KARNATAKA SECONDARY EDUCATION EXAMINATION BOARD

STATE LEVEL PREPARATORY EXAM- MARCH 2023

MODEL KEY ANSWERS

1. If 10, x, 18 are in Arithmetic progression, then the value of x is
   A) 12 B)13 C)14 D) 16

   Answer: C)14

   Explanation: x-10=18-x (common difference = common difference)

   2x=28 then x=14

2. The Highest Common Factor (HCF) of 3 and 5 is ..
   A)1 B)3 C) 5 D) 15
   Answer: A) 1
3. The discriminant of a quadratic equation ax2+bx+c=0 is.......
   A) a^2-4bc B) b^2-4ac C) a^2+4bc D) b^2+4ac
   Answer: B) b^2-4ac
4. The value of $\frac{\sin(90^0 - \theta)}{\cos\theta}$ is ....

   A)0 B)1 C) 2 D) $\frac{1}{2}$

   Answer: B)1
5. The coordinates of the midpoint of the line segment joining the points P(4, 7) & Q(10, 3) are ....
   A)(14, 10) B) (5, 7) C) (3, -5) D) (7, 5)
   Answer: D) (7, 5)
   Explanation: (x, y) = $\left[\frac{x1+x2}{2}, \frac{y2+y1}{2}\right] = \left[\frac{4+10}{2}, \frac{7+3}{2}\right]$ = (7, 5)
6. The equation of the line which is parallel to the line represented by the equation 4x-8y=11 is ......
   A) 4x-16y=22 B) 2x+4y=6 C) 8x-16y=21 D)8x+16y=22
   Answer: C) 8x-16y=21
   Explanation: On comparing 4x-8y=11 & **8x-16y=21,** $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$
7. In which figure, ΔABC ~ ΔPQR, then correct relation among the following is
   A) $\frac{ar\ of\ (\Delta ABC)}{ar\ of\ (\Delta PQR)} = \frac{BC^2}{QR^2}$ B) $\frac{ar\ of\ (\Delta ABC)}{ar\ of\ (\Delta PQR)} = \frac{AB^2}{PN^2}$

C) $\frac{ar\ of\ (\Delta ABC)}{ar\ of(\Delta PQR)} = \frac{BC^2}{QN^2}$ D) $\frac{ar\ of\ (\Delta ABC)}{ar\ of(\Delta PQR)} = \frac{QR^2}{BC^2}$

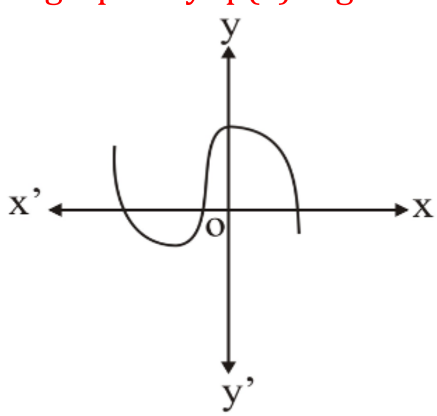
Answer: A)$\frac{ar\ of\ (\Delta ABC)}{ar\ of(\Delta PQR)} = \frac{BC^2}{QR^2}$

8. The curved surface area of the solid hemisphere of radius ‘r’ units is …..
A)$4\pi r^2$ sq. units B) $3\pi r^2$ sq. units C) πr^2 sq. units D) $2\pi r^2$ sq. units
Answer: D) $2\pi r^2$ sq. units

Answer the following questions 8x1= 8

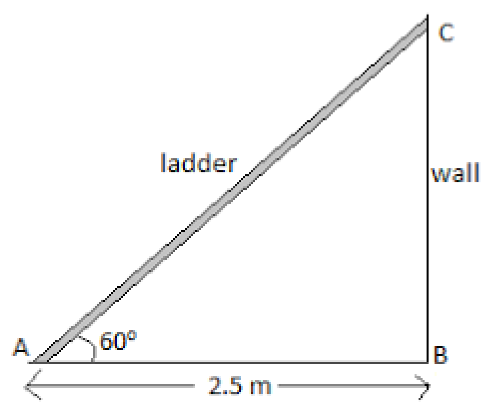
9. State whether the rational number $\frac{35}{50}$ has a terminating decimal expansion or non-terminating recurring decimal expansion.
Solution: it is a terminating decimal expansion,
Explanation: $\frac{35}{50} = \frac{7}{10} = 0.7$

10. The graph of y=p(x) is given below. Write the number of zeroes of p(x).



Solution: The number of times graph touches the x-axis is 3, hence the number of zeroes is 3.

