
Ans : (A)
Hints : Electrton microscope is based on principle of electromagnetic theory

20. Citric acid cycle is the alternate name of which of the following?
(A) HMP shunt (B) Glycolysis (C) TCA cycle (D) Calvin cycle
Ans : (C)
Hints : Citric acid cycle or Krebs'cycle or Tricarboxylic acid cycle is alternative names.

21. Vascular tissue in higher plants develop from which of the following :
(A) Procambium (B) Protoderm (C) Periblem (D) Cortex
Ans : (A)
Hints : Procambium forms vascular tissue in higher plants

22. Which element is cause of etai etai disease
(A) Hg (B) Pb (C) Cd (D) As
Ans : (C)
Hints : Etai etia is caused by Cd

23. Chromosomes can be stained with one of the following chemicals
(A) Acetocarmine (B) Safranine (C) Light green (D) Eosin
Ans : (A)
Hints : Acetocarmine is used to stain chromosome

24. Which one of the following is the American Poultry breed
(A) Australop (B) Minovca (C) Assel (D) Rhod Island Red
Ans : (D)
Hints : Rhod island Red is the American Poultry Breed

25. Which part of the human brain is largest :
(A) Cerebellum (B) Thlamus (C) Cerebrum (D) Medulla
Ans : (C)
Hints : Cerebrum is the largest part of brain

26. When the other floral parts are arranged at the base of the gynoecium, the flower is called :
(A) Hypogynous flower (B) Perigynous flower (C) Epigynous flower (D) Agynous flower
Ans : (A)
Hints : Hypogynous flower/Superior ovary

27. In a CAM plant the concentration of organic acid :
(A) increases during the day (B) decreases or increases during the day
(C) increases during night (D) decreases during any time
Ans : (C)
Hints : In a CAM plant the concentration of organic acid increases during night

28. Protein coat of virus is known as :
(A) Capsid (B) Virion (C) Virioid (D) Bacterial wall
Ans : (A)
Hints : Protein coat of virus is called capsid

29. Net yield of aerobic respiration during Krebs' cycle per glucose molecule is :
(A) 2 ATP molecules (B) 8 ATP molecules (C) 36 ATP molecules (D) 38 ATP molecules
Ans : (A)
Hints : Net yield of 2ATP for two Krebs' cycle (1 glucose molecule) is produced at SLP

30. Feedback inhibition of enzymes is affected by which of the following

(A) enzyme (B) substrate (C) end products (D) intermediate end products

Ans : (C)

Hints : Feedback inhibition is affected by end products

31. The discovery of gibberellins is related with one of the following :

(A) Blast disease of rice (B) Rust disease of wheat

(C) 'Bakanae' disease of rice (D) Early blight disease of potato

Ans : (C)

Hints : Bakanae disease of rice/foolish seedling disease, discovered in Japan

32. Ornithophily refers to the pollination by which of the following :

(A) Insects (B) Birds (C) Snails (D) Air

Ans : (B)

Hints : Pollination by bird is called ornithophily.

33. Which f the following is an example of man-made ecosystem?

(A) Herbarium (B) Aquarium (C) Tissue culture (D) Forest

Ans : (B)

Hints : Aquarium is man-made ecosystem

34. Respiratory enzymes are present in the following organelle :

(A) Peroxysome (B) Chloroplast (C) Mitochondrion (D) Lysosome

Ans : (C)

Hints : Mitochondrion has respiratory enzymes for food oxidation

35. Pellagra is caused due to deficiency of the vitamin :

(A) Thiamin (B) Niacin (C) Pyridoxin (D) Biotin

Ans : (B)

Hints : Pellagra is caused by Niacin (nicotinic acid)

36. Which one of the following Leucocytes transforms into macrophages?

(A) Eosinophil (B) Basophil (C) Monocyte (D) Lymphocyte

Ans : (C)

Hints : Monocytes transforms to form macrophages

37. Mention the "Incubation Period" of P.vivax :

(A) 10–14 days (B) 20–25 days (C) 30 days (D) 45 days

Ans : (A)

Hints : Incubation period of P.vivax is 10-14 days.

38. The specific region of Hypothalamus, responsible for physiological sweat secretion, is

(A) Para-ventricular nucleus (B) Supra-Optic nucleus (C) Median Eminence (D) Pars Distalis

Ans : (A)

Hints : Paraventricular nucleus of hypothalamus is related to sweat secretion

39. The duration of cardiac cycle is :

(A) 0.8 sec (B) 0.8 μ sec (C) 0.08 sec (D) 0.008 sec

Ans : (A)

Hints : The duration of cardiac cycle is 0.8 sec

40. The intensity levels of whispering noise is :

(A) 10 – 15 dB (B) 20 – 40 dB (C) 45 – 50 dB (D) 50 – 55 dB

Ans : (A)

41. The wildlife Protection Act was introduced in :
(A) 1974 (B) 1981 (C) 1986 (D) 1991
Ans : (A)

42. In honey the percentage of Maltose and other sugar is
(A) 9.2 (B) 8.81 (C) 10.5 (D) 11.2
Ans : (B)

43. Identify the correct type of food chain :
dead animal $\longrightarrow$ blow fly maggots $\longrightarrow$ common frog $\longrightarrow$ snake
(A) Grazing food chain (B) Detrital food chain (C) Decomposer food chain (D) Predator food chain
Ans : (B)
Hins : It is Detritus food chain. Always starts from dead organic material.

44. Which is *not* applicable to the Biological species concept ?
(A) Hybridization (B) Natural population (C) Reproductive isolation (D) Gene Pool
Ans : (A)
Hints : Hybridization is not applicable to the bilogical species concept.

45. DNA sequence that code for protein are known as —
(A) Introns (B) Exons (C) Control regions (D) Intervening sequences
Ans. (B)
Hints : Exon is a part of DNA which codes for a protein

46. Which one of the following is a systemic insecticide ?
(A) Malathion (B) Parathion (C) Endrin (D) Furadan
Ans : (D)
Hints : The systemic insecticide is parathion.

47. The resolving power of a compound microscope will increase with —
(A) decrease in wave length of light and increase in numerical aperture
(B) increase in wave length of light and decrease in numerical aperture
(C) increase in both wave length of light and numerical aperture
(D) decrease in both wave length of light and numerical aperture
Ans : (A)
Hints : Decrease in wavelength of light and increase in numerical aperature is responsible.

48. Osteomalacia is a disease caused by the deficiency of —
(A) Calciferol (B) Retinol (C) Tocopherol (D) Phylloquinone
Ans : (A)
Hints : Osteomalacia is caused by calciferol deficiency in body

49. Which is the correct sequence of arrangement of types of W.B.C. in decreasing order in terms of number per mm^3 of human blood ?
(A) Eosinophils > Basophils > Neutrophils (B) Basophils > Eosinophils > Neutrophils
(C) Neutrophils > Eosinophils > Basophils (D) Eosinophils > Neutrophils > Basophils
Ans : (C)

50. Cells in G_0 phase of cell cycle
(A) Exit cell cycle (B) Enter cell cycle (C) Suspend cell cycle (D) Terminate cell cycle
Ans : (C)
Hints : G_o is the arrest / suspended phase of cell cycle.

51. Choose the correct non-protein amino acid
(A) Hydroxyproline (B) hydroxylysine (C) cystine (D) γ amino butyric acid
Ans : (D)

52. Seedless Banana is
(A) Parthenocarpic fruit (B) Multiple fruit (C) Drupe fruit (D) True fruit
Ans : (A)
Hints : It is formed by parthenocarpy (i.e. without fertilization)

53. The major site of protein breakdown to form free amino acids is in the
(A) Kidney (B) Spleen (C) Liver (D) Bone-Marrow
Ans : (C)

54. Collagen is a
(A) Phosphoprotein (B) Globulin (C) Derived Protein (D) Scleroprotein
Ans : (D)
Hints : Collagen is scleroprotein that requires vit-C for synthesis

55. The "Repeating Unit" of glycogen is
(A) Fructose (B) Mannose (C) Glucose (D) Galactose
Ans : (C)
Hints : Glycogen is a homopolymer of glucose

56. Graham's Law is correlated with
(A) Diffusion (B) Osmoregulation (C) Osmosis (D) Adsorption
Ans : (A)
Hints : Graham's law of diffusion, rate of diffusion $\alpha \dfrac{1}{\sqrt{\text{Density of particle}}}$

57. Which of the following does not act as a neurotransmitter ?
(A) Acetyl-choline (B) Glutamic acid (C) Epinephrine (D) Tyrosine
Ans : (D)
Hints : Tyrosine is not a neurotransmitter, it is an amono acid.

58. The generation of excitation-contraction coupling involves all the following events except :
(A) Generation of end-plate potential (B) Release of calcium from troponin
(C) Formation of cross-linkages between actin and myosin (D) Hydrolysis of ATP to ADP
Ans : (B)
Hints : During generation of excitation contraction coupling calcium is attached to troponin.

59. In AIDS, HIV kills :
(A) Antibody molecule (B) T_{HELPER} cell (C) Bone-Marrow cells (D) $T_{\text{Cytotoxic}}$ cell
Ans : (B)
Hints : HIV kills helper T cells.

