Mode Exam (2022 – 23)
CLASS – X – MATHEMATICS

Time Allowed : 3 Hrs   Maximum Marks : 100

PART – I

I. Answer ALL the questions.   14x1 = 14

1) If $f(x) = 2x^2$ and $g(x) = \dfrac{1}{3x}$, then $f \circ g$ is

(A) $\dfrac{3}{2x^2}$ (B) $\dfrac{2}{3x^2}$ (C) $\dfrac{2}{9x^2}$ (D) $\dfrac{1}{6x^2}$

2) If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is

(A) 3   (B) 2   (C) 4   (D) 8

3) If 6 times of 6th term of an A.P. is equal to 7 times the 7th term, then the 13th term of the A.P. is

(A) 0   (B) 6   (C) 7   (D) 13

4) The next term of the sequence $\dfrac{3}{16}, \dfrac{1}{8}, \dfrac{1}{12}, \dfrac{1}{18}, \cdots$ is

(A) $\dfrac{1}{24}$ (B) $\dfrac{1}{27}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{1}{81}$

5) The square root of $\dfrac{256x^8 y^4 z^{10}}{25x^6 y^6 z^6}$ is equal to

(A) $\dfrac{16}{5}\left|\dfrac{x^2 z^4}{y^2}\right|$ (B) $16\left|\dfrac{y^2}{x^2 z^4}\right|$ (C) $\dfrac{16}{5}\left|\dfrac{y}{xz^2}\right|$ (D) $\dfrac{16}{5}\left|\dfrac{xz^2}{y}\right|$

6) Transpose of a column matrix is

(A) unit matrix   (B) diagonal matrix
(C) column matrix   (D) row matrix

7) Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?

(A) 13 m   (B) 14 m   (C) 15 m   (D) 12.8 m

8) The slope of the line joining $(12,3)$, $(4,a)$ is $\dfrac{1}{8}$. The value of 'a' is

(A) 1   (B) 4   (C) −5   (D) 2

9) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\theta)$ is equal to

(A) 0   (B) 1   (C) 2   (D) −1

10) The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in metres) is

(A) 20, $10\sqrt{3}$ (B) 30, $5\sqrt{3}$ (C) 20, 10   (D) 30, $10\sqrt{3}$

11) The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be

(A) 12 cm   (B) 10 cm   (C) 13 cm   (D) 5 cm

12) A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is

(A) 2:1   (B) 1:2   (C) 4:1   (D) 1:4

13) Variance of first 20 natural numbers is

(A) 32.25   (B) 44.25   (C) 33.25   (D) 30

14) If a letter is chosen at random from the English alphabets $\{a,b,...,z\}$, then the probability that the letter chosen precedes x

(A) $\dfrac{12}{13}$ (B) $\dfrac{1}{13}$ (C) $\dfrac{23}{26}$ (D) $\dfrac{3}{26}$

PART – II

II. Answer any TEN questions   Question No. 28 is compulsory   10x2 = 20

15) If $A = \{-2,-1,0,1,2\}$ and $f : A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

16) 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

17) The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

18) If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

19) A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.

20) Find the equation of a line which passes through (5,7) and makes intercepts on the axes equal in magnitude but opposite in sign.

21) Prove the following identity.

$$\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} = \sec\theta + \tan\theta$$

22) From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.

23) The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

24) If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

25) Two coins are tossed together. What is the probability of getting different faces on the coins?

26) Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values.

27) The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.

28) The difference of two natural numbers is 5 and the difference of their reciprocals is 1/10. Find the numbers.

PART – III

II. Answer any SEVEN questions. Question 42 is compulsory   **10x5 =50**

29) If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k.

30) If l^{th}, m^{th} and n^{th} terms of an A.P. are x, y, z respectively, then show that

   (i) $x(m - n) + y(n - l) + z(l - m) = 0$ (ii) $(x - y)n + (y - z)l + (z - x)m = 0$

31) Sivamani is attending an interview for a job and the company gave two offers to him.

Offer A: ₹20,000 to start with followed by a guaranteed annual increase of 6% for the first 5 years.

Offer B: ₹22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.

What is his salary in the 4th year with respect to the offers A and B?

32) A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of passenger train is less than that of an express train by $20\,km$ per hour. Find the average speed of both the trains.

33) If α, β are the roots of $7x^2 + ax + 2 = 0$ and if $\beta - \alpha = \dfrac{-13}{7}$. Find the values of a.

34) Two poles of height 'a' metres and 'b' metres are 'p' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\dfrac{ab}{a+b}$ metres.

35) State and prove Angle Bisector Theorem.

36) You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by $y = -0.1x + 1$.
   (i) find the total MB of the song.
   (ii) after how many seconds will 75% of the song gets downloaded?
   (iii) after how many seconds the song will be downloaded completely?

37) Find the equation of a straight line joining the point of intersection of $3x + y + 2 = 0$ and $x - 2y - 4 = 0$ to the point of intersection of $7x - 3y = -12$ and $2y = x + 3$

38) From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\dfrac{4h}{\sqrt{3}}$ m.

39) A pole 5m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is 60° and the angle of depression to the point 'A' from the top of the tower is 45°. Find the height of the

40) From a solid cylinder whose height is 2 4 cm and diameter 1.4 cm, a conical cavity of the same height and base is hollowed out (Fig. 40). Find the total surface area of the remaining solid.
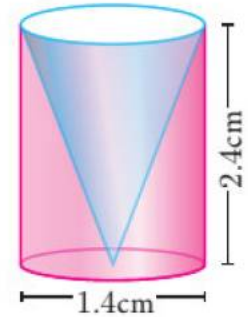

Fig. 40

41) In a game, the entry fee is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

42) In a three-digit number, when the tens and the hundreds digit are interchanged the new number is 54 more than three times the original number. If 198 is added to the number, the digits are reversed. The tens digit exceeds the hundreds digit by twice as that of the tens digit exceeds the unit digit. Find the original number.

PART – IV

IV. Answer ALL questions.   **2x8 = 16**

43) a) Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5} > 1$).

OR

b) Draw a triangle ABC of base $BC = 5.6$ cm, $\angle A = 40°$ and the bisector of $\angle A$ meets BC at D such that $CD = 4$ cm.

44) a) Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$

OR

b) Graph the following quadratic equations and state their nature of solutions.
$$x^2 + x + 7 = 0$$