11. A ladder is placed against a wall such that its foot is at a distance of 2.5m from the wall and it is inclined at an angle of 60^0 with the ground. Find the length of the ladder.



Solution: from figure, BC is wall and AC is ladder.

Apply Cos60=$\frac{adj}{hyp}$ = $\frac{2.5}{AC}$ AC=2X2.5 = 5m

12. Write the formula to find the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3).

Solution: A=$\frac{1}{2}\{ x_1(y_2\text{-}y_3)+\{ x_2(y_3\text{-}y_1)+\{ x_3(y_1\text{-}y_2)$

13. For any event E, if P(E)=0.7, then find P(E).

Solution: P(E)= 0.3

14. If tanΘ=1, then find the value of $sec^2Θ$.

Solution: If tanΘ=1, then Θ=45^0. Then $sec^2Θ=(\sqrt{2})^2 = 2$

15. In the figure, AB is a tangent to a circle with centre 'O'. 'P' is the point of contact. If ∟OAP=50^0, then find ∟AOP.

Solution: 40^0

Explanation: OP is tangent, therefore ∟OPA=90^0, & ∟OAP=50^0.

In triangle, the sum of all interior angles should be 180^0. Hence remaining angle is 40^0

16. Write the formula to find the total surface area of a cylinder of radius r and height is h.

Solution: Total surface area of cylinder is = 2πr(r+h).

Answer the following questions 8x2=16

17. Prove that 3+$\sqrt{2}$ is an irrational number.

OR

The H.C.F and L.C.M of two numbers are 3 and 60 respectively. If one of the numbers is 12, then find the other number.

Solution: Let us assume that 3 +√2 is a rational number with p and q as co-prime integer and q ≠ 0

⇒ 3 +√2 = p / q

⇒ √2 = p / q - 3

⇒ √2 = (3q - p) / 3q

⇒ (3q - p) / 3q is a rational number

However, √2 is in irrational number

This leads to a contradiction that 3 +√2 is a rational number was wrong.

Hence 3 +√2 is an irrational number.

OR

We know HCF XLCM= AXB
Where A & B are numbers.
According to given question 3x60=12XB
Then we have to find B here, therefore B=15.

18.Solve the pair of linear equations by elimination method
2x-y=4
x+y=11

Solution: let the equations are 2x-y=4 --------→(1)
x+y=11 --------→(2)
add above equations for eliminating the value of y, then we get x
2x-y=4
x+y=11
3x= 15, then $\boxed{\mathbf{x=5}}$ **&** $\boxed{\mathbf{y= 6}}$

19. If the lines represented by the pair of linear equations, 2x+3y-8=0 and ax+by-16=0 are coincident then find the values of a and b.

Solution: we know for coincident lines $\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$
Two equations are 2x+3y-8=0 & ax+by-16=0
Then value of a is 2 and b is 3. (any value that can $\frac{a1}{a2}=\frac{b1}{b2}$ **possible)**

20.Find the 26^{th} term of the arithmetic progression 3, 7, 11, ………… using the formula.

Solution: a= 3 and d= 4, 26^{th} term is a+25d= 3+25x4
= 3+100
$\boxed{26^{th} \text{ term is } 103}$

21.Solve $3x^2-5x+2=0$ by using quadratic formula.
OR
Find the roots of the quadratic equation $x^2-5x+6=0$ by the method of completing the square.

