ANSWER KEY

1. (i) $A \cup B = B$

(ii) Required Set $= \{0, 1, 2, 3, 4, 5, 6\}$

(iii) The subsets of $\{2\}$ are :

$\phi, \{2\}$

2 $3(1-x) < 2(x+4)$

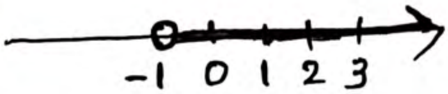
$\Rightarrow 3 - 3x < 2x + 8$

$\Rightarrow -3x - 2x < 8 - 3$

$\Rightarrow -5x < 5$

$\Rightarrow x > -1$ $\therefore$ solution set $= (-1, \infty)$

Numberline representation



3. (i) $(x+1, y-4) = (3,7)$

$\Rightarrow x+1 = 3$ and $y-4 = 7$

$\Rightarrow x = 3-1$ and $y = 7+4$

$\Rightarrow x = 2$ and $y = 11$

(ii) $n(A \times A) = 9$

$\Rightarrow n(A) \times n(A) = 9$

$\Rightarrow [n(A)]^2 = 9$

$\Rightarrow n(A) = 3$

$(-a,0)$ & $(0,a) \in A \times A$

$\therefore A = \{-a, 0, a\}$

4. Total no. of arrangements $= \dfrac{n!}{P_1!\, P_2!}$

$= \dfrac{9!}{2!\, 3!}$

$= 30,240$

Fix 'N' in first place

$\boxed{N}\ \square\ \square\ \square\ \square\ \square\ \square\ \square\ \square$

no. of arrangements in which N comes first $= \dfrac{8!}{3!\,2!}$

$= 3360$

5. $f(x) = \begin{cases} 2x+3 & ; \text{if } x \leq 0 \\ 3(x+1) & ; \text{if } x > 0 \end{cases}$

$L \cdot H \cdot L = \lim\limits_{x \to 0^-} f(x)$

$= \lim\limits_{x \to 0} (2x+3)$

$= 2 \times 0 + 3$

$= 3$

$RHL = \lim\limits_{x \to 0^+} f(x)$

$= \lim\limits_{x \to 0^+} 3(x+1)$

$= 3(0+1)$

$= 3$

$LHL = RHL \Rightarrow \lim\limits_{x \to 0} f(x) = 3$

6. (i) (b) YZ plane

(ii) Let $A(2,-3,-1)$ & $B(-2,4,3)$ be the points

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}.$$

$$= \sqrt{(-2-2)^2+(4+3)^2+(3+1)^2}$$

$$= \sqrt{(-4)^2+7^2+4^2}$$

$$= \sqrt{16+49+16}$$

$$= \sqrt{81}$$

$$= 9 \text{ units}$$

7. $P(A) = 0.35$

$P(A \cap B) = 0.25$

$P(A \cup B) = 0.6$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 0.6 = 0.35 + P(B) - 0.25$$

