1. (i) $y = x+1$; $x \in A$ & $y \in B$

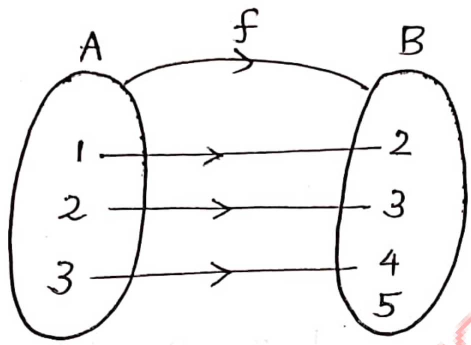
$x=1 \Rightarrow y = 1+1 = 2 \in B \quad \therefore (1,2) \in f$

$x=2 \Rightarrow y = 2+1 = 3 \in B \quad \therefore (2,3) \in f$

$x=3 \Rightarrow y = 3+1 = 4 \in B \quad \therefore (3,4) \in f$

$\therefore f = \{(1,2), (2,3), (3,4)\}$

(ii)



Every element in A has distinct images in B

$\therefore$ f is one-one

$5 \in B$ has no preimage in A under f

$\therefore$ f is not onto

2. $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \longrightarrow (1)$

$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \longrightarrow (2)$

$(1) + (2) \Rightarrow 2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

$$① \Rightarrow \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

3. $(x_1, y_1) = (1, 3)$

$(x_2, y_2) = (0, 0)$

Equation is $\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$$\Rightarrow \frac{1}{2}(y - 3x) = 0$$

$$\Rightarrow y - 3x = 0$$

$$\Rightarrow y = 3x$$

4 f is a continuous function

$\Rightarrow$ f is continuous at $x = 3$ & $x = 4$.

f is continuous at $x=3$

$$\Rightarrow \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3} 10 = 10$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3} (ax+b) = 3a+b$$

$$\therefore \quad 3a+b = 10 \rightarrow ①$$

f is continuous at $x=4$

$$\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4)$$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4} (ax+b) = 4a+b$$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4} (20) = 20$$

$$\therefore \quad 4a+b = 20 \rightarrow ②$$

$$② - ① \Rightarrow \quad a = 10$$

$$① \Rightarrow 3\times 10 + b = 10$$

$$\Rightarrow 30 + b = 10$$

$$\Rightarrow b = -20$$

5. $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

Let $f'(x) = 0$

$\Rightarrow \cos x - \sin x = 0$

$\Rightarrow \cos x = \sin x$

$\Rightarrow x = \frac{\pi}{4}$ $\left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

$f''(x) = -\sin x - \cos x$

$f''\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} - \cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= -\frac{2}{\sqrt{2}} < 0$

$\therefore x = \frac{\pi}{4}$ is a point of local maxima

$\therefore$ local maxima $= f\left(\frac{\pi}{4}\right)$

$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

6. (i) $\vec{a} \circ \vec{b} = 1 \times 3 + 2 \times 2 + 3 \times 1$

$= 3+4+3$

$= 10$

(ii) $|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + 2^2 + 1^2}$

$= \sqrt{9+4+1}$

$= \sqrt{14}$

$$\cos\theta = \frac{\vec{a} \circ \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{10}{\sqrt{14} \times \sqrt{14}}$$

$$= \frac{10}{14}$$

$$= \frac{5}{7}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{5}{7}\right)$$

7 Vector equation

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\vec{a} = P.V(1,2,3) = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\therefore \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$$

Cartesian Equation

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\Rightarrow \frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$$

8 (i) $P(A) = \frac{1}{2}$

$P(B) = \frac{7}{12}$

$P(A' \cup B') = \frac{1}{4}$

$\Rightarrow P((A \cap B)') = \frac{1}{4}$

$\Rightarrow 1 - P(A \cap B) = \frac{1}{4}$

$\Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}.$

(ii) $P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12}$

$= \frac{7}{24}$

$\neq P(A \cap B)$

$\therefore$ A & B are not independent

9. (i) (A) $(2,4) \in R$

(ii) Reflexivity

Let $a \in Z$

$a - a = 0$

clearly. 2 divides 0

$\Rightarrow (a,a) \in R$

$\therefore$ R is reflexive

Symmetry

Let $(a,b) \in R$

$\Rightarrow$ 2 divides $a-b$

$\Rightarrow$ 2 divides $b-a$

$\Rightarrow (b,a) \in R$

$\therefore$ R is symmetric

Transitivity

Let (a,b) & $(b,c) \in R$

$\Rightarrow$ 2 divides $a-b$ & 2 divides $b-c$

$$a-c = (a-b)+(b-c)$$
$$= Mul(2) + Mul(2)$$

$\Rightarrow$ 2 divides $a-c$

$\Rightarrow (a,c) \in R$

$\therefore$ R is transitive

Hence R is an equivalence relatn.