60. Generally artificial Pacemaker consists of one battery made up of
(A) Nickel (B) Dry Cadmium (C) Photo Sensitive Material (D) Lithium
Ans : (D)
Hints : Lithium halide battery is used in artificial pacemaker

61. Goitre can occur as a consequence of all the following except :
(A) Iodine deficiency (B) Pituitary Adenoma
(C) Grave's disease (D) Excessive intake of exogenous thyroxine
Ans : (D)
Hints : Excessive intake of exogenous thyroxine will not produce the symptoms of Goitre.

62. Pernicious anaemia results due to deficiency of
(A) Vit B_1 (B) Vit A (C) Vit B_{12} (D) Iron
Ans : (C)

Hints : Pernicious anaemia is caused by deficiency of vit B_{12} or Cyanocobalamine.

63. Which of the following substances yield less than 4 Kcal/mol when its phosphate bond is hydrolysed
(A) Creatine Phosphate (B) ADP (C) Glucose-6-Phosphate (D) ATP
Ans : (C)

64. The Genetic deficiency of ADH-receptor leads to
(A) Diabetes mellitus (B) Glycosuria (C) Diabetes Insipidus (D) Nephrogenic Diabetes
Ans : (D)
Hints : Nephrogenic diabetes is due to genetic deficiency of ADH-receptor linked to x-chromosome.

65. Out of A-T, G-C pairing, bases of DNA may exist in alternate valency state owing to arrangement called
(A) Tautomerisational mutation (B) Analogue substitution
(C) Point mutation (D) Frameshift mutation
Ans : (A)
Hints : Tautomers are isomers of organic compund that readily interconvert by a chemical reaction. Commonly this reaction result in the formed migration of a H-atom or proton.

66. Cellular Totipotency was first demonstrated by
(A) F.C. Steward (B) Robert Hooke (C) T.Schwann (D) A.V. Leeuwenhock
Ans : (A)

67. Molecular scissors which cut DNA at specific site is
(A) Pectinase (B) Polymerase
(C) Restriction endo nuclease (D) Ligase
Ans : (C)
Hints : Restriction endonuclease is used to cut DNA at specific site (molecular scissor).

68. SO_2 pollution is indicated by
(A) *Desmodium* (Grasses) (B) *Sphagnum* (Mosses) (C) *Usnea* (Lichens) (D) *Cucurbita* (Climbers)
Ans : (C)
Hints : Lichon is the indicator of SO_2 pollution

69. Sporopollenin is chemically
(A) Homopolysaccharide (B) Fatty substance (C) Protein (D) Heteropolysaccharide
Ans : (B)
Hints : Sporopollenin is chemically a fatty substance that persits in fossil state.

70. During replication of DNA, Okazaki fragments are formed in the direction of :
(A) $3' \to 5'$ (B) $5' \to 3'$ (C) $5' \to 5'$ (D) $3' \to 3'$
Ans : (B)
Hints : Okazaki fragments are formed in the direction of $5' \to 3'$, they join after wards.

71. The chemical nature of chromatin is as follows :
(A) Nucleic acids (B) Nucleid acid & histone proteins
(C) Nucleic acids, histone & non histone proteins (D) Nucleic acids & non-histone proteins
Ans : (C)
Hints : Chromatin = nucleic acid + histone proteins + non - histone proteins.

72. Choose the minor carp from the following :
(A) *Cyprinus carpio* (B) *Labeo calbasu*
(C) *Labeo bata* (D) *Ctenopharyngodon idella*
Ans : (C)
Hints : *Laveo bata* is a minor carp., it size is smaller and growth rate slower.

73. The scientific name of Asian tiger mosquito :
(A) *Aedes aegypti* (B) *Aedes albopictus* (C) *Aedes taeniorhynchus* (D) *Aedes albolineatus*

Ans : (B)

Hints : *Aedes albopictus* is an Asian tiger mosquito.

74. The size of filtration slits of Glomerulus :

(A) 10 nm (B) 15 nm (C) 20 nm (D) 25 nm

Ans : (D)

Hints : Average size of filteration slit of glomerulus is 25 nm.

75. *Ornithorhynchus* is an example of :

(A) Dinosaur (B) Monotreme mammal (C) Marsupial mammal (D) Eutherian mammal

Ans : (B)

Hints : *Ornithorhynchus* (Duckbilled platypus) is monotreme.

76. *Scirpophage incertulus* is an example of :

(A) Monophagus pest (B) Diphagus pest (C) Oligophagus pest (D) Polyphagus pest

Ans : (A)

Hints : *Scrirpophaga incertelus* is a monophagus pest that feeds on a single plant.

77. Which one of the following ancestors of man first time showed bipedal movement ?

(A) Australopithecus (B) Cro-magnon (C) Java apeman (D) Peking man

Ans : (A)

78. Trophic levels in ecosystem is formed by :

(A) only bacteria (B) only plants

(C) only herbivores (D) Organisms linked in food chain

Ans : (D)

Hints : Trophic levels in ecosystem is formed by organisms linked in the food chain.

79. The life span of Honey bee drone is :

(A) 3 – 4 months (B) 1 – 2 months (C) 6 – 7 months (D) 10 – 12 months

Ans : (A)

80. Name of a gaseous plant hormone is

(A) IAA (B) Gibberellin (C) Ethylene (D) Abscisic acid

Ans. : (C)

Hints : Ethylene is a gaseous plant hormone that acts for ripening.

BIOLOGY

SECTION-II

1. Name one each specific plant hormone which perform the following exclusive physiological roles :
   a. Maintenance of apical dominance of shoots
   b. Internodal elongation
   c. Enhancement of cell division
   d. Change of sex in flowers

A. a) Apical dominance of shoot is maintained by Auxin
b) Internodal elongation by gibberellin
c) Enhancement of cell division by cytokinin
d) Change of sex in flowers G.A/Auxin/CK

2. Mention the function of the enzyme aconitase in Kreb's cycle

A.

$$Citrate \xrightarrow{Aconitase\ ,\ Fe^{2+}} Cis\ aconitate$$

$$Cis\ aconitate \xrightarrow{Aconitase} Isocitrate$$

3. Write down the scientific names of potato and tomato plants

A.

Name	Scientific name	family
Patato	Solanum tuberosum	Solanaceae
Tomato	Lycopersicum esculentum	Solanaceae

4. Why honey bee is regarded as social insect?

A. In bee hive labour based division in found, each having specific function. Queen bee lays eggs, while sterile females act as workers to perform all works of the hive including collection of nectar, formation of honey, rearing of young etc. Drone or male bees only act during the process of mating to provide spermatozoa

5. What are biopesticides ? Give two examples.

A. Biopestisides are those biological agents that are used for control of weeds, insects and pathogens
a) Nicotine-tobaco
b) Azadirachtin-Neem

6. What is Biosphere Reserve? State the main functions of biosphere reserve

A. Biosphere Reserve are multipurpose protected areas which are meant for preserving genetic diversity. It has 3 zones.
1) Core or Natural zone
2) Buffer zone
3) Transition zone or Manupulation zone.

Function
a) Restoration
b) Conservation
c) Development
d) Monitoring
e) Education and Research

7. What are stem cells ?

A. Stem cells are cells found in most, if not all, multicellular orginism. They are characterised by the ability to renew themselves through mitotic cell division and differentiating into diverse range of specialised cell types.

Example : Bone marrow cells

8. How ADH increases Blood Pressure?

A. ADH hormone is associated with water absorption by kidney. Hyposecretion of ADH leads to low water absorption and volume of urine is increased so. vol of blood will decrease and finally BP will decrease. More ADH leads to increased blood volume and consequently high B.P. ADH also related to vasoconstriction leading to high B.P.

9. Name two end-products of β -oxidation of fatty acid

A. Two products of - β Oxidation

a) Acetyl CoA

b) $FADH_2$

c) $NADH_2$

10. Mention of transformation event of immature sperm to matured spermatozoa. State the specific location of Sertoli cell within Testis.

A. Cell membrane and nuclear membrane start dissociation. Golgi structure modifies to form acrosome cap to contain the enzymes. Mitochondria increases in number and arrange in the middle piece. Distal centriole acts as basal body to give rise to flagella.

