Kerala SSLC Mathematics Question Bank

Version: 1

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Contents

Chapter 1

Arithmetic Sequences

Question 1:

2023 Model (3)

The sum of first 5 terms of an arithmetic sequence is 145.

(a) Find the third term

(b) If the common difference of this sequence is 4, write the terms.

Answer:

(a) If n is an odd number, then:

$$S_n = n \times (middle\ term)$$

$$S_5 = 5 \times (x_3) = 145$$

$$x_3 = 145/5 = 29$$

(b) Since the common difference is 4, the terms of the arithmetic sequence are:

$$x_1 = 29 - 2 \times 4 = 21$$

$$x_2 = 29 - 1 \times 4 = 25$$

$$x_3 = 29$$

$$x_4 = 29 + 1 \times 4 = 33$$

$$x_5 = 29 + 2 \times 4 = 37$$

Note: The sum of an odd number of terms in arithmetic sequence is equal to the middle term multiplied by the number of terms.

Question 2:

2023 Final (2)

$7, 13, 19, ...$ is an arithmetic sequence.

(a) What is its common difference?

(b) Find its 11^{th} term.

Answer:

(a) $13 - 7 = 6$

(b) To calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$

$$x_{11} = 7 + (11-1) \times 6 = 67$$

Question 3:

2023 Model (4)

A sequence is written by adding 3 to the multiples of 4.

(a) Write the algebraic form of the sequence.

(b) Find the tenth term of the sequence.

(c) Is 100 a term of this sequence? Why?

Answer:

(a) The first multiple of 4 is $1 \times 4 = 4$. So, the first term:

$$4 + 3 = 7$$

The second multiple of 4 is $2 \times 4 = 8$. So, the second term:

$$8 + 3 = 11$$

The common difference:

$$11 - 7 = 4$$

To calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$

$$x_n = 7 + (n-1) \times 4$$

$$x_n = 7 + 4n - 4$$

$$x_n = 4n + 3$$

(b) $x_{10} = 4 \times 10 + 3 = 43$

(c) Suppose 100 is a term, then:

$$4n + 3 = 100$$

$$4n = 100 - 3$$

$$4n = 97$$

$$n = 97/4 = 24.25$$

Since n is not a natural number, 100 is not a term in the sequence.

Question 4:

2023 Final (4)

(a) Find the sum of first 20 natural numbers.

(b) Write the algebraic expression of the arithmetic sequence $5, 9, 13, ...$

(c) Find the sum of first 20 terms of the arithmetic sequence $5, 9, 13, ...$

Answer:

(a) To calculate sum of first n natural numbers:

$$S_n = \frac{n(n+1)}{2}$$

$$S_{20} = \frac{20(20+1)}{2}$$

$$S_{20} = 10 \times 21 = 210$$

(b) First term:

$$5$$

Common difference:

$$9 - 5 = 4$$

Formula to calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$

$$x_n = 5 + (n-1) \times 4$$

$$x_n = 5 + 4n - 4$$

$$x_n = 4n + 1$$

(c) First term:

$$5$$

Common difference:

$$9 - 5 = 4$$

To calculate sum of first n terms:

$$S_n = \frac{1}{2}n(x_n + x_1)$$

$$S_n = \frac{1}{2}n(4n + 1 + x_1)$$

$$S_n = \frac{1}{2} \times 20 \times (4 \times 20 + 1 + 5)$$

$$S_n = 860$$

Question 5:

2023 Final (3)

6^{th} term of an arithmetic sequence is 46. Its common difference is 8.

(a) What is its 16^{th} term?

(b) Find its 21^{st} term.

Answer:
Given:

$$x_6 = 46$$

$$d = 8$$

Formula to calculate n^{th} term:

$$x_n = x_1 + (n - 1)d$$

$$x_6 = x_1 + (6 - 1) \times 8 = 46$$

$$x_1 + 5 \times 8 = 46$$

$$x_1 + 40 = 46$$

$$\therefore x_1 = 46 - 40 = 6$$

(a) 16^{th} term:

$$x_{16} = 6 + (16 - 1) \times 8$$

$$x_{16} = 6 + 120 = 126$$

(b) 21^{st} term:

$$x_{21} = 6 + (21 - 1) \times 8$$

$$x_{21} = 6 + 160 = 166$$

Question 6:

2023 Final (4)

Sum of first 7 terms of an arithmetic sequence is 140.
Sum of first 11 terms of the same arithmetic sequence is 440.

(a) What is the 4^{th} term of this arithmetic sequence?

(b) Find its 6^{th} term.

(c) What is the common difference?

(d) Find the first term of this sequence.

Answer:

(a) If n is an odd number, then:

$$S_n = n \times (middle\ term)$$

$$S_7 = 7 \times (x_4) = 140$$

$$x_4 = 140/7 = 20$$

(b) If n is an odd number, then:

$$S_n = n \times (middle\ term)$$

$$S_{11} = 11 \times (x_6) = 440$$

$$x_6 = 440/11 = 40$$

(c) Common difference:

$$\frac{x_6 - x_4}{6 - 4} = \frac{40 - 20}{2}$$

$$= \frac{20}{2} = 10$$

(d) Using the formula for x_4:

$$x_4 = x_1 + (4 - 1) \times 10 = 20$$

$$x_1 + 3 \times 10 = 20$$

$$x_1 = 20 - 30 = -10$$

Question 7:

2023 Model (5)

The sum of the 8^{th} and 19^{th} terms of an arithmetic sequence is 125.

(a) What is the sum of the 7^{th} and 20^{th} terms?

(b) If the 6^{th} term is 40, then find the 21^{st} term.

(c) Find the sum of first 26 terms.