10. (i) $\frac{\pi}{6}$

(ii) $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2\times\frac{1}{2}\right]$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

11 (i) (C)

(ii) $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$.

$A + A' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$(A+A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$

$\therefore (A+A')' = A + A'$

$\Rightarrow A + A'$ is symmetric

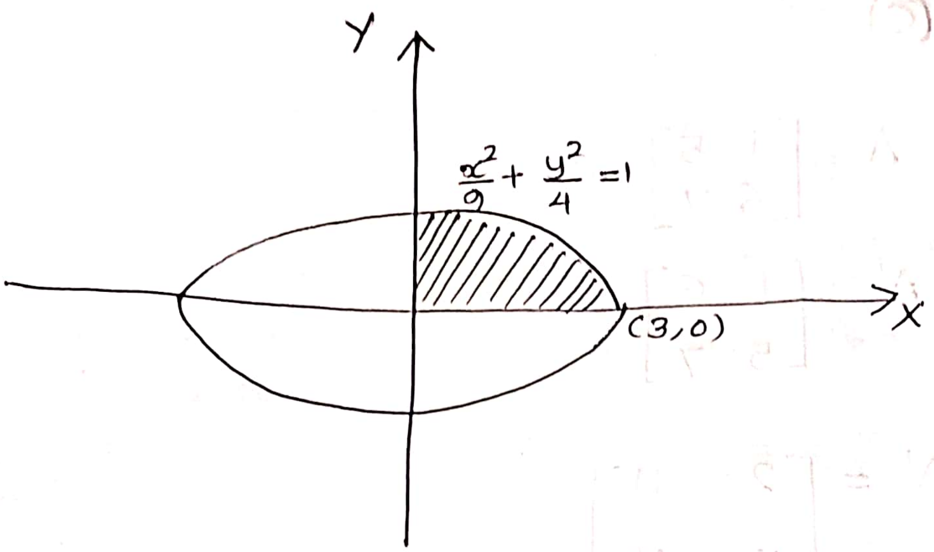
$A - A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$(A-A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -(A - A')$

$\therefore (A-A')' = -(A-A')$

$\Rightarrow A - A'$ is skew symmetric

12.



we have $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\Rightarrow \frac{y^2}{4} = 1-\frac{x^2}{9}$

$\Rightarrow \frac{y^2}{4} = \frac{9-x^2}{9}$

$\Rightarrow y^2 = \frac{4}{9}(9-x^2)$

$\Rightarrow y = \frac{2}{3}\sqrt{9-x^2}$

Area $= 4\times\int_a^b (y \text{ of curve})\, dx$

$= 4\int_0^3 \frac{2}{3}\sqrt{9-x^2}\, dx$

$= \frac{8}{3}\int_0^3 \sqrt{9-x^2}\, dx$

$$= \frac{8}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$$

$$= \frac{8}{3}\left[\frac{3}{2}\sqrt{9-9} + \frac{9}{2}\sin^{-1}(1) - (0+0)\right]$$

$$= \frac{8}{3}\left[\frac{9}{2} \times \frac{\pi}{2}\right]$$

$$= 2 \times 3\pi$$

$$= 6\pi \text{ sq. units.}$$

12 (i) (C)

(ii) $\frac{dy}{dx} = (1+x^2)(1+y^2)$

$$\Rightarrow \frac{dy}{1+y^2} = (1+x^2)\,dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$

14 (i) $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$

$$= \hat{i}(-14+14) - \hat{j}(2-21) + \hat{k}(-2+21)$$

$$= 19\hat{j} + 19\hat{k}$$

(ii) unit vector $\perp$r to $\vec{a}$ & $\vec{b}$ $= \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$$= \frac{19\hat{j}+19\hat{k}}{\sqrt{19^2+19^2}}$$

$$= \frac{19\hat{j}+19\hat{k}}{\sqrt{722}}$$

$$= \frac{19\hat{j}+19\hat{k}}{19\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

