M

COMMON FIRST MIDTERM TEST - 2021

Reg. No. | | | | | | | |

XII STANDARD

Time: 1.30 Hrs. **Marthematics** Marks:50

Part - I

$10 \times 1 = 10$

Note:- (i) Answer all the questions.

(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|\text{adj}(AB)| =$

a) -40 b) -80 c) -60 d) -20

2) If $A^T A^{-1}$ is symmetric then $A^2 =$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$ then λ is

a) 17 b) 14 c) 19 d) 21

4) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

a) 0 b) 1 c) -1 d) i

5) If $\frac{z-1}{z+1}$ is purely imaginary then $|z|$ is

a) ½ b) 1 c) 2 d) 3

6) If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to

a) 2 b) -1 c) 1 d) 0

7) If $\vec{a} = \hat{i} + \hat{j}\ \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda a + \mu b$ then the value of $\lambda + \mu$ is

a) 0 b) 1 c) 6 d) 3

8) The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$

a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

9) Subtration is not a binary operation in

a) R b) z c) N d) Q

10) If a compound statement involves 3 simple statements, then the number of rows in the truth table is

a) 9 b) 8 c) 6 d) 3

Part - II

$4 \times 2 = 8$

Answer any Four questions. Question No. 16 is compulsory.

11) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

12) If $\text{adj}\,A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Find A^{-1}

13) Simplify $\sum_{n=1}^{10} i^{n+50}$

14) A particle acted upon by constant forces $2\hat{i} + 5\hat{j} + 6\hat{k}$ and $-\hat{i} - 2\hat{j} - \hat{k}$ is diplaced from the point $(4, -3, -2)$ to the point $(6, 1, 3)$. Find the total workdone by the forces.

15) Construct the truth table for $(\neg p \ \lambda \ \neg q)$

16) Let $A = \{a + \sqrt{5}b : a, b \in z\}$. Check whether the usual multiplication is a binary operation on A.

Part - III **4×3=12**

Answer any Four questions. Question No. 22 is compulsory.

17) Find the rank of the matrix by Minor Method.

$$\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$$

18) Find the square root of $6 - 8i$

19) If $|z| = 2$ show that $3 \leq |z \ 4i| \leq 7$

20) The volume of the parallelopiped whose coterminus edges are $7\hat{i} + m\hat{j} - 3\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-3\hat{i} + 7\hat{j} + 5\hat{k}$ is 90 Cubic units. Find the value of λ.

21) Show that $7(p \to q) \equiv p \wedge 7q$

22) Find the angle between the straight line $\frac{x+3}{2} = \frac{y-1}{2} = -z$ with coordinate axes.

Part - IV **4×5=20**

Answer all the following questions.

23) a) Solve by Cramer's rule, the system of equations

$x_1 - x_2 = 3,\ 2x_1 + 3x_2 + 4x_3 = 17,\ x_2 + 2x_3 = 7$ **(or)**

b) Given the complex number $z = 3 + 2i$ represent the complex numbers z, iz and z + iz is one Argand plane. Show that these Complex numbers form the vertices of an isosceles right triangle.

24) a) Prove by vector Method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ **(or)**

b) The upward speed v(t) of a rocket at time t is aproximated by $v(t) = at^2 + bt + c$, $0 \leq t \leq 100$ where a, b and c are constants. It has been found that the speed at times $t = 3$ $t = 6$ and $t = 9$ seconds are respectively 64, 133 and 208 Miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian Elimination Method).

25) a) Verify i) Closure property ii) Commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation X_{11} on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(or)

b) Find the vector parametric, vector non parametric and Cartesian form of the equation of the plane passing through the points (−1, 2, 0) (2, 2, −1) and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

26) a) If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$ show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$ **(or)**

b) Let A be Q\{1}. Define * on A by $x * y = x + y - xy$. It is * binary on A?

i) If so, examine the commutative and associative properties statisfied by * on A.

ii) If so, examine the existence of identity, existence of inverse properties for the operation * on A.