Answer:

(a) Sum of term positions:

$$8 + 19 = 7 + 20 = 27$$

$$\therefore x_7 + x_{20} = x_8 + x_{19} = 125$$

(b) Given:

$$x_6 = 40$$

Sum of term positions:

$$6 + 21 = 8 + 19 = 27$$

$$\therefore x_6 + x_{21} = x_8 + x_{19} = 125$$

$$x_{21} = 125 - x_6 = 125 - 40 = 85$$

(c) Sum of term positions:

$$1 + 26 = 8 + 19 = 27$$

$$\therefore x_1 + x_{26} = x_8 + x_{19} = 125$$

To calculate sum of n terms:

$$S_n = \frac{n}{2}(x_1 + x_n)$$

$$S_{26} = \frac{26}{2}(x_1 + x_{26})$$

$$= 13 \times 125$$

$$\therefore S_{26} = 1625$$

Note: In an arithmetic sequence, if the sums of positions of two pairs of terms are equal, then the sums of the pairs of the terms are also equal.

Question 8:

2022 Final (2)

$5, 8, 11, ...$ is an arithmetic sequence.

(a) What is 20^{th} term?

(b) What is the algebraic expression for this sequence?

Answer:

Common difference:

$$8 - 5 = 3$$

(a) To calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$

$$x_{20} = 5 + (20-1) \times 3 = 62$$

(b) Formula to calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$

$$x_n = 5 + (n-1)3$$

$$x_n = 5 + 3n - 3$$

$$x_n = 3n + 2$$

Question 9:

2022 Final (2)

The algebraic expression for the sum of n terms of an arithmetic sequence is $n^2 + n$.

(a) Find the first term of this arithmetic sequence.

(b) Find the sum of first 10 terms of this arithmetic sequence.

Answer:

(a) We have the formula to calculate sum:

$$S_n = n^2 + n$$

Sum of the first term:

$$S_1 = 1^2 + 1 = 1 + 1 = 2$$

$$\therefore x_1 = 2$$

(b) We have the formula to calculate sum:

$$S_n = n^2 + n$$

Sum of the first term:

$$S_{10} = 10^2 + 10 = 100 + 10 = 110$$

Question 10:

2022 Final (8)

$6, 10, 14, ...$ is an arithmetic sequence.

(a) Find the sum of the first 15 terms of this arithmetic sequence.

(b) What is the difference between the first term and the 16^{th} term?

(c) Find the ditference between the sum of first 15 terms and sum of the next 15 terms.

Answer:

(a) We have:

$$x_1 = 6$$

$$x_2 = 10$$

$$d = x_2 - x_1 = 10 - 6 = 4$$

To calculate 15^{th} term, using the formula:

$$x_n = x_1 + (n-1)d$$

$$x_{15} = 6 + (15-1) \times 4 = 62$$

To calculate sum of 15 terms, using the formula:

$$S_n = \frac{n}{2}(x_1 + x_n)$$

$$S_{15} = \frac{15}{2}(6 + 62) = 510$$

(b) To calculate the difference, use:

$$x_{16} - x_1$$

$$x_{16} = 6 + (16-1) \times 4 = 66$$

$$difference = 66 - 6 = 60$$

(c) To calculate the difference, use:

$$S_{30} - S_{15} - S_{15}$$

$$S_{30} = \frac{30}{2}(6 + 6 + (30-1) \times 4) = 1920$$

$$difference = 1920 - 510 - 510 = 900$$

Question 11:

2022 Final (1)

What is the common difference of the Arithmetic sequence $3, 7, 11, ...$

Question 12:

2022 Model (1)

Write the first three terms of the arithmetic sequence with first term 6 and common difference 4.

Question 13:

2023 Model (2)

Write the next two terms of the arithmetic sequence $5, 12, 19, ...$

Question 14:

2022 Model (2)

The sum of first seven terms of an arithmetic sequence is 84. Find its 4^{th} term.

Answer:

If n is an odd number, then:

$$S_n = n \times (middle\ term)$$

$$S_7 = 7 \times (x_4) = 84$$

$$x_4 = 84/7 = 12$$

Question 15:

2022 Model (2)

The expression for the sum of first $'n'$ terms of an arithmetic sequence is $2n^2 + 4n$. Find the first term and common difference of this sequence.

Answer:

We have the formula to calculate sum:

$$S_n = n^2 + 4n$$

Sum of the first term:

$$S_1 = 2 \times 1^2 + 4 \times 1 = 2 + 4 = 6$$

$$\therefore x_1 = 6$$

Sum of the first and second term:

$$S_2 = 2 \times 2^2 + 4 \times 2 = 8 + 8 = 16$$

$$x_1 + x_2 = 16$$

$$\therefore x_2 = 16 - x_1 = 16 - 6 = 10$$

Common difference:

$$d = x_2 - x_1 = 10 - 6 = 4$$

Question 16:

2022 Model (8)

(a) Consider the arithmetic sequence $4, 7, 10, ...$ what is the algebraic expression for this sequence?

(b) Write the 20^{th} term of this sequence. What is the smallest three digit number which is a term of this sequence?

(c) Find the sum of first 20 terms of this sequence.
What is the difference between the sum of first 20 terms of this sequence and sum of first 20 terms of the arithmetic sequence with algebraic form $3n + 2$.

Answer:

(a) We have:

$$x_1 = 4$$

$$x_2 = 7$$

$$\therefore d = x_2 - x_1 = 7 - 4 = 3$$

To derive the algebraic expression, let's use the formula to calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$

$$x_n = 4 + (n-1)3 = 4 + 3n - 3$$

$$x_n = 3n + 1$$

(b)

$$x_{20} = 3 \times 20 + 1 = 61$$

Suppose 100 is the smallest term, then:

$$3n + 1 = 100$$

$$3n = 100 - 1 = 99$$

$$n = \frac{99}{3} = 33$$

Since 33 is a natural number, 100 is the smallest three digit term.