WB-JEE - 2009

MATHEMATICS
QUESTIONS & ANSWERS

1. If C is the reflecton of A (2, 4) in x-axis and B is the reflection of C in y-axis, then |AB| is

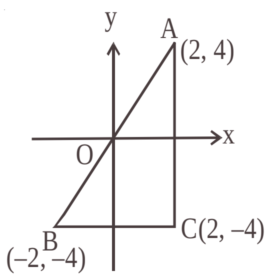
(A) 20 (B) $2\sqrt{5}$ (C) $4\sqrt{5}$ (D) 4

Ans : (C)

Hints : $A \equiv (2,4)$; $C \equiv (2,-4)$; $B \equiv (-2,-4)$

$|AB| = \sqrt{(2-(-2))^2 + (4-(-4))^2} = \sqrt{4^2+8^2}$

$= \sqrt{16+64} = \sqrt{80} = \sqrt{16\times 5} = 4\sqrt{5}$



2. The value of $\cos 15° \cos 7\frac{1°}{2} \sin 7\frac{1°}{2}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{16}$

Ans : (B)

Hints : $\cos 15° \cos 7\frac{1}{2}^0 \sin 7\frac{1}{2}^0 = \frac{1}{2}\left(2\sin 7\frac{1}{2}^0 \cos 7\frac{1}{2}^0\right).(\cos 15°)$

$\frac{1}{2}(\sin 15°)(\cos 15°) = \frac{1}{4}(2\sin 15° \cos 15°) = \frac{1}{4}\times \sin 30^0 = \frac{1}{8}$

3. The value of integral $\int_{-1}^{1} \frac{|x+2|}{x+2} dx$ is

(A) 1 (B) 2 (C) 0 (D) –1

Ans : (B)

Hints : $I = \int_{-1}^{1} \frac{|x+2|}{x+2} dx$, $x+2 = v \Rightarrow dx = dv$

$\therefore I = \int_{1}^{3} \frac{|v|}{v} dv = \int_{1}^{3} \frac{v}{v} dv = \int_{1}^{3} dv = 2$

4. The line y = 2t² intersects the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ in real points if

(A) $|t|\le 1$ (B) $|t|<1$ (C) $|t|>1$ (D) $|t|\ge 1$

Ans : (A)

Hints : $\frac{x^2}{9}+\frac{y^2}{4}=1$; $y = 2t^2$

$\frac{x^2}{9}+\frac{4t^4}{4}=1 \Rightarrow \frac{x^2}{9}+t^4=1 \Rightarrow x^2=9(1-t^4)$

$x^2 \ge 0 \Rightarrow 9(1-t^4)\ge 0 \Rightarrow t^4-1\le 0$

$\Rightarrow (t^2-1)(t^2+1)\le 0$

$\Rightarrow t^2-1\le 0\ \ (\because t^2+1>0)$

$\Rightarrow |t|\le 1$

5. General solution of sin x + cosx = $\min\limits_{a\in IR}\{1, a^2-4a+6\}$ is

(A). $\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}$ (B) $2n\pi+(-1)^n\frac{\pi}{4}$ (C) $n\pi+(-1)^{n+1}\frac{\pi}{4}$ (D) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$

Ans : (D)

Hints : $\sin x+\cos x=\min\limits_{a\in IR}\{1, a^2-4a+6\}$

$a^2 - 4a + 6 = (a-2)^2 + 2$ $\therefore \min\limits_{a\in IR}(a^2-4a+6)=2$

$\therefore \min\limits_{a\in IR}\{1, a^2-4a+6\}=\min\{1,\ 2\}=1$

sinx + cosx = 1 $\Rightarrow \frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(x+\frac{\pi}{4}\right)=\sin\frac{\pi}{4}$, $\Rightarrow x+\frac{\pi}{4}=n\pi+(-1)^n.\frac{\pi}{4}$

$\Rightarrow x=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$

6. If A and B square matrices of the same order and AB = 3I, then A^{-1} is equal to

(A) 3B (B) $\frac{1}{3}B$ (C) $3B^{-1}$ (D) $\frac{1}{3}B^{-1}$

Ans : (B)

Hints : $AB = 3I,\ A^{-1}.AB=3.A^{-1}I \Rightarrow B=3A^{-1}\Rightarrow A^{-1}=\frac{1}{3}B$

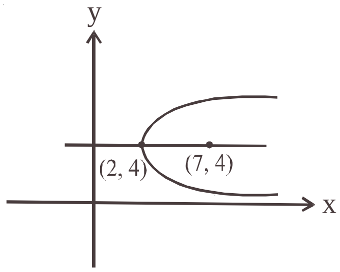
7. The co-ordinates of the focus of the parabola described parametrically by $x = 5t^2 + 2$, $y = 10t + 4$ are

(A) (7, 4) (B) (3, 4) (C) (3, –4) (D) (–7, 4)

Ans : (A)

Hints : $x = 5t^2 + 2$; $y = 10t + 4$, $\left(\frac{y-4}{10}\right)^2 = \left(\frac{x-2}{5}\right)$

or, $(y-4)^2 = 20(x-2)$



8. For any two sets A and B, A – (A – B) equals

(A) B (B) A – B (C) $A \cap B$ (D) $A^C \cap B^C$

Ans : (C)

Hints : $A-(A-B) = A-(A \cap B^c) = A \cap (A \cap B^c)^c = A \cap (A^c \cup B) = (A \cap A^c) \cup (A \cap B) = A \cap B$

9. If $a = 2\sqrt{2}$, b = 6, A = 45°, then

(A) no triangle is possible (B) one triangle is possible

(C) two triangle are possible (D) either no triangle or two triangles are possible

Ans : (A)

Hints : $a = 2\sqrt{2}$; $b = 6$; $A = 45^0$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin B = \frac{b}{a}\sin A$$

$$\Rightarrow \sin B = \frac{6}{2\sqrt{2}}\sin 45^\circ = \frac{3}{\sqrt{2}}.\frac{1}{\sqrt{2}} = \frac{3}{2} \Rightarrow \text{No triangle is possible since } \sin B > 1$$

10. A Mapping from IN to IN is defined as follows :

$f : IN \to IN$

$f(n) = (n + 5)^2$, $n \in IN$

(IN is the set of natural numbers). Then

(A) f is not one-to-one (B) f is onto

(C) f is both one-to-one and onto (D) f is one-to-one but not onto

Ans : (D)

Hints : $f : IN \to IN$; $f(n) = (n + 5)^2$

$(n_1 + 5)^2 = (n_2 + 5)^2$

$\Rightarrow (n_1 - n_2)(n_1 + n_2 + 10) = 0$

$\Rightarrow n_1 = n_2 \rightarrow$ one-to-one

There does not exist $n \in IN$ such that $(n + 5)^2 = 1$

Hence f is not onto

11. In a triangle ABC if $\sin A \sin B = \frac{ab}{c^2}$, then the triangle is

(A) equilateral (B) isosceles (C) right angled (D) obtuse angled

Ans : (C)

Hints : $\sin A \sin B = \frac{ab}{c^2}$

$$\Rightarrow c^2 = \frac{ab}{\sin A \sin B} = \left(\frac{a}{\sin A}\right)\left(\frac{b}{\sin B}\right)$$

$$\Rightarrow c^2 = \left(\frac{c}{\sin C}\right)^2 \Rightarrow \sin^2 C = 1 \Rightarrow \sin C = 1 \Rightarrow C = 90^o$$

12. $\int \frac{dx}{\sin x + \sqrt{3}\cos x}$ equals

(A) $\frac{1}{2}\ln\left|\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)\right|+c$ (B) $\frac{1}{2}\ln\left|\tan\left(\frac{x}{4}-\frac{\pi}{6}\right)\right|+c$ (C) $\frac{1}{2}\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)\right|+c$ (D) $\frac{1}{2}\ln\left|\tan\left(\frac{x}{4}+\frac{\pi}{3}\right)\right|+c$

where c is an arbitrary constant

Ans : (C)

Hints : $$\int \frac{dx}{\sin x + \sqrt{3}\cos x} = \int \frac{dx}{2\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)} = \frac{1}{2}\int \frac{dx}{\sin\left(x + \frac{\pi}{3}\right)}$$

$$= \frac{1}{2}\int \text{cosec}\left(x + \frac{\pi}{3}\right)dx = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)\right| + c$$

$$= \frac{1}{2}\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)\right| + c$$

13. The value of $\left(1+\cos \pi/6\right)\left(1+\cos \pi/3\right)\left(1+\cos 2\pi/3\right)\left(1+\cos 7\pi/6\right)$ is

(A) $\frac{3}{16}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$

Ans : (A)

Hints : $$\left(1+\cos\frac{\pi}{6}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{2\pi}{3}\right)\left(1+\cos\frac{7\pi}{6}\right)$$

$$= \left(1+\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{\sqrt{3}}{2}\right) = \left(1-\frac{3}{4}\right)\left(1-\frac{1}{4}\right) = \frac{1}{4}\times\frac{3}{4} = \frac{3}{16}$$

14. If $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta$ then

(A) $\frac{1}{3} \le P \le \frac{1}{2}$ (B) $P \ge \frac{1}{2}$ (C) $2 \le P \le 3$ (D) $-\frac{\sqrt{13}}{6} \le P \le \frac{\sqrt{13}}{6}$

Ans : (A)

Hints : $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta = \frac{1}{2}\sin^2\theta + \frac{1}{3}\left(1 - \sin^2\theta\right) = \frac{1}{3} + \frac{1}{6}\sin^2\theta$

$0 \le \sin^2\theta \le 1 \Rightarrow \frac{1}{3} \le \frac{1}{3} + \frac{1}{6}\sin^2\theta \le \frac{1}{3} + \frac{1}{6}$