Solution: here a=3, b=-5 and c=2
Quadratic formula is

$$x= \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\mathbf{x}= \frac{-(-5)\pm\sqrt{5^2-4x3x2}}{2x3} = \frac{5\pm\sqrt{25-24}}{6} = \frac{5\pm\sqrt{1}}{6} = \frac{5\pm1}{6} = \frac{6}{6} \text{ or } \frac{4}{6}$$

x= 1 or x= 2/3 .
OR

Given equation is x^2-5x+6=0, divide by 2 we get

x^2-5/2x+6/2=0

x^2-5x+6=0

$(x-\frac{5}{2})^2= (\frac{-5}{2})^2-6$

$(x-\frac{5}{2})^2 =+ \text{ or } - \frac{1}{2}$

X=3 or x=2

22. In the given figure, ΔABC is a right angled triangle. Find the value of the following:
i)sin α
ii)tan β

Solution: from figure, sin $\alpha=\frac{OPP}{HYP} = \frac{4}{5}$

Tan $\beta=\frac{OPP}{ADJ} = \frac{3}{4}$

Hence the roots are **two distinct real roots.**

23. A box contains 20 cards, which are numbered from 1 to 20. If one card is drawn at random from the box, then find the probability of getting a perfect cube number.

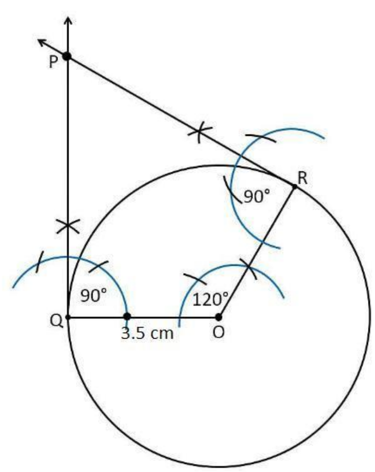
Solution: Possible outcomes of card are (1 to 20) = 20

A perfect cube numbers (1, 8) = 2

Therefore probability = $\frac{2}{20} = \frac{1}{10}$

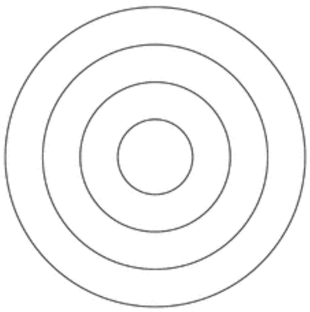
24.Construct a pair of tangents to the circle of radius 3cm, which are inclined at an angle of 60^0.

Solution:



Answer the following question 9x3= 27

25. A student prepares a model of atomic structure that consists of four concentric circular rings. The length of circumferences of these rings are in A.P. The sum of the circumferences of the first three rings is 66cm and the circumference of the fourth ring is 44cm. find the circumference of the third ring using formula.



OR

The sum of the first four terms of an A.P is 38 and the sum of the first seven terms is 98. Find the first term and common difference of the A.P.

Solution: circumference of first three rings are a, a+d, a+2d

Sum of first three rings are a+a+d+a+2d= 66

3a+3d=66

a+d=22--------(1)

fourth ring is 44cm, a+3d=44------------(2)

subtract above two equations we get 2d=22 then d=11

put this d=11 in any one equation above we get a=11

then circumference of the third ring is a+2d=11+2x11= 11+22 =33

Hence circumference of the third ring is 33cm

OR

Let sum of first 4 terms is 38 ----→ $S_n=\frac{n}{2}(2a+(n-1)d)$

$38=2(2a+3d)$

$19=2a+3d$

$2a+3d=19$----------(1)

And the sum of first seven terms is 98. -------→ $S_n=\frac{n}{2}(2a+(n-1)d)$

$98=\frac{7}{2}(2a+6d)$

$2x14=2(a+3d)$

a+3d=14-------(2)

Subtract above two equation, we get $\boxed{a=5}$ and $\boxed{d=3}$

Therefore first term is 5 and common difference is 3

Verification: $S_n=\frac{n}{2}(2a+(n-1)d) = \frac{4}{2}(2x5+(4-1)x3) = 2(10+9) = 38$

26.Divide $p(x)=x^3-3x^2+3x-5$ by $g(x)=x^2-x+1$ and find the quotient q(x) amd the remainder r(x).

OR

Find the zeroes of the quadratic polynomial $p(x)= x^2-2x-8$ and verify the relationship between the zeroes and coefficients.