(c) To calculate sum of first 20 terms, using the formula:

$$S_n = \frac{n}{2}(x_1 + x_n)$$

$$S_{20} = \frac{20}{2}(4 + 3 \times 20 + 1)$$

$$S_{20} = 10 \times 65 = 650$$

For the algebraic form:

$$x_n = 3n + 2$$

$$x_1 = 3 \times 1 + 2 = 5$$

$$x_2 = 3 \times 2 + 2 = 8$$

$$d = 8 - 5 = 3$$

$$S_{20} = \frac{20}{2}(5 + 3 \times 20 + 2)$$

$$S_{20} = 10 \times 67 = 670$$

$$\therefore difference = 670 - 650 = 20$$

Question 17:

2021 Final (1)

Arithmetic sequence with common difference 2 is:

$$[7, 10, 13, ... \quad ; \quad 7, 5, 3, ...$$

$$7, 9, 11, ... \quad ; \quad 2, 5, 8, ...]$$

Question 18:

2021 Final (2)

Write the first term and common difference of the arithmetic sequence $3n + 2$.

Hint: See the question 16 last part.

Question 19:

2021 Final (2)

(a) What is the tenth term of the arithmetic sequence $a+1, a+2, a+3, ...$?

(b) What is its common difference?

(c) Write the algebraic form of the abve sequence.

Answer:

(a) We have:

$$x_1 = a + 1$$

$$x_2 = a + 2$$

$$\therefore d = a + 2 - (a + 1) = 1$$

To calculate 10^{th} term, using the formula:

$$x_n = x_1 + (n-1)d$$

$$x_{10} = a + 1 + (10 - 1) \times 1 = a + 10$$

(b) We have:

$$x_1 = a + 1$$

$$x_2 = a + 2$$

$$\therefore d = a + 2 - (a + 1) = 1$$

(c) To derive the algebraic expression, let's use the formula to calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$

$$= a + 1 + (n-1) \times 1$$

$$= a + 1 + n - 1$$

$$\therefore x_n = a + n$$

Question 20:

2021 Final (4)

(a) Write the 20^{th} term of the arithmetic sequence $5, 10, 15, ...$

(b) Find the sum of the first 20 terms of the arithmetic sequence $5, 10, 15, ...$

(c) What is the sum of the first 20 terms of the arithmetic sequence $4, 9, 14, ...$?

Question 21:

2021 Final (5)

1
2 3
4 5 6
7 8 9 10
....

(a) Write the fifth line of the pattern.

(b) How many numbers are there in the tenth line?

(c) How many numbers are there in the first ten line altogether?

(d) What is the first number in the eleventh line?

Answer:

(a) Fifth line:
11 12 13 14 15

(b) Each line has the same number as that of the position. So the answer is 10

(c) Each line ends with the sum of the natural numbers corresponding to the position. To calculate sum of first n natural numbers:

$$S_n = \frac{n(n+1)}{2}$$

$$S_{10} = \frac{10 \times (10+1)}{2} = 55$$

(d) Since the 10^{th} line ends with 55, the eleventh line should starts with: $55+1 = 56$

Question 21:

2021 Final (5)

(a) What is the remainder on dividing the terms of the arithmetic sequence $100, 109, 118, ...$ by 9?

(b) Write the sequence of three digit numbers, which are multiples of 9.

(c) What is the position of 999 in the arithmetic sequence of three digit numbers which are multiples of 9?

Answer:

(a) 1

(b) 100 has a remainder of 1, so the first number is:

$$100 - 1 + 9 = 108$$

The multiples of 9:

$$108, 117, 126, ...$$

(c) To derive the algebraic expression, let's use the formula to calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$
$$= 108 + (n-1) \times 9$$
$$= 108 + 9n - 9$$
$$\therefore x_n = 9n + 99$$

To find position of 999:

$$9n + 99 = 999$$
$$9n = 999 - 99 = 900$$
$$\therefore n = \frac{900}{9} = 100$$

Question 22:

2021 Model (1)

What is the common difference of the arithmetic sequence $4, 10, 16, ...$?

$$[4, 5, 6, 10]$$

Question 23:

2021 Model (2)

Algebraic form of an arithmetic sequence is $3n + 2$.

(a) What is its first term?

(b) Find its 10^{th} term.

Question 24:

2021 Model (3)

The 5^{th} term of an arithmetic sequence is 20 and the 8^{th} term is 32.

(a) What is the common difference of this sequence?

(b) Find its 11^{th} term.

Question 25:

2021 Model (4)

The 10^{th} term of an arithmetic sequence is 20 and its 20^{th} term is 10.

(a) What is its common difference?

(b) What is its 30^{th} term?

(c) Which is the first negative term of this sequence?

Question 26:

2021 Model (4)

(a) What is its 20^{th} term?

(b) Find the sum of first 20 terms of this sequence.

(c) What is the sum of first 20 terms of the arithmetic sequence $6, 8, 10, ...$?

Question 27:

2021 Model (5)

Look at the following number pattern.

$$
\begin{array}{ccccc}
 & & 1 & & \\
 & 2 & 3 & 4 & \\
5 & 6 & 7 & 8 & 9
\end{array}
$$

....

....

(a) Write the next line of this pattern.

(b) Write the sequence of last numbers in each line.

(c) What will be the last number in the 9^{th} line.

(d) Write the first and last numbers of the 10^{th} line.

Answer:

(a) From the given pattern, the line ends with the square of the line number. So, the 4^{th} line should end with 16:

$$10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16$$

(b) The sequence follows the algebraic form of n^2:

$$1, 4, 9, 16, ...$$

(c) Last number of 9^{th} line:

$$9^2 = 81$$

(d) Since the 9^{th} line ends with 81, the 10^{th} line should start with:

$$81 + 1 = 82$$

Last number of 10^{th} line:

$$10^2 = 100$$

Question 28:

2020 Final (2)

(a) Write the 6^{th} term of the arithmetic sequence $1, 25, 49, 73, 97, ...$

(b) How many perfect square terms are there in the arithmetic sequence $97, 73, 49, ...$?