$$\Rightarrow 0.6 = 0.1 + P(B)$$

$$\Rightarrow P(B) = 0.6 - 0.1$$

$$= 0.5$$

$$P(\text{not } B) = P(B')$$
$$= 1 - P(B)$$
$$= 1 - 0.5$$
$$= 0.5$$

8. $x^2 + y^2 + 8x + 10y - 8 = 0$

$\Rightarrow (x^2 + 8x) + (y^2 + 10y) = 8$

$\Rightarrow (x^2 + 8x + 16) + (y^2 + 10y + 25) = 8 + 16 + 25$

$\Rightarrow (x+4)^2 + (y+5)^2 = 49$

$\Rightarrow (x-(-4))^2 + (y-(-5))^2 = 7^2$

$\therefore$ centre : $(h,k) = (-4,-5)$

radius : $r = 7$

9. $U = \{1,2,3,4,5,6\}$

$A = \{2,3\}$

$B = \{3,4,5\}$

(i) $A \cup B = \{2,3,4,5\}$

(ii) $A' = \{1,4,5,6\}$

$B' = \{1,2,6\}$

(iii) $(A \cup B)' = \{1, 6\}$

$A' \cap B' = \{1, 6\}$

$\therefore (A \cup B)' = A' \cap B'$

10 (i) $f(x) = x+1$

$g(x) = 2x-3$

$(f+g)(x) = f(x) + g(x)$

$= x+1 + 2x-3$

$= 3x-2$

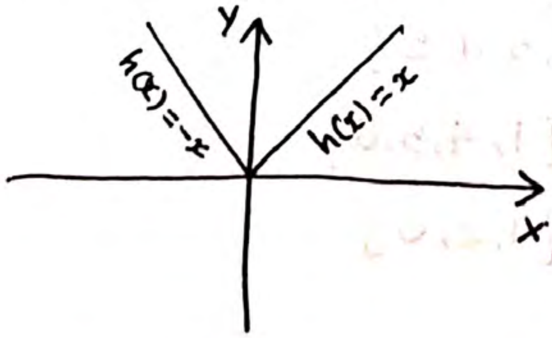
$(fg)(x) = f(x) \cdot g(x)$

$= (x+1)(2x-3)$

$= 2x^2 - 3x + 2x - 3$

$= 2x^2 - x - 3$

(ii) $h: \mathbb{R} \rightarrow \mathbb{R}$ given by $h(x) = |x|$



Domain $= \mathbb{R}$

Range $= [0, \infty)$

11. (i) $i^{-35} = \frac{1}{i^{35}}$

$= \frac{1}{i^3}$

$= \frac{1}{-i}$

$= -(-i)$

$= i$

(ii) $z = \frac{1+i}{1-i}$

$= \frac{(1+i)(1+i)}{(1-i)(1+i)}$

$= \frac{1+2i+i^2}{1+1}$

$= \frac{2i}{2}$

$= i = 0+i$

$$\bar{z}^{-1} = \frac{\bar{z}}{|z|^2}$$

$$\bar{z} = -i$$

$$|z|^2 = 1$$

$$\therefore \bar{z}^{-1} = \frac{-i}{1}$$

$$= -i$$

$$= 0 - i$$

12. (i) no. of ways $= {}^{52}C_4$

$$= \frac{52 \times 51 \times 50 \times 49}{1 \times 2 \times 3 \times 4}$$

$$= 270725$$

(ii) no. of ways $= {}^{26}C_2 \times {}^{26}C_2$

$$= \frac{26 \times 25}{1 \times 2} \times \frac{26 \times 25}{1 \times 2}$$

$$= 105625$$

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(ii) $\left(x-\frac{1}{x}\right)^4 = {}^4C_0\, x^4 - {}^4C_1\, x^3\left(\frac{1}{x}\right) + {}^4C_2\, x^2\left(\frac{1}{x}\right)^2 - {}^4C_3\, x\left(\frac{1}{x}\right)^3 + {}^4C_4\left(\frac{1}{x}\right)^4$

$= 1\times x^4 - 4\, x^3\times\frac{1}{x} + 6x^2\times\frac{1}{x^2} - 4x\cdot\frac{1}{x^3} + \frac{1}{x^4}$

$= x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$

14. Let G_1, G_2, G_3 be the numbers

$\Rightarrow 1, G_1, G_2, G_3, 256$ are in G.P

$\therefore a = 1$

$ar = G_1$

$ar^2 = G_2$

$ar^3 = G_3$

$ar^4 = 256$

$ar^4 = 256 \Rightarrow r^4 = 256$

$\Rightarrow r = \pm 4.$

$\therefore r = 4 \Rightarrow G_1 = 4,\ G_2 = 16,\ G_3 = 64$

$r = -4 \Rightarrow G_1 = -4,\ G_2 = 16,\ G_3 = -64$

15. $a^2 = 9 \Rightarrow a = 3$
$b^2 = 16 \Rightarrow b = 4.$

$a^2 + b^2 = c^2$
$9 + 16 = c^2$
$c^2 = 25 \Rightarrow c = 5$

Foci : $(\pm c, 0) = (\pm 5, 0)$

Vertices : $(\pm a, 0) = (\pm 3, 0)$

eccentricity, $e = \frac{c}{a}$

$= \frac{5}{3}$

Length of latus rectum $= \frac{2b^2}{a}$

$= \frac{2 \times 16}{3}$

$= \frac{32}{3}.$

16. $n(S) = 9$

(i) Let A - event 'red disc'

