First Year Higher Secondary Model Examination, February 2023
Mathematics (Science)

Answer Key

<u>Answer any 6 questions. Each carries 3 scores.</u>

1. i) option B) (-2, 3] (1)

ii) {}, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c} (2)

2. i) R={(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6) } (2)

ii) No, 1 has more than one images. (1)

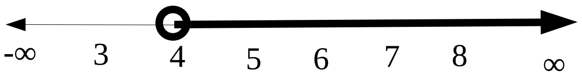
3. i) $2(x-1)<3(x-2)$ (2)

$2x-2<3x-6$

$2x-3x<2-6$

$-x<-4$

$x>4$ ie (4, ∞)

ii) 

(1)

4. i) n = 3+7 = 10

Therefore $^{n}C_2={}^{10}C_2=45$ (2)

ii) No. Of shake hands = $^{10}C_2=45$ (1)

5. i) $y^2=10x$, $y^2=4ax$ (1)

So 4a = 10, $a=\frac{5}{2}$

Length of latus rectum of the parabola = 4a = 10

ii) Here a = 3 (2)

Focus of the parabola in negative y-axis and directrix on positive y-axis. Therefore the equation of the parabola is of the form $x^2=-4ay$

ie, $x^2=-12y$

6. i) option B) (1, 0, -2) (1)

ii) Here $A(0,7,-10), B(1,6,-6), C(4,9,-6)$ (2)

$AB=\sqrt{(1-0)^2+(6-7)^2+(-6--10)^2}$

$=\sqrt{1^2+(-1)^2+4^2}$

$=\sqrt{1+1+16} = \sqrt{18}=3\sqrt{2}$

$BC=\sqrt{(4-1)^2+(9-6)^2+(-6--6)^2}$

$=\sqrt{3^2+3^2+0}$

$=\sqrt{9+9} = \sqrt{18}=3\sqrt{2}$

$AC=\sqrt{(4-0)^2+(9-7)^2+(-6--10)^2}$

$=\sqrt{4^2+2^2+4^2}$

$=\sqrt{16+4+16}=\sqrt{36}=6$

Here AB = BC ≠AC

Therefore ABC are the vertices of an isosceles triangle.

7. i) 1 (1)

ii) $\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{\frac{\pi}{2}-x}\right)$ (2)

$x\to\frac{\pi}{2}$, $\frac{\pi}{2}-x\to 0$ take $h=\frac{\pi}{2}-x$ and $x=\frac{\pi}{2}-h$

Now $\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{\frac{\pi}{2}-x}\right) = \lim_{h\to 0}\left(\frac{\cos\left(\frac{\pi}{2}-h\right)}{h}\right)$

$= \lim_{h\to 0}\left(\frac{\sin h}{h}\right)=1$

8. i) $P(A')=1-P(A)$ (1)

$= 1-\frac{1}{4}=\frac{3}{4}$

ii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ (1)

$= \frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}$

iii) $P(A'\cap B')=P((A\cup B)')$ (1)

$= 1-P(A\cup B)$

$= 1-\frac{5}{8}=\frac{3}{8}$

<u>Answer any 6 questions. Each carries 4 scores.</u>

9. i) A (1)

ii) $A\cup B=\{1,2,3,4,5,6,8,10\}$ (3)

$(A\cup B)'=\{7,9\}$

$A'=\{6,7,8,9,10\}$

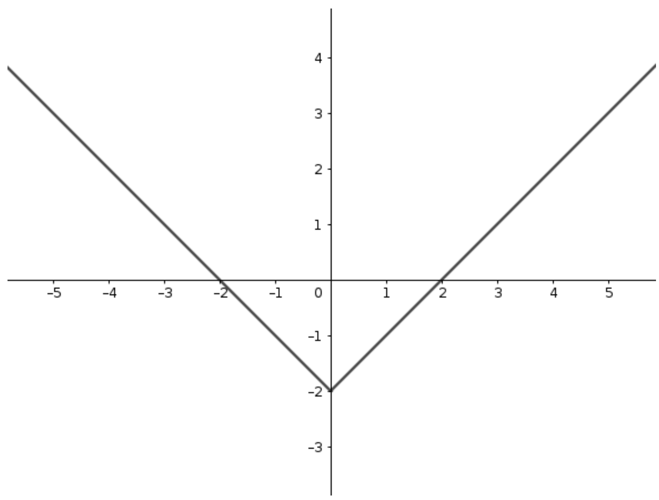
$B'=\{1,3,5,7,9\}$

$A'\cap B'=\{7,9\}$

Therefore $(A\cup B)'=A'\cap B'$

10. i) (2)

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	2	1	0	-1	-2	-1	0	1	2



ii) Range = [-2, ∞) (1)

iii) $9-x\geq 0$ then $x\leq 9$ (1)

So domain of g = (- ∞, 9]

11. i) $Z=i^9+i^{18}=i+i^2=i-1$ (1)

Z = - 1 + i

ii) $\overline{Z}$ = - 1 – i (1)

iii) Multiplicative inverse = $\frac{a}{a^2+b^2}+i\frac{-b}{a^2+b^2}$ (2)

= $\frac{2}{2^2+(-3)^2}+i\frac{--3}{2^2+(-3)^2}$

= $\frac{2}{13}+i\frac{3}{13}$

12. i) No. Of arrangements when all vowels occur together = 3! x 6! = 4320 (2)

ii) No. Of arrangements when all vowels do not occur together = Total no. of arrangements – No. of arrangements when all vowels occur together = 8! - 3! x 6! (2)

= 40320 – 4320 = 36000

13. $(a+b)^n={}^nC_0a^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+...+{}^nC_nb^n$