Answer:

(a) (*Please refer previous answers*)

(b) The given sequence has these terms:

$$97, 73, 49, 25, 1, ...$$

The perfect square numbers below 97 are:

$$1, 4, 9, 16, 25, 36, 49, 64, 81$$

So, the perfect squares in the given sequence are:

$$1, 25, 49$$

$\therefore$ The number of perfect squares terms is 3.

Question 29:

2020 Final (3)

(a) Write the first term and the common difference of the arithmetic sequence whose algebraic expression is $3n + 5$.

(b) First term of an arithmetic sequence is 8 and the common difference is 5. Write its algebraic form.

Question 30:

2020 Final (3)

(a) What is the common difference of the arithmetic sequence $x-1, x, x+1, ...$?

(b) If $x-1$ is an even number, which is the next even number?

(c) Prove that the product of two consecutive even numbers added to 1 gives a perfect square.

Answer:

(a) Common difference:

$$x-(x-1)=x-x+1=1$$

(b) Next even number:

$$x-1+2=x+1$$

(c) Let consecutive even number are $x-1$ and $x+1$, then:

$$(x-1)(x+1)+1=x^2-x+x-1+1=x^2$$

$\therefore x^2$ is a perfect square.

Question 31:

2020 Final (4)

(a) What is the sum of the first 5 terms of the arithmetic sequence $1, 3, 5, 7, ...$?

(b) What is the sum of the first n terms of the arithmetic sequence $1, 3, 5, 7, ...$?

(c) Find the sum of the first n terms of the arithmetic sequence $\frac{1}{n}, \frac{3}{n}, \frac{5}{n}, \frac{7}{n}, ...$

(d) What is the sum of first 2020 terms of the arithmetic sequence $\frac{1}{2020}, \frac{3}{2020}, \frac{5}{2020}, ...$?

Answer:

(a) Sum of odd numbers:

$$S_n=n^2$$

$$S_5=5^2=25$$

(b) Sum of odd numbers:

$$S_n=n^2$$

(c) Factor out $\frac{1}{n}$:

$$\begin{aligned} S_n &= \frac{1}{n}(1+3+5+7+...) \\ &= \frac{1}{n}n^2 \\ \therefore S_n &= n \end{aligned}$$

(d) Factor out $\frac{1}{2020}$:

$$\begin{aligned} S_{2020} &= \frac{1}{2020}(1+3+5+7+...) \\ &= \frac{1}{2020} \times 2020^2 \\ \therefore S_{2020} &= 2020 \end{aligned}$$

Question 32:

2020 Final (5)

Sum of the first 4 terms of an arithmetic sequence is 72. Sum of the first 9 terms is also 72.

(a) What is the 5^{th} term of the sequence?

(b) Find the sum if the first five terms.

(c) Write the sequence.

Answer:

(a) If n is an odd number, then:

$$S_n = n \times (middle\ term)$$

Given:

$$\begin{aligned} S_9 &= 72 \\ \therefore x_5 &= \frac{S_9}{9} = \frac{72}{9} = 8 \end{aligned}$$

(b) Sum of the first 5 terms:

$$S_5 = S_4 + x_5 = 72 + 8 = 80$$

(c) Since:

$$\begin{aligned} S_5 &= 80 \\ x_3 &= \frac{S_5}{5} = \frac{80}{5} = 16 \\ d &= \frac{x_5 - x_3}{5-3} = \frac{8-16}{2} = -4 \\ x_2 &= x_3 - d = 16 - (-4) = 16 + 4 = 20 \\ x_1 &= x_2 - d = 20 - (-4) = 20 + 4 = 24 \\ &\therefore 24, 20, 16, ... \end{aligned}$$

Question 33:

2020 Final (6)

Read the following Passage. Understand the Mathematical concept in it and answer the questions that follow. Each question carries 1 score.

The common difference of the arithmetic sequence $15, 14, 13, 12, ...$ is $14 - 15 = -1$. First term of the sequence is 15 and the 15^{th} term is $15 + 14 \times -1 = 15 - 14 = 1$.

Similarly the 4^{th} term is 12 and the 12^{th} term is 4.

Its 16^{th} term is, $x_{16} = 15 + 15 \times -1 = 15 - 15 = 0$. So the sum of the first 31 terms is also zero. That is if the n^{th} term of an arithmetic sequence with common difference -1 is m, then the m^{th} term is n and the $(m+n)^{th}$ term is zero.

(a) Seventh term of an arithmetic sequence is 10 and the 10^{th} term is 7. What is the common difference?

(b) What is the 21^{st} term of the arithmetic sequence $21, 20, 19, ...$?

(c) 5^{th} term of an arithmetic sequence is 17 and the $17^t h$ term is 5. Which term of the sequence is zero?

(d) 5^{th} term of an arithmetic sequence is 17 and the $17th$ term is 5. What is the 44^{th} term?

(e) First term of an arithmetic sequence is n and the n^{th} term is 1. What is the $(n+1)^{th}$ term?

(f) The first term of an arithmetic sequence is n and the n^{th} term is 1. Sum of how many terms, starting from the first term, of this sequence is zero?