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{4}{9}$

(ii) Let B - event 'yellow disc'

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{2}{9}$$

(iii) Let C - event 'blue disc'

$$P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{3}{9}$$

(iv) Let D - event 'not blue disc'

$$P(D) = \frac{n(D)}{n(S)}$$

$$= \frac{6}{9}$$

17. (i) $25^{\circ} = 25 \times \frac{\pi}{180}$ radian

$= \frac{5\pi}{36}$ rad.

(ii) $\sin 15^{\circ} = \sin(45^{\circ} - 30^{\circ})$

$= \sin 45^{\circ} \cos 30^{\circ} - \cos 45^{\circ} \sin 30^{\circ}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

(iii) L.H.S $= \frac{\sin 3x + \sin x}{\cos 3x + \cos x}$

$= \frac{2 \sin\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right)}{2 \cos\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right)}$

(xiv) $= \frac{\sin 2x}{\cos 2x}$

$= \tan 2x$

$= R.H.S$

18 (i) $(x_1, y_1) = (-4, 3)$

$m = \frac{1}{2}$

Equn: $y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = \frac{1}{2}(x+4)$

$\Rightarrow 2(y-3) = x+4$

$\Rightarrow 2y - 6 = x+4$

$\Rightarrow x - 2y + 10 = 0$

(ii) Equn is : $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\Rightarrow \frac{y+1}{5+1} = \frac{x-1}{3-1}$

$\Rightarrow \frac{y+1}{6} = \frac{x-1}{2}$

$$\Rightarrow \quad 2(y+1) = 6(x-1)$$

$$\Rightarrow \quad 2y+2 = 6x-6$$

$$\Rightarrow \quad 6x-2y-8=0$$

(iii) Slope of line (i) ; $m_1 = \frac{1}{2}$

Slope of line (ii) ; $m_2 = \frac{5+1}{3-1}$

$$= \frac{6}{2}$$

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$$

$$= \left|\frac{3-1/2}{1+3/2}\right|$$

$$= \left|\frac{5/2}{5/2}\right|$$

$$= 1$$

$$\therefore \theta = \tan^{-1}(1) = \frac{\pi}{4} = 45^\circ$$

$$\phi = 180^\circ - \theta = 180^\circ - 45^\circ = \underline{\underline{135^\circ}}$$

$\therefore$

19. (i) $f(x) = \tan x$

$$\frac{d}{dx}\left[f(x)\right] = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\left[\frac{\tan(x+h) - \tan x}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\sin(x+h)\cdot\cos x - \cos(x+h)\sin x}{h\ \cos(x+h)\ \cos x}\right]$$

$$= \lim_{h\to 0}\left[\frac{\sin(x+h-x)}{h\cdot\cos(x+h)\cos x}\right]$$

$$= \lim_{h\to 0}\frac{\sin h}{h}\cdot\lim_{h\to 0}\frac{1}{\cos(x+h)\cos x}$$

$$= 1 \times \frac{1}{\cos(x)\cdot\cos x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

(ii) $y = x \cdot \sin x$

$$\frac{dy}{dx} = x \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x)$$

$$= x \cdot \cos x + \sin x \cdot 1$$

$$= x \cdot \cos x + \sin x$$

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x_i	f_i	$x_i f_i$	$f_i x_i^2$
5	5	25	125
15	8	120	1800
25	15	375	9375
35	16	560	19600
45	6	270	12150
55	50	1350	43050

(i) Mean , $\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{1350}{50} = 27$

(ii) Varience , $\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - (\bar{x})^2$

$$= \frac{43050}{50} - 27^2 = 132$$

(iii) $S \cdot D = \sqrt{\text{Varience}} = \sqrt{132}$