Solution: by division, $x^2-x+1)\ x^3-3x^2+3x-5(x-2$

x^3-x^2+1x

$-2x^2+2x-5$

$-2x^2+2x-2$

$0x-3$

Quotient is q(x)=x-2 and r(x)=-3

OR

$P(x)= x^2-2x-8$

ax^2+bx+c

a=1, b=-2 and c=-8

$\alpha+\beta=-\frac{b}{a} = \frac{2}{1}$ & $= -\frac{8}{1}$

we know $x^2-(\alpha+\beta)x+(\alpha x\beta)$

$x^2-2x+-8$

x^2-2x-8

verification : x^2-2x-8

$x^2-4x+2x-8$

$x(x-4)+2(x-4)$

x=-2 & x=4

$\alpha=-2$ & $\beta=4$

$\alpha+\beta = -2$ & $\alpha x\beta= -8$

27.Find the coordinates of the point P on x axis, which is equidistant from the points A(2, -5) & B(-2, 9).

OR

Find the coordinates of the point P on AB which divides the line joining the points A(-2, -2) and B(5, 12) in the ratio 3:4.

Solution: let the point P(x, 0) lies on the x-axis.

Since P(x, 0) is equidistance from the points A(-2, 5) and B(-2, 9)

$PA^2=PB^2$

$(2-x)^2+(-5-0)^2=(-2-x)^2+9^2$.

$4+x^2-4x+25=4+x^2-4x+81$

$-8x=81-25=56$

$\boxed{x=-7}$

OR

Given coordinates are (-2, -2) and (5, 12) in the ratio of 3:4

$$(x, y) = \left[\frac{mx2+nx1}{m+n}, \frac{my2+ny1}{m+n}\right]$$

$$= \left[\frac{3(5)+4(-2)}{3+4}, \frac{3(12)+4(-2)}{3+4}\right]$$

$$= \left[\frac{15+8}{7}, \frac{36-8}{7}\right]$$

$$= \left[\frac{7}{7}, \frac{28}{7}\right]$$

(x, y) **=(1, 4)**

$\boxed{\textbf{Hence the coordinates are (1, 4)}}$

28. Find the arithmetic mean for the following grouped data.

C.I	f
0-2	2
2-4	6
4-6	8
6-8	3
8-10	1
N=20	

OR

Find the mode of the following grouped data.

C.I	f
0-6	2
6-12	9
12-18	15
18-24	12
24-30	5

Solution:

C.I	f	x	fx
0-2	2	1	2
2-4	6	3	18
4-6	8	5	40
6-8	3	7	21
8-10	1	9	9
N	20		$\sum fx$ = 90