Answer:

(a) If we have two terms of m^{th} and n^{th} positions, then Common difference:

$$d = \frac{x_m - x_n}{m - n}$$

$$= \frac{5 - 17}{17 - 5} = -1$$

(b) To calculate 21^{st} term, using the formula:

$$x_n = x_1 + (n-1)d$$

$$x_{21} = 21 + (21 - 1) \times -1 = 21 + 20 \times -1 = 21 - 20 = 1$$

(c) We have:

$$x_5 = 17$$

$$x_{17} = 5$$

$$d = \frac{5-17}{17-5} = \frac{-7}{7} = -1$$

To calculate n^{th} term, using the formula:

$$x_n = x_1 + (n-1)d$$

$$x_5 = x_1 + (5-1) \times -1 = 17$$

$$x_1 + 4 \times -1 = 17$$

$$x_1 - 4 = 17$$

$$x_1 = 17 + 4 = 21$$

$$x_n = x_1 + (n-1)d$$

$$= 21 + (n-1) \times -1 = 0$$

$$= 21 - n + 1 = 0$$

$$\therefore n = 22$$

(d) To calculate 44^{th} term, using the formula:

$$x_n = x_1 + (n-1)d$$

$$x_{44} = 21 + (44-1) \times -1$$

$$= 21 - 43 = -22$$

(e) To calculate $(n+1)^{th}$ term, using the formula:

$$x_n = x_1 + (n-1)d$$

$$x_{n+1} = n + ((n+1)-1) \times -1$$

$$= n + (n+1-1) \times -1$$

$$= n + n \times -1 = n - n = 0$$

$$\therefore x_{n+1} = 0$$

(f) $2n+1$

Question 34:

2020 Model (2)

The second term of an arithmetic sequence is 8 and common difference is 3.

(a) Write the sequence.

(b) What is the 12^{th} term of the sequence?

Question 35:

2020 Model (3)

The 5^{th} term of an arithmetic sequence is 38 and 8^{th} term in 62.

(a) What is the common difference of the sequence?

(b) Will the difference between any two terms of the sequence be 100? Why?

Answer:

(a) $d = \frac{62-38}{8-5} = \frac{24}{3} = 8$

(b) $\frac{100}{8} = 12.5$
Since 8 is not a multiple of 100, 100 cannot become a difference.

Question 36:

2020 Model (4)

The sum of first n terms of an arithmetic sequence is $n^2 + 2n$.

(a) What is the first term?

(b) Find the common difference.

(c) What is the sum of first 20 terms of this sequence?

(d) Prove that the sum of continuous terms starting from the first of the sequence $3, 5, 7, ...$ added to 1 gives a perfect square.

Answer:

(a) (*Please refer previous answers*)

(b) (*Please refer previous answers*)

(c) (*Please refer previous answers*)

(d) $S_n + 1 = n^2 + 2n + 1 = (n+1)^2$ is a perfect square.

Question 37:

2020 Model (5)

The sum of first and 21^{st} terms of an arithmetic sequence is 140.

(a) What is the 11^{th} term?

(b) Write the sequence, if the first term is 10.

(c) Calculate the sum of first 11 terms of this sequence.

(d) Find the sum of first 11 terms of the sequence $20, 25, 30, ...$

Answer:

(a) We have:

$$x_1 + x_{21} = x_{22} = 140$$
$$\therefore x_{11} + x_{11} = 140$$
$$= 2x_{11} = 140$$
$$x_{11} = 70$$

(b) (*Please refer previous answers*)

(c) (*Please refer previous answers*)

(d) (*Please refer previous answers*)

Question 38:

2020 Model (6)

Read the following, understand the mathematical idea in it and answer the questions that follow. Each question carries 1 score.

We know that a sequence got by starting with any number and adding a fixed number repeatedly is called an arithmetic sequence. Example: $1, 3, 5, 7, ...$ Like this we can form sequences by starting with any number and multiplying by a fixed non-zero number repeatedly. For example $1, 2, 4, 8, ...$ In this sequence, one number multiplied by 2 gives the next number. Such sequences are called geometric sequences. The common number used for repeated multiplication is called common ratio.

(a) What is the 5^{th} term of the geometric sequence $1, 2, 4, 8, ...$?

(b) Write the geometric sequence with first term 2 and common ratio 3.

(c) What is the common ratio of the geometric sequence $3, 12, 48, ...$?

(d) Write the 10th term of the geometric sequence $1, -1, 1, ...$

(e) What is the sum of 10 consecutive terms of the geometric sequence $1, -1, 1, ...$?

(f) Which of the following numbers will not be a term of any geometric sequence? $(\pi, 0, \sqrt{2}, \frac{1}{\pi})$

Answer:

(a) 16

(b) $2, 6, 18, ...$

(c) 4

(d) -1

(e) 0

(f) 0

Question 39:

2019 Final (2)

(a) Write the first integer term of the arithmetic sequence $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, ...$

(b) What is the sum of the first 7 terms of this sequence?

Answer:

(a) Given:

$$x_1 = \frac{1}{7}$$

$$x_2 = \frac{2}{7}$$

$$\therefore d = \frac{2}{7} - \frac{1}{7} = \frac{2-1}{7} = \frac{1}{7}$$

7^{th} term:

$$x_7 = \frac{1}{7} + 6 \times \frac{1}{7} = \frac{1}{7} + \frac{6}{7}$$

$$= \frac{6+1}{7} = \frac{7}{7} = 1$$

(b)

$$S_7 = \frac{7}{2}(\frac{1}{7} + 1) = \frac{7}{2}(\frac{1+7}{7})$$

$$= \frac{7}{2}(\frac{8}{7}) = \frac{8}{2} = 4$$

Question 40:

2019 Final (3)

(a) What is the remainder on dividing the terms of the arithmetic sequence $100, 107, 114, ...$ by 7?

(b) Write the sequence of all three digit numbers. Which leaves remainder 3 on division by 7? Which is the last term of this sequence?