$(a+b)^4={}^4C_0a^4+{}^4C_1a^3b+{}^4C_2a^2b^2+{}^4C_3ab^3+{}^4C_4b^4$

$= a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$(a-b)^4=a^4-4a^3b+6a^2b^2-4ab^3+b^4$

$(a+b)^4-(a-b)^4=8a^3b+8ab^3$

$= 8ab(a^2+b^2)$ (3)

$(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4=8\sqrt{3}\sqrt{2}((\sqrt{3})^2+(\sqrt{2})^2)$

$= 8\sqrt{6}(5)=40\sqrt{6}$ (1)

14. (4)

$3+33+333+...+n\,terms=3[1+11+111+...+n\,terms]$

$= \frac{3}{9}[9+99+999+...+n\,terms]$

$= \frac{3}{9}[10-1+10^2-1+10^3-1+...+n\,terms]$

$= \frac{3}{9}[10+10^2+10^3+...+n\,terms-n]$

$= \frac{3}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$

15. $\frac{x^2}{25}+\frac{y^2}{9}=1$ (4)

Here $a^2=25, b^2=9$

ie, a = 5 and b = 3

Now $a^2=b^2+c^2$

$25=9+c^2$

$c^2=25-9=16$ ie, c = 4

focii are (±c , 0) ie, (± 4, 0)

vertices are (± a, 0) ie, (± 5, 0)

eccentricity, $e=\frac{c}{a}=\frac{4}{5}$

length of latus rectum = $\frac{2b^2}{a}=\frac{18}{5}$

16. S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT } (1)

i) P(getting 3 heads) = $\frac{1}{8}$ (1)

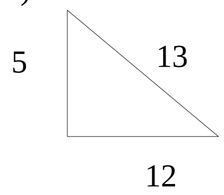
ii) P(getting exactly two heads) = $\frac{3}{8}$ (1)

iii) P(getting one head and two tails) = $\frac{3}{8}$ (1)

<u>Answer any 3 questions. Each carries 6 scores.</u>

17. i) $240.\frac{\pi}{180}=\frac{4\pi}{3}$ (1)

ii) (2)



$\sqrt{12^2+5^2}=\sqrt{144+25}=\sqrt{169}=13$

sin x = $\frac{5}{13}$

cos x = $\frac{-12}{13}$

iii) $\cos x+\cos y=2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$ (3)

$\sin x-\sin y=2\cos(\frac{x+y}{2})\sin(\frac{x-y}{2})$

$\frac{\cos 7x+\cos 5x}{\sin 7x-\sin 5x}=\frac{2\cos(\frac{7x+5x}{2})\cos(\frac{7x-5x}{2})}{2\cos(\frac{7x+5x}{2})\sin(\frac{7x+5x}{2})}$

$= \frac{2\cos(6x)\cos x}{2\cos(6x)\sin x}$

$= \frac{\cos x}{\sin x}$

= cot x

18. i) slope of the line passing through (2, 5) and (-3, 6) is m = $\frac{y_2-y_1}{x_2-x_1}=\frac{6-5}{-3-2}=\frac{1}{-5}$ (4)

If the two lines are perpendicular, then

$m_1m_2=-1$

Therefore $m_2=5$

Now the equation of the line passing through (-3,5) with slope 5 is given by

$y-y_1=m(x-x_1)$

y – 5 = 5 (x - -3)

y – 5 = 5x + 15

5x – y + 20=0

ii) $d=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$ (2)

$= \left|\frac{3.(-3)+(-4).5+26}{\sqrt{3^2+(-4)^2}}\right|$

$= \left|\frac{-9-20+26}{\sqrt{9+16}}\right|$

$= \left|\frac{-3}{5}\right|=\frac{3}{5}$

19. i) $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ (3)

$f(x)=\cos x$

$f(x+h)=\cos(x+h)$

$\frac{d}{dx}\cos x=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$

$= \lim_{h\to 0}\frac{-2\sin(\frac{x+h+x}{2})\sin(\frac{x+h-x}{2})}{h}$

$= \lim_{h\to 0}\frac{-2\sin(\frac{2x+h}{2})\sin(\frac{h}{2})}{h}$

$= \lim_{h\to 0}-\sin(\frac{2x+h}{2}).\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$

$= -\sin\left(\frac{2x}{2}\right)=-\sin x$

ii) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x).\frac{d}{dx}(f(x))-f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$ (3)

$\frac{d}{dx}\left(\frac{1+\sin x}{\cos x}\right)=\frac{\cos x.\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}(\cos x)}{(\cos x)^2}$

$= \frac{\cos x.(0+\cos x)-(1+\sin x)(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x+\sin x+\sin^2 x}{\cos^2 x}$

$= \frac{1+\sin x}{\cos^2 x}$

20. (6)

class	f_i	x_i	$f_i x_i$	x_i^2	$x_i^2 f_i$
0 - 10	3	5	15	25	75
10 - 20	7	15	105	225	1575
20 - 30	12	25	300	625	7500
30 - 40	15	35	525	1225	18375
40 - 50	8	45	360	2025	16200
50 - 60	3	55	165	3025	9075
60 - 70	2	65	130	4225	8450
	50		1600		61250

Mean , $\overline{X}=\frac{\sum_{i=1}^{n} x_i f_i}{N}=\frac{1600}{50}=32$

Variance, $\sigma^2=\frac{\sum_{i=1}^{n} x_i^2 f_i}{N}-(\overline{X})^2$

$= \frac{61250}{50}-32^2=1225-1024=201$

S.D, $\sigma=\sqrt{Variance}=\sqrt{201}=14.177$

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