$\Rightarrow \frac{1}{3} \le P \le \frac{1}{2}$

15. A positive acute angle is divided into two parts whose tangents are $\frac{1}{2}$ and $\frac{1}{3}$. Then the angle is

(A) $\pi/4$ (B) $\pi/5$ (C) $\pi/3$ (D) $\pi/6$

Ans : (A)

Hints : Angle $\theta = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}\right)$

$= \tan^{-1}\left(\frac{5/6}{5/6}\right) = \tan^{-1}(1) = \pi/4$

16. If $f(x) = f(a-x)$ then $\int_0^a x\, f(x)dx$ is equal to

(A) $\int_0^a f(x)dx$ (B) $\frac{a^2}{2}\int_0^a f(x)dx$ (C) $\frac{a}{2}\int_0^a f(x)dx$ (D) $-\frac{a}{2}\int_0^a f(x)dx$

Ans : (C)

Hints : $f(x) = f(a-x)$, $I = \int_0^a xf(x)dx = \int_0^a (a-x)f(a-x)dx$

$= \int_0^a (a-x)f(x)dx = a\int_0^a f(x)dx - I$

$\therefore 2I = a\int_0^a f(x)dx \Rightarrow I = \frac{a}{2}\int_0^a f(x)dx$

17. The value of $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$ is

(A) $\frac{\pi}{60}$ (B) $\frac{\pi}{20}$ (C) $\frac{\pi}{40}$ (D) $\frac{\pi}{80}$

Ans : (A)

Hints : $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} = \int_0^{\pi/2} \frac{\sec^2\theta}{(\tan^2\theta+4)(\tan^2\theta+9)} d\theta$ (putting $x = \tan\theta$)

$$= \frac{1}{5}\int_0^{\pi/2} \frac{\{(9+\tan^2\theta)-(4+\tan^2\theta)\}\sec^2\theta}{(\tan^2\theta+4)(\tan^2\theta+9)} d\theta$$

$$= \frac{1}{5}\left[\int_0^{\pi/2} \frac{\sec^2\theta}{4+\tan^2\theta} d\theta - \int_0^{\pi/2} \frac{\sec^2\theta}{9+\tan^2\theta} d\theta\right]$$

$$= \frac{1}{5}\left[\frac{1}{2}\tan^{-1}\left(\frac{\tan\theta}{2}\right)\Big|_0^{\pi/2} - \frac{1}{3}\tan^{-1}\left(\frac{\tan\theta}{3}\right)\Big|_0^{\pi/2}\right]$$

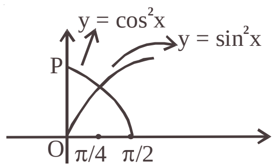
$$= \frac{1}{5}\left[\frac{1}{2}.\frac{\pi}{2} - \frac{1}{3}.\frac{\pi}{2}\right] = \left(\frac{\pi}{2}\right)\left(\frac{1}{5}\right)\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{\pi}{2}.\frac{1}{5}.\frac{1}{6} = \frac{\pi}{60}$$

18. If $I_1 = \int_0^{\pi/4} \sin^2 x dx$ and $I_2 = \int_0^{\pi/4} \cos^2 x dx$, then,

(A) $I_1 = I_2$ (B) $I_1 < I_2$ (C) $I_1 > I_2$ (D) $I_2 = I_1 + \pi/4$

Ans : (B)

Hints : $I_1 = \int_0^{\pi/4} \sin^2 x dx$; $I_2 = \int_0^{\pi/4} \cos^2 x dx$



In $\left(0, \frac{\pi}{4}\right)$, $\cos^2 x > \sin^2 x$ $\therefore \int_0^{\pi/4} \cos^2 x dx > \int_0^{\pi/4} \sin^2 x dx$

$I_2 > I_1$ i.e. $I_1 < I_2$

19. The second order derivative of a $\sin^3 t$ with respect to a $\cos^3 t$ at $t = \frac{\pi}{4}$ is

(A) 2 (B) $\frac{1}{12a}$ (C) $\frac{4\sqrt{2}}{3a}$ (D) $\frac{3a}{4\sqrt{2}}$

Ans : (C)

Hints : $y = a\sin^3 t$; $x = a\cos^3 t$

$$\frac{dy}{dt} = 3a\sin^2 t\cos t; \quad \frac{dx}{dt} = -3a\cos^2 t\sin t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\frac{\sin t}{\cos t} = -\tan t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-\tan t) = \frac{d}{dt}(-\tan t).\frac{dt}{dx}$$

$$= (-\sec^2 t)\frac{1}{-3a\cos^2 t\ \sin t} \quad = \frac{1}{+3a\cos^4 t\sin t}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\pi/4} = \frac{1}{3a\left(\frac{1}{\sqrt{2}}\right)^4 \cdot \left(\frac{1}{\sqrt{2}}\right)} = \frac{\left(\sqrt{2}\right)^5}{3a} = \frac{4\sqrt{2}}{3a}$$

20. The smallest value of $5\cos\theta + 12$ is

(A) 5 (B) 12 (C) 7 (D) 17

Ans : (C)

Hints : $5\cos\theta + 12,\ -1 \le \cos\theta \le 1$

$\Rightarrow -5 \le 5\cos\theta \le 5$

$\therefore 5\cos\theta + 12 \ge -5 + 12 \Rightarrow 5\cos\theta + 12 \ge 7$

21. The general solution of the differential equation $\frac{dy}{dx} = e^{y+x} + e^{y-x}$ is

(A) $e^{-y} = e^x - e^{-x} + c$ (B) $e^{-y} = e^{-x} - e^x + c$ (C) $e^{-y} = e^x + e^{-x} + c$ (D) $e^y = e^x + e^{-x} + c$

where c is an arbitrary constant

Ans : (B)

Hints : $e^{-y}dy = (e^x + e^{-x})dx$ Integrate

$$-e^{-y} = e^x - e^{-x} + c,\quad e^{-y} = e^{-x} - e^{+x} + c$$

22. Product of any r consecutive natural numbers is always divisible by

(A) $r!$ (B) $(r+4)!$ (C) $(r+1)!$ (D) $(r+2)!$

Ans : (A)

Hints : $(n+1)(n+2)\ldots\ldots\ldots(n+r)$

$$= \frac{(n+r)!}{n!}$$

$$= \frac{(n+r)!}{n!\,r!}\ r! = r!\ {}^{n+r}C_n$$

23. The integrating factor of the differential equation $x\log x\frac{dy}{dx} + y = 2\log x$ is given by

(A) e^x (B) $\log x$ (C) $\log(\log x)$ (D) x

Ans : (B)

Hints : $\frac{dy}{dx} + \frac{1}{x\log x}.y = \frac{2}{x}$

$$\text{If} = e^{\int\frac{1}{x\log x}dx} = e^{\int\frac{1/x}{\log x}dx}$$

$$= e^{\log(\log x)} = \log x$$

24. If $x^2 + y^2 = 1$ then

(A) $yy'' - (2y')^2 + 1 = 0$ (B) $yy'' + (y')^2 + 1 = 0$ (C) $yy'' - (y')^2 - 1 = 0$ (D) $yy'' + (2y')^2 + 1 = 0$

Ans : (B)

Hints : $2x + 2yy' = 0$

$x + yy' = 0$

$1 + yy'' + (y')^2 = 0$

25. If $c_0, c_1, c_2, \ldots\ldots\ldots\ldots\ldots\ldots, c_n$ denote the co-efficients in the expansion of $(1 + x)^n$ then the value of $c_1 + 2c_2 + 3c_3 + \ldots.. + nc_n$ is

(A) $n.2^{n-1}$ (B) $(n + 1)2^{n-1}$ (C) $(n + 1)\,2^n$ (D) $(n + 2)\,2^{n-1}$

Ans. (A)

Hints : $(1+x)^n = c_0 + xc_1 + x^2c_2 + \ldots\ldots.x^nc_n$

$n(1+x)^{n-1} = c_1 + 2xc_2 + \ldots\ldots\ldots nx^{n-1}c_n$

Put $x = 1$

$n(2)^{n-1} = c_1 + 2c_2 + 3c_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + nc_n$

26. A polygon has 44 diagonals. The number of its sides is

(A) 10 (B) 11 (C) 12 (D) 13

Ans : (B)

Hints : ${}^nC_2 - n = 44$

$$\frac{n(n-1)}{2} - n = 44$$

$$n\left[\frac{n-1}{2} - 1\right] = 44$$

$n(n-3) = 88$

$n(n-3) = 11 \times 8$

$n = 11$

27. If α, β be the roots of $x^2 - a(x - 1) + b = 0$, then the value of $\dfrac{1}{\alpha^2 - a\alpha} + \dfrac{1}{\beta^2 - a\beta} + \dfrac{2}{a+b}$

(A) $\dfrac{4}{a+b}$ (B) $\dfrac{1}{a+b}$ (C) 0 (D) –1

Ans : (C)