Answer:

(a)

$$\begin{array}{r} 14 \\ 7\,\big)\overline{\,100} \\ \underline{7} \\ 30 \\ \underline{28} \\ 2 \end{array}$$

$\therefore$ the remainder is 2

(b) 100 has a remainder of 2, so the first number is:

$$x_1 = 100 - 2 + 3 = 101$$

$$x_2 = 101 + 7 = 108$$

$$x_3 = 108 + 7 = 115$$

$\therefore$ the sequence is:

$$101, 108, 115, ...$$

Suppose 999 is a term, then:

$$\begin{array}{r} 142 \\ 7\,\big)\overline{\,999} \\ \underline{7} \\ 29 \\ \underline{28} \\ 19 \\ \underline{14} \\ 5 \end{array}$$

999 has a remainder of 5, so the last term is:

$$999 - 5 + 3 = 997$$

Question 41:

2019 Final (4)

Find the following sums:

(a) $1+2+3+......+100$

(b) $1+3+5+......+99$

(c) $2+4+6+......+100$

(d) $3+7+11+......+199$

Answer Keys:

(a) 5050

(b) 2500

(c) 2550

(d) 5050

Question 42:

2019 Final (5)

If the terms of the arithmetic sequence $\frac{2}{9},\frac{3}{9},\frac{4}{9},\frac{5}{9},...$ are represented as $x_1, x_2, x_3, ...$ then

(a) $x_1+x_2+x_3=......$

(b) $x_4+x_5+x_6=......$

(c) Find the sum of first 9 terms.

(d) What is the sum of first 300 terms?

Answer:

(a)

$$\begin{aligned}\frac{2}{9}+\frac{3}{9}+\frac{4}{9}&=\frac{2+3+4}{9}\\&=\frac{9}{9}=1\end{aligned}$$

(b)

$$\begin{aligned}\frac{5}{9}+\frac{6}{9}+\frac{7}{9}&=\frac{5+6+7}{9}\\&=\frac{18}{9}=2\end{aligned}$$

(c) Common difference:

$$d = x_1 - x_2 = \frac{3}{9} - \frac{2}{9} = \frac{3-2}{9}$$
$$= \frac{1}{9}$$

To calculate sum of first 9 terms, using the formula:

$$S_n = \frac{n}{2}(2x_1 + (n-1)d)$$

$$S_9 = \frac{9}{2}(2 \times \frac{2}{9} + (9-1) \times \frac{1}{9})$$
$$= \frac{9}{2}(\frac{4}{9} + \frac{8}{9}) = \frac{9}{2} \times \frac{12}{9}$$
$$= \frac{12}{2}$$
$$\therefore S_9 = 6$$

(d) To calculate sum of first 300 terms, using the formula:

$$S_n = \frac{n}{2}(2x_1 + (n-1)d)$$

$$S_{300} = \frac{300}{2}(2 \times \frac{2}{9} + (300-1) \times \frac{1}{9})$$
$$= \frac{300}{2}(\frac{4}{9} + \frac{299}{9}) = 150 \times \frac{303}{9}$$
$$= \frac{45450}{9} = 5050$$
$$\therefore S_{300} = 5050$$

Question 43:

2019 Final (6)

Read the following. Understand the Mathematical concepts in it and answer the questions that follow.

The remainders obtained on dividing the powers of two by 7 have an interesting property. We can understand it from the table given below.

Number	2^1	2^2	2^3	2^4	2^5	2^6	2^7	......
Remainder	2	4	1	2	4	1	2	......

If the powers are $1, 4, 7, ...$ the remainder is 2
If the powers are $3, 6, 9, ...$ the remainder is 1

(a) What is the remainder on dividing 2^8 by 7?

(b) Write the sequence of powers of 2 leaving remainder 1 on division by 7.

(c) Check whether 2019 is a term of the arithmetic sequence $3, 6, 9, ...$

(d) What is the remainder on dividing 2^{2019} by 7?

(e) Write the algebraic form of the arithmetic sequence $1, 4, 7, ...$

(f) Write the algebraic form of the sequence $2^1, 2^4, 2^7, ...$ (powers of two leaving remainder 2 on division by 7).

Answer:

(a) 4

(b) Given:

$$x_1 = 3$$

$$x_2 = 6$$

$$d = x_2 - x_1 = 6 - 3 = 3$$

$$x_3 = 6 + 3 = 9$$

$$\therefore 3, 6, 9, ...$$

(c) Using the formula to calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$

Suppose 2019 is term, then:

$$3 + (n-1) \times 3 = 2019$$

$$3 + 3n - 3 = 2019$$

$$3n = 2019$$

$$n = \frac{2019}{3}$$

$$
\begin{array}{r}
673 \\
3\,\overline{)\,2019} \\
\underline{18} \\
21 \\
\underline{21} \\
09 \\
\underline{9} \\
0
\end{array}
$$

Since the remainder is 0, 2019 is a term.

(d) 2019 is part of the sequence which makes the remainder 1.
$\therefore$ the remainder is 1.

(e) Given:

$$x_1 = 1$$

$$x_2 = 4$$

$$d = x_2 - x_1 = 4 - 1 = 3$$

To calculate n^{th} term, using the formula:

$$x_n = x_1 + (n-1)d$$

$$= 1 + (n-1) \times 3$$

$$= 1 + 3n - 3$$

$$= 3n - 2$$

(f)

$$2^{3n-2}$$

Question 44:

2019 Model (2)

Consider the arithmetic sequence $13, 23, 33, ...$

(a) What is its common difference?

(b) What is the first three digit term of this sequence?

Answer:

(a) Given:

$$x_1 = 13$$

$$x_2 = 23$$

$$d = x_2 - x_1 = 23 - 13 = 10$$

(b) Suppose 100 is the term, then:

$$x_n = x_1 + (n-1)d = 100$$

$$= 13 + (n-1) \times 10 = 13 + 10n - 10 = 100$$

$$= 10n + 3 = 100$$

$$\therefore 10n = 100 - 3 = 97$$

$$\therefore n = \frac{97}{10} = 9.7 \approx 10$$

The first three digit term:

$$x_{10} = 13 + 9 \times 10 = 103$$

Question 45:

2019 Model (3)

The algebraic form of an arithmetic sequence is $5n + 4$.