Mean by direct method, x= $\frac{\sum fx}{n}$ = $\frac{90}{20}$ = 4.5

Mean = 4.5

OR

C.I	f
0-6	2
6-12	9
12-18	15
18-24	12
24-30	5

Here LRL=12, f_1=15, f_0=9, f_2=12 and h=6

Mode= LRL+ $\left\{\frac{f1-f0}{2f1-f0-f2}\right\}$x h

= 12+$\left\{\frac{15-9}{30-9-12}\right\}$x 6

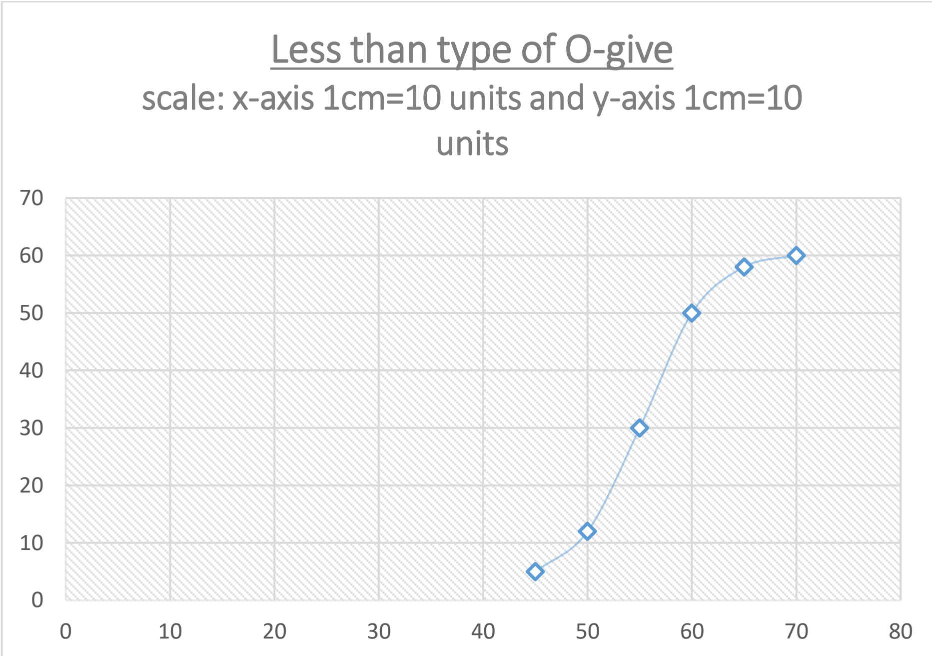
= 12+4

Mode=16

29. During the medical check up of 60 students of a class, their weights were recorded as follows. Draw 'less than type of O-give' for the given data:

Weight (in kg)	Number of students (cf)
Less than 45	5
Less than 50	12
Less than 55	30
Less than 60	50
Less than 65	58
Less than 70	60

Solution:



30. In the figure, ABC is a right angled triangle and $DP \perp AB$. If BP=6cm, DP=8cm and AC=16cm then find the length of AB.

Solution:

In triangle ABC & ΔDPB

$\llcorner C = \llcorner = 90^0$

Therefore Δ ABC ~ ΔDPB

$$\frac{AC}{DP} = \frac{BC}{PB}$$

$$\frac{8}{16} = \frac{BC}{6}$$

BC=12cm,

In triangle ABC, $AB^2 = BC^2 + AC^2$

$$AB^2 = \sqrt{256 + 144}$$

$$AB = 20cm$$

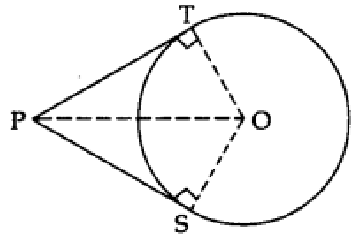
31. Prove that "the length of the tangents drawn from an external point to a circle are equal".

Solution:

Given: PT and PS are tangents from an external point P to the circle with centre O.

To prove: PT = PS

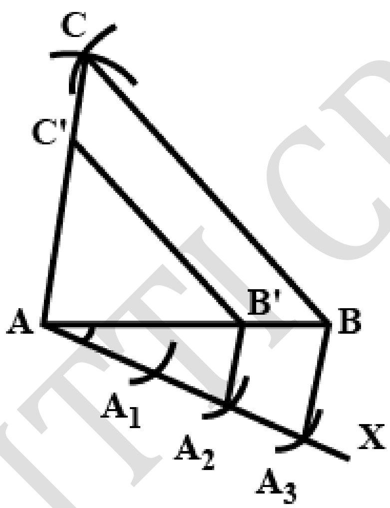
Construction: Join O to P, T and S.



Proof: In ΔOTP and ΔOSP.
OT = OS …[radii of the same circle]
OP = OP …[common]
∠OTP = ∠OSP …[each 90°]
ΔOTP = ΔOSP …[R.H.S.]
PT = PS …[c.p.c.t.]

32. Construct a triangle ABC, with AB=9cm, BC=5cm and AC=6cm, and then construct another triangle similar to it whose sides are 2/3 of the corresponding sides of the first triangle ABC.

Solution:



33. In the figure, OAPB is a quadrant of a circle and OAP is a sector. If ∟AOP=30^0, and the area of the shaded region is 462 cm^2. Then find the length of the arc PB.

Solution: given
sector OAP, $\theta=30^0$. Shaded region = 462cm^2.

Area of sector OAP= $\frac{\theta}{360}$X πr^2.

$$462=\frac{60}{360}X\frac{22}{7}r^2.$$

r=42cm

length of PB = ?

from fig, $\llcorner AOB - \llcorner POB = 90-30=60$

length of PB = $\frac{\theta}{360} X\ 2\pi r$

$= \frac{60}{360} X\ 2x\frac{22}{7}\ x\ 42$

$= 14x\frac{22}{7}$

$= 2x22$

$= 42cm$

length of arc PB=42cm

34. Find the solution of the given pair of linear equations by graphical method: x+y=5 & 3x-y=3

Solution:

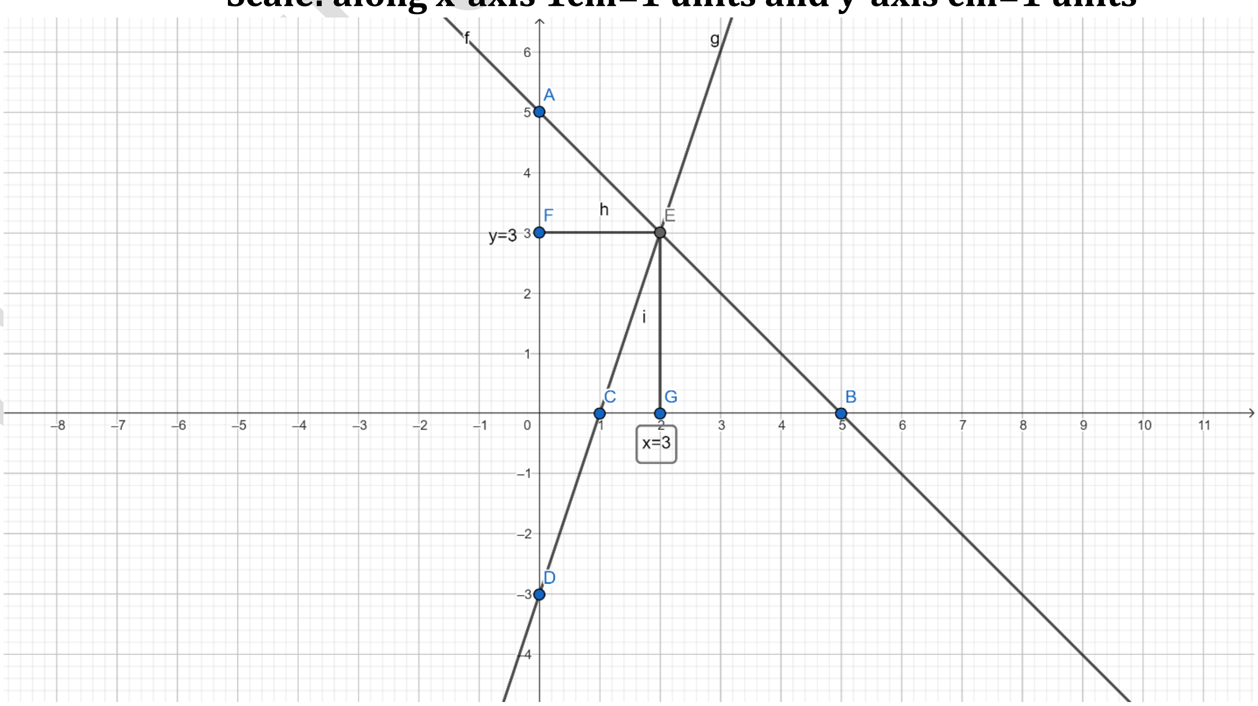
We have x+y=5

x	**0**	**5**
y	**5**	**0**

3x-y=3

x	**0**	**1**
y	**-3**	**0**

Scale: along x-axis 1cm=1 units and y-axis cm=1 units



35. A motor boat whose speed is 18km/hr in still water takes one hour more to go 24km upstream than the time taken to return downstream to the same spot. Find the speed of the stream.

OR

Person A is 26 years older than person B. The product of their ages (in years), 3 years froom now will be 360. Find the present ages of person A and person B.

Solution: Given:- Speed of boat =18km/hr
Distance =24km
Let x be the speed of stream.Let t1 and t2 be the time for upstream and downstream.
As we know that,
speed=time/distance
⇒time=speed/distance
For upstream,
Speed =(18−x)km/hr
Distance =24km
Time =t1
Therefore,
t1=18−x/24
For downstream,
Speed =(18+x)km/hr
Distance =24km
Time =t2
Therefore,
t2=18+x/24
Now according to the question-
t1=t2+1
18−x/24=18+x/24+1
After converting this as quadratic equation,
$\Rightarrow x^2+48x-324=0$
$\Rightarrow x^2+54x-6x-324=0$
$\Rightarrow x(x+54)-6(x+54)=0$
$\Rightarrow (x+54)(x-6)=0$
$\Rightarrow x=-54$ or $x=6$
Since speed cannot be negative.