Hints : $x^2 - ax = a + 3$ $\alpha\beta = a + b$

$\alpha + \beta = a$

$\alpha^2 - a\alpha = -(a + b)$

$\beta^2 - a\alpha = -(a + b)$

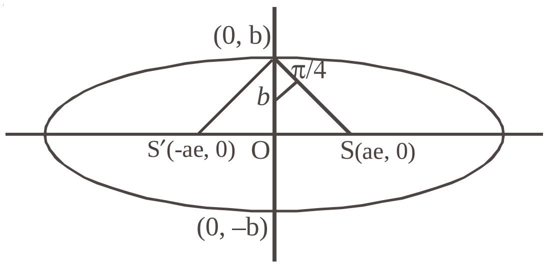
$$-\frac{1}{a+b} - \frac{1}{a+b} + \frac{2}{a+b} = 0$$

28. The angle between the lines joining the foci of an ellipse to one particular extremity of the minor axis is 90°. The eccentricity of the ellipse is

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{\sqrt{3}}$ (C) $\sqrt{\dfrac{2}{3}}$ (D) $\sqrt{\dfrac{1}{2}}$

Ans : (D)

Hints : $\dfrac{b}{ae} = \tan\dfrac{\pi}{4}$



$b = ae \Rightarrow \dfrac{b}{a} = e$

$e^2 = 1 - \dfrac{b^2}{a^2}$

$e^2 = 1 - e^2$

$e^2 = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{\sqrt{2}}$

29. The order of the differential equation $\dfrac{d^2y}{dx^2} = \sqrt{1 - \left(\dfrac{dy}{dx}\right)^2}$ is

(A) 3 (B) 2 (C) 1 (D) 4

Ans : (B)

30. The sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$

(A) 7 (B) 4 (C) 1 (D) 5

Ans : (B)

Hints : Put $1x - 21 = y$

$y^2 + y - 2 = 0$

$(y-1)(y+2) = 0$

$y = 1$ $\quad$ $y = -2$

$|x - 2| = 1$ $\quad$ (Not possible)

$x - 2 = \pm 1$

$x = 2 \pm 1$

$x = 3, 1$

Sum $= 4$

31. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4} \{3 - f(x)\}dx = 7$ then the value of $\int_{-1}^{2} f(x)dx$

(A) –2 (B) 3 (C) 4 (D) 5

Ans : (D)

Hints : $\int_{-1}^{4} f(x)\,dx = 4$

$3(4-2) - \int_{2}^{4} f(x)dx = 7$

$\int_{2}^{4} f(x)dx = -1$

$\int_{-1}^{2} f(x)dx = \int_{-1}^{4} f(x)dx + \int_{4}^{2} f(x)dx = 4 - \int_{2}^{4} f(x)dx = 4 - (-1) = 5$

32. For each $n \in$ N, $2^{3n} - 1$ is divisible by

(A) 7 (B) 8 (C) 6 (D) 16

where N is a set of natural numbers

Ans : (A)

Hints : $2^{3n} = (8)^n = (1+7)^n = 1 + {}^nC_1 7 + {}^nC_2 7^2 \ldots + {}^nC_n 7^n$

$2^{3n} - 1 = 7[{}^nC_1 + {}^nC_2 7 + \ldots.]$

33. The Rolle's theorem is applicable in the interval $-1 \le x \le 1$ for the function

(A) $f(x) = x$ (B) $f(x) = x^2$ (C) $f(x) = 2x^3 + 3$ (D) $f(x) = |x|$

Ans : (B)

Hints : $f(x) = x^2$ | and $f(1) = f(-1)$ for $f(x) = |x|$ but at $x = 0$, $f(x) = |x|$ is not differentiable hence (B) is the correct option.

$f(1) = 1 = f(-1)$

34. The distance covered by a particle in t seconds is given by $x = 3 + 8t - 4t^2$. After 1 second velocity will be

(A) 0 unit/second (B) 3 units/second (C) 4 units/second (D) 7 units/second

Ans : (A)

Hints : $v = \dfrac{dx}{dt} = 8 - 8t$

$t = 1, v = 8 - 8 = 0$

35. If the co-efficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ be same, then the value of 'a' is

(A) $\dfrac{3}{7}$ (B) $\dfrac{7}{3}$ (C) $\dfrac{7}{9}$ (D) $\dfrac{9}{7}$

Ans : (D)

Hints : $(3+ax)^9 = {}^9C_0 3^9 + {}^9C_1 3^8(ax) + {}^9C_2 3^7(ax)^2 + {}^9C_3 3^6(ax)^3$

${}^9C_2 3^7 a^2 = {}^9C_3 3^6 a^3$

$\dfrac{9}{7} = a$

36. The value of $\left(\dfrac{1}{\log_3 12} + \dfrac{1}{\log_4 12}\right)$ is

(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) 2

Ans : (C)

Hints : $\log_{12} 3 + \log_{12} 4 = \log_{12} 12 = 1$

37. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then the value of $\dfrac{1}{1+x} + \dfrac{1}{1+y} + \dfrac{1}{1+z}$ will be

(A) $x + y + z$ (B) 1 (C) $ab + bc + ca$ (D) abc

Ans : (B)

Hints : $1 + x = \log_a a + \log_a bc = \log_a abc$

$\dfrac{1}{1+x} = \log_{abc} a$, Similarly $\dfrac{1}{1+y} = \log_{abc} b$

$\dfrac{1}{1+z} = \log_{abc} c$, Ans. $= \log_{(abc)} abc = 1$

38. Using binomial theorem, the value of $(0.999)^3$ correct to 3 decimal places is

(A) 0.999 (B) 0.998 (C) 0.997 (D) 0.995

Ans : (C)

Hints : ${}^3C_0 - {}^3C_1(.001) + {}^3C_2(.001)^2 - {}^3C_3(.001)^3$

$= 1 - .003 + 3(.000001) - (.000000001) = 0.997$

39. If the rate of increase of the radius of a circle is 5 cm/.sec., then the rate of increase of its area, when the radius is 20 cm, will be

(A) 10π (B) 20π (C) 200π (D) 400π

Ans : (C)

Hints : $A = \pi r^2$ $\quad \frac{dr}{dt} = 5$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi\, 20\,(5)$$

$= 200\,\pi$

40. The quadratic equation whose roots are three times the roots of $3ax^2 + 3bx + c = 0$ is

(A) $ax^2 + 3bx + 3c = 0$ (B) $ax^2 + 3bx + c = 0$ (C) $9ax^2 + 9bx + c = 0$ (D) $ax^2 + bx + 3c = 0$

Ans : (A)

Hints : $3a\alpha^2 + 3b\alpha + c = 0$

$$x = 3\alpha \Rightarrow \alpha = \frac{x}{3}$$

$$3a\frac{x^2}{9} + 3b.\frac{x}{3} + c = 0$$

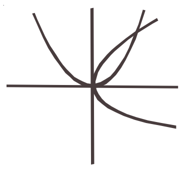
$ax^2 + 3bx + 3c = 0$

41. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(A) $2\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{4}{3}\right)$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Ans : (C)

Hins : Angle between axes (since co-ordinate axes are the tangents for the given curve).



42. In triangle ABC, $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$, then B is equal to

(A) 30º (B) 60º (C) 90º (D) 120º

Ans : (C)

Hints : $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\sin B = \frac{b}{a}.\sin A = \frac{3}{2}\cdot\frac{2}{3} = 1$$

$$B = \frac{\pi}{2}$$

43. $\int_0^{1000} e^{x-[x]}$ is equal to

(A) $\frac{e^{1000}-1}{e-1}$ (B) $\frac{e^{1000}-1}{1000}$ (C) $\frac{e-1}{1000}$ (D) 1000 (e – 1)

Ans : (D)

Hins : $I = 1000\int_0^1 e^{x-[x]}$

$= 1000\int_0^1 e^x dx = 1000(e^x)_0^1 = 100(e-1)$

Period of function is 1

44. The coefficient of x^n, where n is any positive integer, in the expansion of $(1 + 2x + 3x^2 + ... \infty)^{½}$ is

(A) 1 (B) $\frac{n+1}{2}$ (C) 2n + 1 (D) n + 1

Ans : (A)

Hints :
$$\begin{aligned} s &= 1 + 2x + 3x^2 ....\infty \\ xs &= x + 2x^2 + ...\infty \\ \hline s(1-x) &= 1 + x + x^2 + ...\infty \end{aligned}$$
$$s = \frac{1}{(1-x)^2}$$

$f(x) = \frac{1}{1-x}$, $f(x) = (1-x)^{-1} = 1 + x + x^2 + x^3 .... = 1$

45. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = a^2$ intersect at two distinct points if

(A) a < 2 (B) 2 < a < 8 (C) a > 8 (D) a = 2

Ans. (B)

Hints : $C_1(5, 0)$ $r_1 = \sqrt{25-16} = 3$

$C_2(0, 0)$ $r_2 = a$

$r_1 \,\&\, r_2 < C_1C_2 < r_1 + r_2$

$|a-3| < \sqrt{25} < a+3$

$|a-3| < 5 < a+3$

$-5 < a-3 < 5$ $\quad$ $2 < a$

$-2 < a < 8$

$2 < a < 8$

46. $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is equal to

(A) $\log(\sin^{-1} x) + c$ (B) $\frac{1}{2}(\sin^{-1} x)^2 + c$ (C) $\log\left(\sqrt{1-x^2}\right) + c$ (D) $\sin(\cos^{-1} x) + c$

where c is an arbitrary constant

Ans : (B)