(a) What is its first term?

(b) What is the difference of its 10^{th} and 20^{th} terms?

(c) Can the difference of any two terms of this sequence be 368? Justify.

Answer:

(a)

$$x_1 = 5 \times 1 + 4 = 5 + 4 = 9$$

(b)

$$x_{10} = 5 \times 10 + 4 = 50 + 4 = 54$$

$$x_{20} = 5 \times 20 + 4 = 100 + 4 = 104$$

$$x_{20} - x_{10} = 104 - 54 = 50$$

(c) Common difference:

$$d = \frac{x_{20} - x_{10}}{20 - 10} = \frac{104 - 54}{10}$$

$$= \frac{50}{10} = 5$$

$$\frac{368}{5} = 73.6$$

368 is not a multiple of 5
$\therefore 368$ cannot be a difference.

Question 46:

2019 Model (4)

(a) Write the sequence of odd numbers greater than 1.

(b) What is the algebraic form of this sequence?

(c) What is the algebraic form of the arithmetic sequence $\frac{3}{6}, \frac{5}{6}, \frac{7}{6}, ...$

(d) Prove that this sequence does not contain any natural number.

Answer:

(a) $3, 5, 7, ...$

(b) Given:

$$x_1 = 3$$
$$x_2 = 5$$
$$d = x_2 - x_1 = 5 - 3 = 2$$

Using the formula to calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$
$$x_n = 3 + (n-1) \times 2 = 3 + 2n - 2$$
$$\therefore x_n = 2n + 1$$

(c) Given:

$$x_1 = \frac{3}{6}$$
$$x_2 = \frac{5}{6}$$
$$d = x_2 - x_1 = \frac{5}{6} - \frac{3}{6}$$
$$= \frac{5-3}{6} = \frac{2}{6}$$

Using the formula to calculate n^{th} term:

$$x_n = x_1 + (n-1)d$$
$$x_n = \frac{3}{6} + (n-1) \times \frac{2}{6}$$
$$= \frac{3}{6} + \frac{2}{6}n - \frac{2}{6} = \frac{3-2}{6} + \frac{2}{6}n$$
$$= \frac{2}{6}n + \frac{1}{6}$$
$$\therefore x_n = \frac{2n+1}{6}$$

(d) For a given natural number n, the expression $2n + 1$ always produce an odd number. And an odd number cannot be a multiple of 6.
$\therefore$ the sequence with algebraic form $\frac{2n+1}{6}$ cannot contain any natural number.

Question 47:

2019 Model (5)

The sum of first 9 terms of an arithmetic sequence is 45 and the sum of first 18 terms is 171.

(a) What is the sum of its 10^{th} to 18^{th} terms?

(b) What is its 5^{th} term?

(c) Find its 14^{th} term.

(d) Find the sum of 5^{th} to 14^{th} terms.

Answer:

(a) Given:

$$S_9 = 45$$

$$S_{18} = 171$$

$$S_{18} = \underbrace{x_1 + x_2 + x_3 + ... + x_9}_{S_9=45} + \overbrace{x_{10} + ... + x_{18}}^{s_{18}-s_9=?} = 171$$

$$S_{18} - S_9 = 171 - 45 = 126$$

(b) If n is an odd number, then:

$$S_n = n \times (middle\ term)$$

$$S_9 = 9 \times (x_5) = 45$$

$$\therefore x_5 = \frac{45}{9} = 5$$

(c) x_{14} is the middle term between x_{10} and x_{18}

$$\therefore x_{14} = \frac{126}{9} = 14$$

(d) For the sequence from x_5 to x_{14}:

$$x_1 = 5$$

$$x_{14} = 14$$

$$n = 10$$

$$S_{10} = \frac{10}{2}(5 + 14) = 95$$

Question 48:

2018 Final (2)

The algebraic form of an arithmetic sequence is $5n + 3$.

(a) What is the first term of the sequence?

(b) What will be the remainder if the terms of the sequence are divided by 5?

Answer Keys:

(a) 8

(b) 3

Question 49:

2018 Final (3)

The algebraic form for the sum of first n terms of an arithmetic sequence is $2n^2+8n$. How many consecutive terms of the sequence, starting from the first, are to be added to get 330?

Answer:

Given:

$$S_n = 2n^2 + 8n = 330$$

Divide both sides with 2:

$$n^2 + 4n = 165$$

Add 4 to both sides:

$$n^2 + 4n + 4 = 165 + 4$$

$$(n+2)^2 = 169$$

$$n + 2 = \sqrt{169} = 13$$

$$\therefore n = 13 - 2 = 11$$

Note: This chapter requires the knowledge from the chapter 4 (Second Degree Equations) of Mathematics textbook.

Question 50:

2018 Final (4)

There are 20 terms in an arithmetic sequence. Sum of the first and last terms is 88.

(a) What is the sum of the 2^{nd} and 19^{th} terms?

(b) If the 10^{th} term is 42, what is the 11^{th} term?

(c) What is the common difference of the sequence?

(d) What is the first term?

Answer:

(a) Sum of term positions:

$$2 + 19 = 1 + 20 = 21$$

$$\therefore x_2 + x_{19} = x_1 + x_{20} = 88$$

(b) Given:

$$x_{10} = 42$$

Sum of term positions:

$$10 + 11 = 1 + 20 = 21$$

$$\therefore x_{10} + x_{11} = x_1 + x_{20} = 88$$

$$\therefore x_{11} = 88 - 42 = 46$$

(c) Common difference:

$$x_{11} - x_{10} = 46 - 42 = 6$$

(d)

$$x_{10} = x_1 + (10 - 1) \times 6 = 42$$

$$x_1 = 42 - 36 = 6$$

Question 51:

2018 Final (5)

Consider the numbers between 100 and 300 which leave remainder 2 on division by 3.