(iii) Area $= |\vec{a}\times\vec{b}|$

$= \sqrt{19^2+19^2}$

$= \sqrt{722}$ sq units

$= 19\sqrt{2}$ sq. units

15. $\vec{a_1} = i + 2j + k$ $\vec{b_1} = i + j + k$

$\vec{a_2} = 2i - j + 4k$ $\vec{b_2} = 2i + j + 2k$

$$d = \frac{|(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= i(2-1) - j(2-2) + k(1-2)$$

$$= i - k$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{1+1} = \sqrt{2}$$

$$\vec{a_2} - \vec{a_1} = i - 3j + 3k$$

$$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = 1 \times 1 - 1 \times 3 = 1 - 3 = -2$$

$$\therefore d = \frac{|-2|}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

16

Bag I	Bag II
3R 4B	5R 6B

E_1 – event 'Bag I'

E_2 – event 'Bag II'

A – event 'getting red ball'

$$P(E_2|A) = \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$

$P(A|E_1) = \frac{3}{7}$

$P(A|E_2) = \frac{5}{11}$

$$\therefore P(E_2|A) = \frac{1/2 \times 5/11}{1/2 \times \frac{3}{7} + 1/2 \times 5/11}$$

$$= \frac{35}{68}$$

17. $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$ $\quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$X = A^{-1} B$$

$$= \frac{\text{adj}A}{|A|} B$$

$$|A| = 1(1+1) - 1(2-2) + 1(-2-2)$$

$$= 1 \times 2 + 1 \times -4$$

$$= -2 \neq 0$$

To find adj A :

$$\begin{array}{c|c|cccc} & c_1 & c_2 & & & \\ R_1 & 1 & 1 & 1 & 1 & 1 \\ \hline R_2 & 2 & 1 & 1 & 2 & 1 \\ & 2 & -1 & 1 & 2 & -1 \\ & 1 & 1 & 1 & 1 & 1 \\ & 2 & 1 & 1 & 2 & 1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 2 & -2 & 0 \\ 0 & -1 & 1 \\ -4 & 3 & -1 \end{bmatrix}$$

$$\therefore X = -\frac{1}{2}\begin{bmatrix} 2 & -2 & 0 \\ 0 & -1 & 1 \\ -4 & 3 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 6-8+0 \\ 0-4+2 \\ -12+12-2 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore x = y = z = 1$$

18 (a) $y = x^x$

$$\Rightarrow \log y = x \cdot \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x$$

$$\Rightarrow \frac{dy}{dx} = y\left[1 + \log x\right]$$

$$= x^x\left[1 + \log x\right]$$

(ii) $\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a$

$\frac{dx}{dt} = \frac{d}{dt}(at^2) = 2at$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$= \frac{2a}{2at}$$

$$= \frac{1}{t}$$

(iii) Given: $\frac{dr}{dt} = 5 cm/s$

$\text{Area} = \pi r^2$

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2)$$

$$= 2\pi r \cdot \frac{dr}{dt}$$

$$= 2\pi r \times 5$$

$$= 10\pi r \ cm^2/s$$

$$\left.\frac{dA}{dt}\right|_{r=8} = 10\pi \times 8 = 80\pi \ cm^2/s$$

19. (i) $\displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$

(ii) $\displaystyle I = \int \frac{dx}{x^2+4x-5}$

we have;

$$x^2+4x-5 = (x^2+4x+4)-4-5$$
$$= (x+2)^2-9$$
$$= (x+2)^2-3^2$$

$\therefore$ $\therefore$ $\displaystyle I = \int \frac{dx}{(x+2)^2-3^2}$

$$= \frac{1}{2\times 3} \log\left|\frac{x+2-3}{x+2+3}\right| + C$$

$$= \frac{1}{6} \log\left|\frac{x-1}{x+5}\right| + C$$

(iii) $\displaystyle I = \int_2^3 \frac{x}{1+x^2}\,dx.$

Put $1+x^2 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \frac{1}{2}dt$

$x = 2 \Rightarrow t = 1+2^2 = 5$

$x = 3 \Rightarrow t = 1+3^2 = 10$

$$\therefore \quad I = \frac{1}{2}\int_{5}^{10} \frac{dt}{t}$$

$$= \frac{1}{2}\Big[\log|t|\Big]_{5}^{10}$$

$$= \frac{1}{2}\left(\log 10 - \log 5\right)$$

$$= \frac{1}{2}\log\left(\frac{10}{5}\right)$$

$$= \frac{1}{2}\log 2$$

20 $\quad 5x + y = 100$

x	0	20
y	100	0

Points $(0,100)$ & $(20,0)$

$x + y = 60$

x	0	60
y	60	0

Points $(0,60)$, $(60,0)$

Point of intersection:

$$5x+y=100 \rightarrow ①$$

$$x+y=60 \rightarrow ②$$

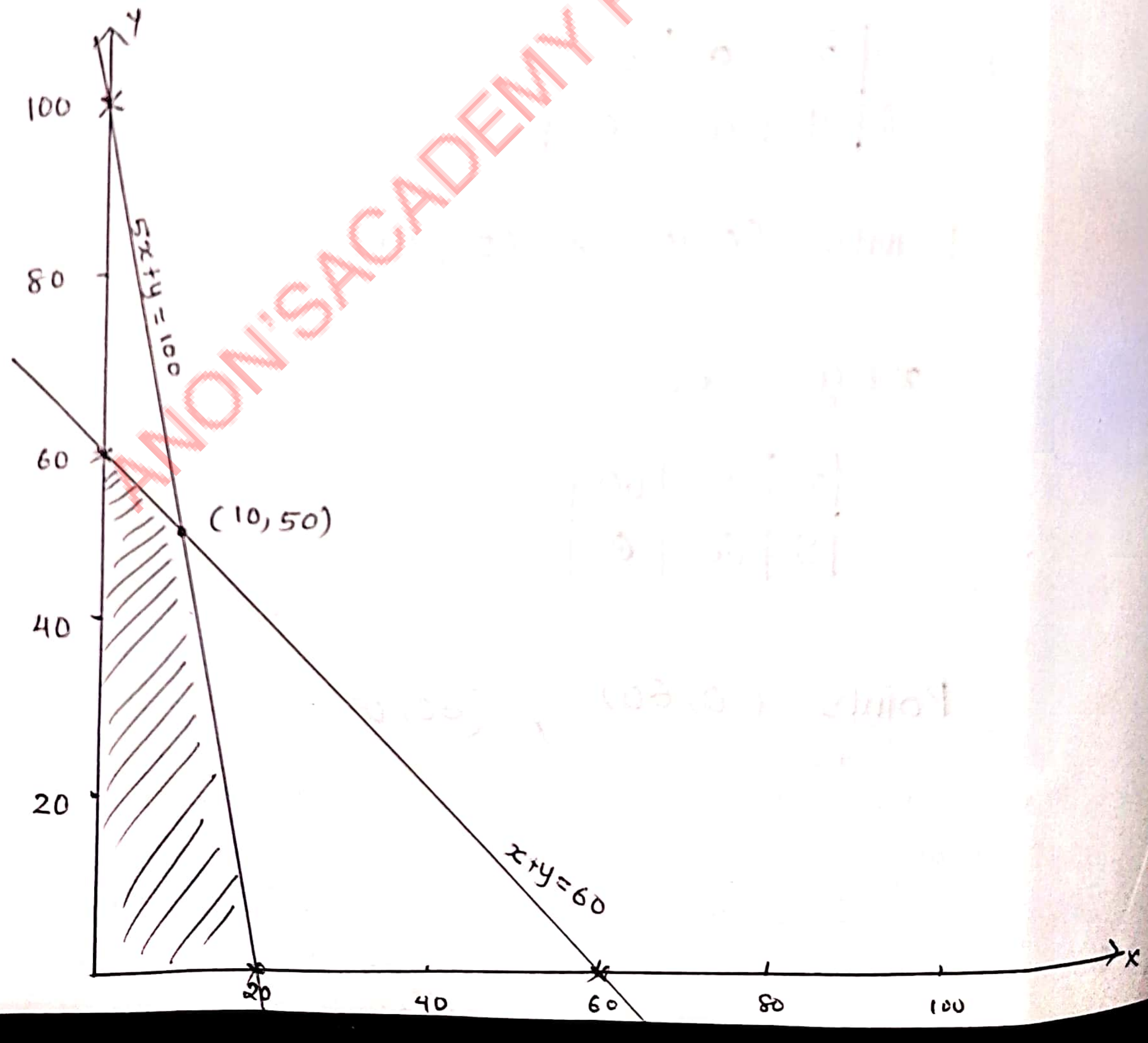
$$① - ② \Rightarrow 4x = 40$$

$$\Rightarrow x = 10$$

$$② \Rightarrow 10 + y = 60$$

$$\Rightarrow y = 50$$

∴ point of intersectn is (10, 50)



Corner Points	$Z = 250x + 75y$
$(0,60)$	$Z = 4500$
$(0,0)$	$Z = 0$
$(20,0)$	$Z = 5000$
$(10,50)$	$Z = 6250$

$$\text{Max } Z = 6250$$