Hints : $I = \int t\,dt$ $\sin^{-1} x = t$

$= \frac{1}{2}t^2 + c$ $\frac{1}{\sqrt{1-x^2}} dx = dt$

$= \frac{1}{2}(\sin^{-1} x)^2 + c$

47. The number of points on the line $x + y = 4$ which are unit distance apart from the line $2x + 2y = 5$ is

(A) 0 (B) 1 (C) 2 (D) Infinity

Ans : (A)

Hints : $x + y = 4$

$x + y = \frac{5}{2}$

$PQ = \frac{4 - 5/2}{\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

48. Simplest form of $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$ is

(A) $\sec\frac{x}{2}$ (B) $\sec x$ (C) $\operatorname{cosec} x$ (D) 1

Ans : (A)

Hints : $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2.2\cos^2 2x}}}} = \frac{2}{\sqrt{2+\sqrt{2+2\cos 2x}}} = \frac{2}{\sqrt{2+\sqrt{2.2\cos^2 x}}}$

$= \frac{2}{\sqrt{2+2\cos x}} = \frac{2}{2\cos\frac{x}{2}} = \sec\frac{x}{2}$

49. If $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) –1

Ans : (A)

Hints : $y=\tan^{-1}\sqrt{\dfrac{1-\cos\left(\dfrac{\pi}{2}-x\right)}{1+\cos\left(\dfrac{\pi}{2}-x\right)}}$

$$=\tan^{-1}\sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}=\tan^{-1}\left|\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|=\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\frac{dy}{dx}=-\frac{1}{2}$$

50. If three positive real numbers a, b, c are in A.P. and $abc = 4$ then minimum possible value of b is

(A) $2^{3/2}$ (B) $2^{2/3}$ (C) $2^{1/3}$ (D) $2^{5/2}$

Ans : (B)

Hints : $(b - d)\, b\, (b + d) = 4$

$(b^2 - d^2)\, b = 4$

$b^3 = 4 + d^2 b$

$b^3 \geq 4 \Rightarrow b \geq (2)^{2/3}$

51. If $5\cos 2\theta + 2\cos^2\dfrac{\theta}{2} + 1 = 0$, when $(0 < \theta < \pi)$, then the values of θ are :

(A) $\dfrac{\pi}{3}\pm\pi$ (B) $\dfrac{\pi}{3}, \cos^{-1}\left(\dfrac{3}{5}\right)$ (C) $\cos^{-1}\left(\dfrac{3}{5}\right)\pm\pi$ (D) $\dfrac{\pi}{3}, \pi-\cos^{-1}\left(\dfrac{3}{5}\right)$

Ans : (D)

Hints : $5\cos 2\theta + 1 + \cos\theta + 1 = 0$

$5(2\cos^2\theta - 1) + \cos\theta + 2 = 0$

$10\cos^2\theta + \cos\theta - 3 = 0$

$(5\cos\theta + 3)(2\cos\theta - 1) = 0$

$\cos\theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{3}$

$\cos\theta = -\dfrac{3}{5}$

$\theta = \cos^{-1}\left(-\dfrac{3}{5}\right)$

$= \pi - \cos^{-1}\left(\dfrac{3}{5}\right)$

52. For any complex number z, the minimum value of $|z| + |z-1|$ is

(A) 0 (B) 1 (C) 2 (D) –1

Ans : (B)

Hints : $1 = |z-(z-1)|$

$1 \leq |z| + |z-1|$

53. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$ there is / are

(A) one pair of common tangents
(B) only one common tangent
(C) three common tangents
(D) no common tangent

Ans : (D)

Hints : $C_1\,(0,0)$ $\quad r_1 = 4$

$C_2\,(0,1)$ $\quad r_2 = \sqrt{0+1} = 1$

$$C_1C_2 = \sqrt{0+1} = 1$$

$$r_1 - r_2 = 3$$

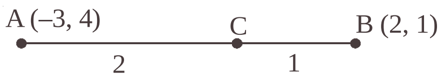
$$C_1C_2 < r_1 - r_2$$

54. If C is a point on the line segment joining A (–3, 4) and B (2, 1) such that AC = 2BC, then the coordinate of C is

(A) $\left(\frac{1}{3},2\right)$
(B) $\left(2,\frac{1}{3}\right)$
(C) $(2,7)$
(D) $(7,2)$

Ans : (A)

Hints :



$$C\left(\frac{4-3}{3},\frac{2+4}{3}\right)$$

$$C\left(\frac{1}{3},2\right)$$

55. If a, b, c are real, then both the roots of the equation $(x-b)(x-c)+(x-c)(x-a)+(x-a)(x-b)=0$ are always

(A) positive
(B) negative
(C) real
(D) imaginary

Ans : (C)

Hints : $3x^2 - 2x(a+b+c) + ab + bc + ca = 0$

$$D = 4(a+b+c)^2 - 4.3(ab+bc+ca)$$

$$= 4(a^2+b^2+c^2-ab-bc-ca)$$

$$= 2[(a-b)^2+(b-c)^2+(c-a)^2]$$

$$= [(a-b)^2+(b-c)^2+(c-a)^2]$$

$$\geq 0$$

56. The sum of the infinite series $1+\frac{1}{2!}+\frac{1.3}{4!}+\frac{1.3.5}{6!}+.......$ is

(A) e
(B) e^2
(C) $\sqrt{e}$
(D) $\frac{1}{e}$

Ans : (C)

Hints : $T_n = \dfrac{1.3.5....(2n-1)}{\lfloor 2n}$

$$= \frac{\lfloor 2n}{\lfloor 2n\,(2.4...2n)}$$

$$= \frac{\lfloor 2n}{2^{n} \lfloor n \lfloor 2n}$$

$$= \frac{x^{n}}{\lfloor n} \qquad \frac{1}{2} = x$$

$$\therefore \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + ... = e^x - 1$$

$$\exp = 1 + e^x - 1 = e^x = e^{1/2}$$

57. The point (–4, 5) is the vertex of a square and one of its diagonals is $7x – y + 8 = 0$. The equation of the other diagonal is

(A) $7x - y + 23 = 0$ (B) $7y + x = 30$ (C) $7y + x = 31$ (D) $x - 7y = 30$

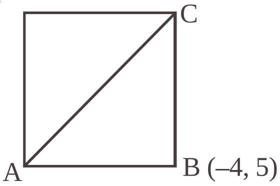
Ans : (C)

Hints : $x + 7y = k$ ......(1)

$-4 + 35 = k$

$31 = k$

$x + 7y - 31 = 0$



58. The domain of definition of the function $f(x) = \sqrt{1 + \log_e(1-x)}$ is

(A) $-\infty < x \le 0$ (B) $-\infty < x \le \frac{e-1}{e}$ (C) $-\infty < x \le 1$ (D) $x \ge 1 - e$

Ans : (B)

Hints : $1 - x > 0 \Rightarrow x < 1$

$1 + \log_e(1-x) \ge 0$

$\log_e(1-x) \ge -1 \Rightarrow 1 - x \ge e^{-1}$

$x \le 1 - \frac{1}{e}$

$x \le \frac{e-1}{e}$

59. For what value of m, $\frac{a^{m+1} + b^{m+1}}{a^m + b^m}$ is the arithmetic mean of 'a' and 'b'?

(A) 1 (B) 0 (C) 2 (D) None

Ans : (B)

Hints : $\frac{a^{m+1} + b^{m+1}}{a^m + b^m} = \frac{a+b}{2}$

$m = 0$ Satisfy.

60. The value of the limit $\lim\limits_{x\to 1} \dfrac{\sin(e^{x-1}-1)}{\log x}$ is

(A) 0 (B) e (C) $\dfrac{1}{e}$ (D) 1

Ans : (D)

Hints : $\operatorname*{Lt}\limits_{h\to 0} \dfrac{\sin(e^h-1)}{\log(1+h)}$ Put $x = 1+h$

$$= \operatorname*{Lt}_{h\to 0} \frac{\sin(e^h-1)}{(e^h-1)} \cdot \frac{(e^h-1)}{\log(1+h)}$$

$$= \operatorname*{Lt}_{h\to 0} \frac{\sin(e^h-1)}{(e^h-1)} \cdot \frac{(e^h-1)}{h} \cdot \frac{h}{\log(1+h)}$$

$= 1.\,1.\,1$

$= 1$

61. Let $f(x) = \dfrac{\sqrt{x+3}}{x+1}$ then the value of $\operatorname*{Lt}\limits_{x\to -3-0} f(x)$ is

(A) 0 (B) does not exist (C) $\dfrac{1}{2}$ (D) $-\dfrac{1}{2}$

Ans : (B)

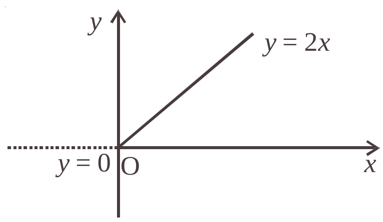
Hints : Because on left hand side of 3 function is not defined.