$\Rightarrow x=-54$
$\therefore x=6$
Thus the speed of stream is 6km/hr
Hence the correct answer is 6km/hr.

OR

Let present age of A be x years. Then, his B's age is (x+26) years.

A's age after 3 years =(x+3) years

After 3 years B's=(x+26+3)years=(x+29) years

It is given that after 3 years from now, the product of A's and B's ages will be 360 years

$\therefore (x+3)(x+29)=360 \Rightarrow x^2+32x-273=0$

This is the required equation.

$\therefore (x+39)(x-7)=0$

$\therefore x=-39$ and $x=7$

$\therefore x=7$.......($\because$ x can not be negative)
$\therefore$B's present age=7 years
$\therefore$B's present age=7+26=33 years.

36. A tower and a building are standing vertically on a level ground. The angles of elevation of the top of the tower from a point on the same ground and from the top of the building are found to be 300 and 600 respectively. If the distance of the point from the foot of the tower is $30\sqrt{3}$m and height of the building is 10m, then find the distance between the foot of the tower and building and also the distance between their tops.

Solution: let height of the building ED is 10m

The distance between the foot of the tower and building is BE=PD=?
The distance between the their tops AD=?

In triangle ACB, $\tan 30 = \frac{AB}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AP+10}{BC}$$

$$AP = 20m$$

In APD, $\tan 60 = \frac{AP}{PD}$

In triangle APD, $\sin 60 = \frac{AB}{AD}$

$$\frac{\sqrt{3}}{2} = \frac{20}{AD}$$

$$AD = 40\sqrt{3}$$

$$\sqrt{3} = \frac{20}{PD}$$

$$PD=20/\sqrt{3}$$

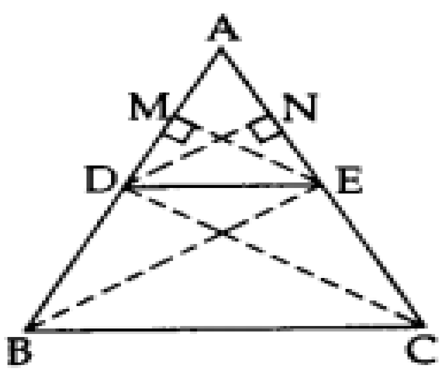
Therefore distance between the foot of the tower and building is $\frac{20}{\sqrt{3}}$ m

37. Prove that "if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio".

Solution:

Statement:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Given: In ΔABC, DE || BC.

To prove: $\frac{AD}{DB}=\frac{AE}{EC}$

Const.: Draw EM ⊥ AD and DN ⊥ AE. Join B to E and C to D.

Proof: In ΔADE and ΔBDE,

$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{1/2 \times AD \times EM}{1/2 \times DB \times EM} = \frac{AD}{DB}$ ……..(i)

[Area of Δ = 1/2 x base x corresponding altitude)

In ΔADE and ΔCDE,

$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{1/2 \times AE \times DN}{1/2 \times EC \times DN} = \frac{AE}{EC}$ ……..(ii)

∵ DE || BC …[Given

∴ ar(ΔBDE) = ar(ΔCDE)….. (iii) [∵ As on the same base and between the same parallel sides are equal in area

From (i), (ii) and (iii),

$\frac{AD}{DB}=\frac{AE}{EC}$

Hence the proof

38. A solid is in the shape of frustum of a cone of height 12cm and radii of its circular ends are 5cm and 10cm. find the total surface area and volume of the solid.

Solution:

Given : R=10cm, r=5cm and h=12cm.

Volume of the frustum of a cone $= \frac{1}{3}\pi h(R^2+r^2+Rr)$

$= \frac{1}{3} x \frac{22}{7} x 12(10^2+5^2+10x5)$

$= \frac{88}{7}(100+25+50)$

$= \frac{88}{7}(175)$

$= 2200cm^3.$

Slant height of the cone$= 10^2+12^2.$

$= \sqrt{(h2+(10-5)2)}$

$= \sqrt{144+25}$

$= \sqrt{169}$

$= 13cm$

TSA of frustum of cone$= \pi l(R+r)+\pi(R^2+r^2)$

$= \pi(13(15)+100+25)$

$= \frac{22}{7}(195+125)$

$= \frac{22}{7}(320)$

$= 1005.71\ cm^2.$