62. $f(x) = x + |x|$ is continuous for

(A) $x \in (-\infty, \infty)$ (B) $x \in (-\infty, \infty) - \{0\}$ (C) only $x > 0$ (D) no value of x

Ans : (A)

Hints : $f(x) = \begin{cases} 2x\,; & x \ge 0 \\ 0\,; & x < 0 \end{cases}$



63. $\tan\left[\dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}\left(\dfrac{a}{b}\right)\right] + \tan\left[\dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}\left(\dfrac{a}{b}\right)\right]$ is equal to

(A) $\dfrac{2a}{b}$ (B) $\dfrac{2b}{a}$ (C) $\dfrac{a}{b}$ (D) $\dfrac{b}{a}$

Ans : (B)

Hints : Let $\dfrac{1}{2}\cos^{-1}\left(\dfrac{a}{b}\right) = \theta$, then $\cos 2\theta = \dfrac{a}{b}$

$$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$$

$$=\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=2\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)=\frac{2}{\cos 2\theta}=\frac{2}{a/b}=\frac{2b}{a}$$

64. If $i=\sqrt{-1}$ and n is a positive integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is euqal to

(A) 1 (B) i (C) i^n (D) 0

Ans : (D)

Hints : $i^n(1+i+i^2+i^3)=i^n(1+i-1-i)=0$

65. $\int \frac{dx}{x(x+1)}$ equals

(A) $\ln\left|\frac{x+1}{x}\right|+c$ (B) $\ln\left|\frac{x}{x+1}\right|+c$ (C) $\ln\left|\frac{x-1}{x}\right|+c$ (D) $\ln\left|\frac{x-1}{x+1}\right|+c$

where c is an arbitrary constant.

Ans : (B)

Hints : $\int\frac{dx}{x(x+1)}=\int\left(\frac{1}{x}-\frac{1}{x+1}\right)dx=\int\frac{dx}{x}-\int\frac{dx}{x+1}=\ln|x|-\ln|x+1|+C=\ln\left|\frac{x}{x+1}\right|+C$

66. If a, b, c are in G.P. ($a>1, b>1, c>1$), then for any real number x (with $x>0, x\neq 1$), $\log_a x$, $\log_b x$, $\log_c x$ are in

(A) G.P. (B) A.P. (C) H.P. (D) G..P. but not in H.P.

Ans : (C)

Hints : a, b, c are in G.P.

$\Rightarrow \log_x a, \log_x b, \log_x c$ are in A.P.

$\Rightarrow \frac{1}{\log_x a}, \frac{1}{\log_x b}, \frac{1}{\log_x c}$ are in H.P.

$\Rightarrow \log_a x, \log_b x, \log_c x$ are in H.P.

67. A line through the point A (2, 0) which makes an angle of 30º with the positive direction of x-axis is rotated about A in clockwise direction through an angle 15º. Then the equation of the straight line in the new position is

(A) $(2-\sqrt{3})x+y-4+2\sqrt{3}=0$ (B) $(2-\sqrt{3})x-y-4+2\sqrt{3}=0$

(C) $(2-\sqrt{3})x-y+4+2\sqrt{3}=0$ (D) $(2-\sqrt{3})x+y+4+2\sqrt{3}=0$

Ans : (B)

Hints : Equation of line in new position :

$y-0=\tan 15^\circ (x-2)$

$\Rightarrow y=\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)(x-2)$

$\Rightarrow y=\frac{(\sqrt{3}-1)^2}{2}(x-2)$

$\Rightarrow 2y = \left(4 - 2\sqrt{3}\right)(x - 2)$

$\Rightarrow y = \left(2 - \sqrt{3}\right)(x - 2)$

$\Rightarrow \left(2 - \sqrt{3}\right)x - y - 4 + 2\sqrt{3} = 0$

68. The equation $\sqrt{3}\sin x + \cos x = 4$ has

(A) only one solution (B) two solutions (C) infinitely many solutions (D) no solution

Ans : (D)

Hints : $\sqrt{3}\sin x + \cos x = 2\sin\left(x + \frac{\pi}{6}\right) \le 2.$ Therefore

$\sqrt{3}\sin x + \cos x = 4$ cannot have a solution

69. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through (–1 , 1). The equation of the curve is

(A) $y = x^3 + 2$ (B) $y = -x^3 - 2$ (C) $y = 3x^3 + 4$ (D) $y = -x^3 + 2$

Ans : (A)

Hints : $\frac{dy}{dx} = 3x^2 \Rightarrow \int dy = \int 3x^2 dx \Rightarrow y = x^3 + C$

Curve passes through (–1, 1). Hence $1 = -1 + C \Rightarrow C = 2$

$\therefore y = x^3 + 2$

70. The modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$ is

(A) $\sqrt{5}$ unit (B) $\frac{\sqrt{11}}{5}$ unit (C) $\frac{\sqrt{5}}{5}$ unit (D) $\frac{\sqrt{12}}{5}$ unit

Ans : (C)

Hints : $\frac{1-i}{3+i} + \frac{4i}{5} = \frac{5-5i+4i(3+i)}{5(3+i)} = \frac{5-5i+12i-4}{5(3+i)} = \frac{1+7i}{5(3+i)} = \frac{(1+7i)(3-i)}{5(9+1)}$

$= \frac{3+21i-i+7}{5\times10} = \frac{10+20i}{5\times10} = \frac{1+2i}{5}$

$\therefore$ Modulus $= \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \sqrt{\frac{1}{25} + \frac{4}{25}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$ unit

71. The equation of the tangent to the conic $x^2 - y^2 - 8x + 2y + 11 = 0$ at (2, 1) is

(A) $x + 2 = 0$ (B) $2x + 1 = 0$ (C) $x + y + 1 = 0$ (D) $x - 2 = 0$

Ans : (D)

Hints : Equation of tangent at (x_1, y_1) is

$xx_1 - yy_1 - 4(x + x_1) + (y + y_1) + 11 = 0$

$x_1 = 2; y = 1$

$\therefore$ Equation of tangent is

$2x - y - 4(x+2) + (y+1) + 11 = 0$

or $-2x - 8 + 12 = 0$

or $-2x+4=0$

or $2x=4$

or $x=2$

or $x-2=0$

72. A and B are two independent events such that P(A∪B′) = 0.8 and P(A) = 0.3. The P(B) is

(A) $\frac{2}{7}$ (B) $\frac{2}{3}$ (C) $\frac{3}{8}$ (D) $\frac{1}{8}$

Ans : (A)

Hints : Let P(B) = x

$P(A\cup B') = P(A) + P(B') - P(A\cap B') = 0.3 + (1-x) - 0.3(1-x)$

or $0.8 = 1 - x + 0.3x$

or $1 - 0.7x = 0.8$

or $0.7x = 0.2$

or $x = \frac{2}{7}$

73. The total number of tangents through the point (3, 5) that can be drawn to the ellipses $3x^2 + 5y^2 = 32$ and $25x^2 + 9y^2 = 450$ is

(A) 0 (B) 2 (C) 3 (D) 4

Ans : (C)

Hints : (3, 5) lies outside the ellipse $3x^2+5y^2=32$ and on the ellipse $25x^2+9y^2=450$. Therefore there will be 2 tangents for the first ellipse and one tangent for the second ellipse.

74. The value of $\lim\limits_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+........\frac{n}{n^2+n^2}\right]$ is

(A) $\frac{\pi}{4}$ (B) log 2 (C) zero (D)1

Ans : (A)

Hints : $\lim\limits_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+...+\frac{n}{n^2+n^2}\right]$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2+r^2}=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\left(\frac{r}{n}\right)^2}=\int_0^1\frac{dx}{1+x^2}=\left[\tan^{-1}x\right]_0^1=\frac{\pi}{4}$$

75. A particle is moving in a straight line. At time t, the distance between the particle from its starting point is given by $x = t - 6t^2 + t^3$. Its acceleration will be zero at

(A) $t = 1$ unit time (B) $t = 2$ unit time (C) $t = 3$ unit time (D) $t = 4$ unit time

Ans : (B)

Hints : $x = t - 6t^2 + t^3$ $\qquad \frac{dx}{dt} = 1 - 12t + 3t^2$

$$\frac{d^2x}{dt^2} = -12 + 6t$$

Acceleration = $\frac{d^2x}{dt^2}$

∴ Acceleration = $0 \Rightarrow 6t - 12 = 0 \Rightarrow t = 2$

76. Three numbers are chosen at random from 1 to 20. The probability that they are consecutive is

(A) $\frac{1}{190}$ (B) $\frac{1}{120}$ (C) $\frac{3}{190}$ (D) $\frac{5}{190}$

Ans : (C)

Hints : Total number of cases ; $^{20}C_3 = \frac{20\times19\times18}{2\times3} = 20 \times 19 \times 3 = 1140$

Total number of favourable cases = 18

$\therefore$ Required probability $= \frac{18}{1140} = \frac{3}{190}$

77. The co-ordinates of the foot of the perpendicular from (0, 0) upon the line $x + y = 2$ are

(A) (2, –1) (B) (–2, 1) (C) (1, 1) (D) (1, 2)

Ans : (C)

Hints : Let P be the foot of the perpendicular. P lies on a line perpendicular to $x + y = 2$.

$\therefore$ Equation of the line on which P lies is of the form : $x - y + k = 0$

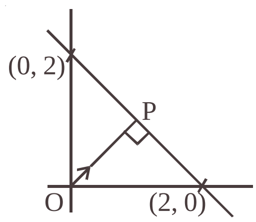
But this line passes through (0, 0).

$\therefore k = 0$

Hence, co-ordinates of P may be obtained by solving $x + y = 2$ and $y = x$

$\therefore x = 1, y = 1$

Hence, P $\equiv$ (1, 1)



78. If A is a square matrix then,

(A) $A + A^T$ is symmetric (B) AA^T is skew - symmetric (C) $A^T + A$ is skew-symmetric (D) A^TA is skew symmetric

Ans : (A)

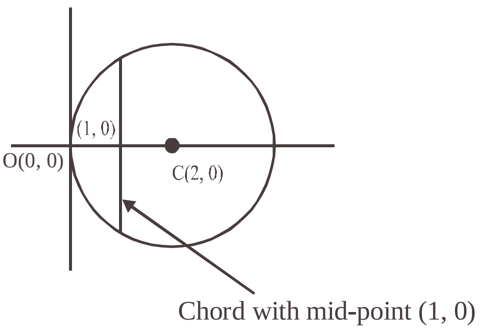
Hints : $(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$

79. The equation of the chord of the circle $x^2 + y^2 - 4x = 0$ whose mid point is (1, 0) is

(A) $y = 2$ (B) $y = 1$ (C) $x = 2$ (D) $x = 1$

Ans : (D)

Hints :



Equation : $x = 1$

80. If $A^2 - A + I = 0$, then the inverse of the matrix A is

(A) $A - I$ (B) $I - A$ (C) $A + I$ (D) A

Ans : (B)

Hints : $A^2 - A + I = 0 \Rightarrow A^2 = A - I \Rightarrow A^2.A^{-1} = A.A^{-1} - A^{-1} \Rightarrow A = I - A^{-1} \Rightarrow A^{-1} = I - A$

MATHEMATICS

SECTION-II

1. A train moving with constant acceleration takes t seconds to pass a certain fixed point and the front and back end of the train pass the fixed point with velocities u and v respectively. Show that the length of the trai is $\frac{1}{2}(u + v)t$.

A. $v = u + at$ $\qquad a = \frac{v-u}{t}$

$$v^2 = u^2 + 2aS$$

$$\frac{v^2-u^2}{2a} = S \Rightarrow S = \frac{(v+u)(v-u)}{2a} = \frac{at(v+u)}{2a} = \frac{u+v}{2}t$$

2. Show that

$$\frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta} = \frac{1}{2}(\tan 27\theta - \tan\theta)$$

A. $T_1 = \frac{2\sin\theta}{2\cos 3\theta}.\frac{\cos\theta}{\cos\theta} = \frac{\sin 2\theta}{2.\cos 3\theta.\cos\theta}$

$$= \frac{1}{2}.\frac{\sin(3\theta-\theta)}{\cos 3\theta.\cos\theta}$$

$$T_1 = \frac{1}{2}(\tan 3\theta - \tan\theta)$$

$$T_2 = \frac{1}{2}(\tan 9\theta - \tan 3\theta)$$

$$T_3 = \frac{1}{2}(\tan 27\theta - \tan 9\theta)$$

$$T_1 + T_2 + T_3 = \frac{1}{2}(\tan 27\theta - \tan\theta)$$

3. If x = sin t, y = sin 2t, prove that

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

A. $y = \sin(2\sin^{-1}x)$

$$\frac{dy}{dx} = \cos(2\sin^{-1}x).\frac{2}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\frac{dy}{dx} = 2\cos(2\sin^{-1}x)$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4.\cos^2(2\sin^{-1}x) = 4[1-\sin^2(2\sin^{-1}x)]$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4[1-y^2]$$

Again differentiate

$$(1-x^2)2.\frac{dy}{dx}.\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2(-2x) = -8y\frac{dy}{dx}$$

Divide by $2\frac{dy}{dx}$

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$$

4. Show that, for a positive integer n, the coefficient of x^k $(0 \le K \le n)$ in the expansion of $1+(1+x)+(1+x)^2+........+(1+x)^n$ is ${}^{n+1}C_{n-k}$.

A. $S = \frac{1-(1+x)^{n+1}}{1-(1+x)} = \frac{(1+x)^{n+1}-1}{x}$

Coefficient of x^k in $\frac{(1+x)^{n+1}}{x}-\frac{1}{x}$ = Coefficient of x^{k+1} in $(1+x)^{n+1} = {}^{n+1}C_{k+1} = {}^{n+1}C_{n-k}$

5. If m, n be integers, then find the value of $\int_{-\pi}^{\pi}(\text{cosmx}-\text{sinnx})^2\text{dx}$

A. $I = \int_{-\pi}^{\pi}(\cos^2 mx+\sin^2 nx-2\sin nx.\cos mx)dx$

$= \int_{-\pi}^{\pi}\cos^2 mx.dx+\int_{-\pi}^{\pi}\sin^2 nx.dx-2\int_{-\pi}^{\pi}\sin nx.\cos mx.dx$

$= 2\int_0^{\pi}\cos^2 mx.dx+2\int_0^{\pi}\sin^2 nx.dx-0$ (Odd .....)

$= 2\int_0^{\pi}(1+\cos 2mx)dx+\int_0^{\pi}(1-\cos 2nx)dx$

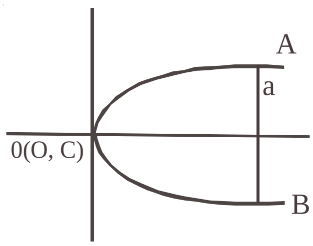
$= \pi+\frac{1}{2m}(\sin 2mx)_0^{\pi}+\pi-\frac{1}{2n}(\sin 2nx)_0^{\pi}$

$= \pi+\pi+\frac{1}{2m}(0-0)-\frac{1}{2n}(0-0)$

$= 2\pi$

6. Find the angle subtended by the double ordinate of length 2a of the parabola $y^2 = ax$ at its vertex.

A. $y^2 = ax$, $a^2 = ax$, $a = x$ [put y = a]



A (a, a), B(a, –a)

Slope OA = $\frac{a}{a} = 1$

Slope of OB = $\frac{-a}{a} = -1$

Ans. = $\frac{\pi}{2}$

7. If f is differentiable at x = a, find the value of

$$\underset{x \to a}{Lt} \frac{x^2 f(a) - a^2 f(x)}{x - a}.$$

A. $\underset{x \to a}{Lt} \frac{x^2 f(a) - a^2 f(x)}{x - a}, \frac{0}{0}$ form by LH

$= \underset{x \to a}{Lt} \frac{2x f(a) - a^2 f^1(x)}{1}$

$= 2af(a) - a^2 f^1(a)$

8. Find the values of 'a' for which the expression $x^2 - (3a - 1)x + 2a^2 + 2a - 11$ is always positve.

A. $x^2 - (3a - 1)x + 2a^2 + 2a - 11 > 0$

$D < 0$

$(3a - 1)^2 - 4(2a^2 + 2a - 11) < 0$

$9a^2 - 6a + 1 - 8a^2 - 8a + 44 < 0$

$a^2 - 14a + 45 < 0$

$(a - 9)(a - 5) < 0$

$5 < a < 9$

9. Find the sum of the first n terms of the series 0.2 + 0.22 + 0.222 + ...

A. $S = \frac{2}{9}[0.9 + 0.99 + 0.999 + ...]$

$= \frac{2}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001).......]$

$= \frac{2}{9}[n - (0.1 + 0.01.......... + n\ terms)]$

$$= \frac{2}{9}n - \frac{2}{9}\frac{(0.1)[1-(0.1)^n]}{[1-(0.1)]}$$

$$\frac{2}{9}n - \frac{2}{9}\frac{(0.1)}{(0.9)}[1-(0.1)^n]$$

$$\frac{2}{9}n - \frac{2}{81} + \frac{2}{81}(0.1)^n$$

10. The equation to the pairs of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5$. Find the equations of its diagonals.

A. $x = 2$ ......(i)

$x = 3$ ...... (ii)

$y = 1$ .... (iii)

$y = 5$ ..... (iv)

A (2, 1), B (3, 1), C (3, 5), D(2, 5)

Equation of AC

$$\frac{x-2}{3-2} = \frac{y-1}{5-1}, \quad x-2 = \frac{y-1}{4}$$

$4x - 8 = y - 1, \; 4x - y - 7 = 0$

Equation of BD $\dfrac{x-3}{2-3} = \dfrac{y-1}{5-1}$

$$\frac{x-3}{-1} = \frac{y-1}{4}, \; -4x + 12 = y - 1$$

$4x + y - 13 = 0$