(a) Which is the first number in this sequence?

(b) Which is the last number in this sequence?

(c) How many such numbers are there in this sequence?

(d) Find the sum of all numbers in the sequence.

Answer:

(a) Suppose 101 is a term, then:

$$\begin{array}{r} 33 \\ 3\,\overline{)\,101} \\ \underline{9} \\ 11 \\ \underline{9} \\ 2 \end{array}$$

101 has a remainder of 2, so the first term:

$$x_1 = 101$$

(b) Suppose 299 is a term, then:

$$\begin{array}{r} 99 \\ 3\,\overline{)\,299} \\ \underline{27} \\ 29 \\ \underline{27} \\ 2 \end{array}$$

299 has a remainder of 2, so the last term:

$$x_n = 299$$

(c)

$$x_2 = 101 + 3 = 104$$

$$d = x_2 - x_1 = 104 - 101 = 3$$

$$x_n = x_1 + (n-1)d = 299$$

$$101 + 3n - 3 = 299$$

$$3n = 299 - 98 = 201$$

$$n = \frac{201}{3} = 67$$

$$S_n = \frac{n}{2}(x_1 + x_n)$$

$$= \frac{67}{2}(101 + 299)$$

$$\therefore S_{67} = 13400$$

Question 52:

2018 Final (6)

Read the following, understand the mathematical idea expressed in it and answer the questions that follow:
$1, 4, 9, 16, ...$ are the squares of the counting numbers. The remainders got by dividing the square numbers with natural numbers have a cyclic property. For example, the remainders on dividing these numbers by 4 are tabulated here.

Number	1	4	9	16	25	–	–	–
Remainder	1	0	1	0	1	–	–	–

On dividing by 4 perfect squares leave only 0 and 1 as remainders. From this, we can conclude that an arithmetic sequence whose terms leaves remainder 2 on dividing by 4 do not have a perfect square.

(a) Which are the possible remainders on dividing any number with 4?

(b) Which are the numbers we would not get on dividing a perfect square by 4?

(c) What is the remainder that leaves on dividing the terms of the arithmetic sequence $2, 5, 8, 11, ...$ by 4?

(d) Does the arithmetic sequence $3, 7, 11, ...$ contain perfect squares?

(e) Write a sequence with common difference 4 which contains many perfect squares.

Question 53:

2018 Model (2)

The 25^{th} term of an arithmetic sequence is 140 and the 27^{th} term is 166. What is its common difference ? What is its 35^{th} term?

Question 54:

2018 Model (4)

Sum of the first five terms of an arithemetic sequence is 45. What is the third term ? The common difference of the sequence is 4. Write the first two terms. Write another arithemetic sequence having the sum of the first five term 45.

Question 55:

2018 Model (5)

(a) Find the least and heighest three digit number which leave a remainder 1 on division by 9.

(b) How many three digit numbers are there, which leave a remainder one on division by 9?

(c) Find the sum of all such numbers.

Question 56:

2017 Model (2)

Consider the arithmetic sequence $12, 23, 34, ...$

(a) What is the 10^{th} term of this sequence?

(b) Is 1111 a term of this sequence? Why?

Question 57:

2017 Model (5)

The first term of an arithmetic sequence is 6 and the sum of the first 6 terms is 66.

(a) What is its 6^{th} term?

(b) What is the common difference of the sequence?

(c) What are the first 6 terms of this sequence?

Question 58:

2017 Model (4)

The first term of an arithmetic sequence is 6 and the common difference is 4.

(a) What is the algebraic form of this sequence?

(b) What is the algebraic expression to find the sum of the first n terms of this sequence?

(c) How many terms of this sequence, starting from the first, are to be added to get 510?

Question 59:

2015 Final (2)

First term of an arithmetic sequence is 10 and its common difference 3. Write the first three terms of the sequence. Verify whether 100 is a term of this sequence.

Question 60:

2015 Final (3)

Sum of first n terms of an arithmetic sequence is $3n^2 + n$. Find the first term and common difference of this sequence.

Question 61:

2015 Final (4)

The terms of an arithmetic sequence with common difference 4 are natural numbers.

(a) If x is a term in this sequence, what is the next term?

(b) If the sum of reciprocals of two consecutive terms of this $\frac{4}{15}$, sequence is find those terms.

Question 62:

2015 Final (5)

Consider the arithmetic sequence $9, 15, 21, ...$

(a) Write the algebraic form of this sequence

(b) Find the twenty fifth term of this sequence.

(c) Find the sum of terms from twenty fifth to fiftieth of this sequence.

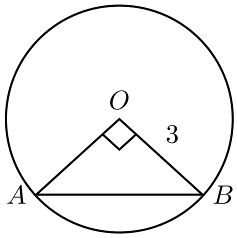
(d) Can the sum of some terms of this sequence be 2015? Why?

Chapter 2

Circles

Question 1:

2023 Final (2)



AB is a chord of a circle of radius 3 centimetres. Chord AB makes a right angle at the centre. What is the length of AB?

Answer:

Given:

$$\angle AOB = 90^\circ$$

$$OB = 3cm$$

$$AB = ?$$

Since radius is 3:

$$OA = 3cm$$

Since two sides are equal and $\angle AOB$ is right, this is an isosceles right triangle.

$$\therefore \angle OAB = 45^\circ$$

$$\angle OBA = 45^\circ$$

Using Pythagorean theorem:

$$c = \sqrt{a^2 + b^2}$$

$$= \sqrt{3^2 + 3^2}$$

$$= \sqrt{3^2 \times 1^2 + 3^2 \times 1^2}$$

$$= \sqrt{3^2(1^2 + 1^2)} = \sqrt{3^2 \times 2}$$

$$= 3\sqrt{2}$$

Question 2:

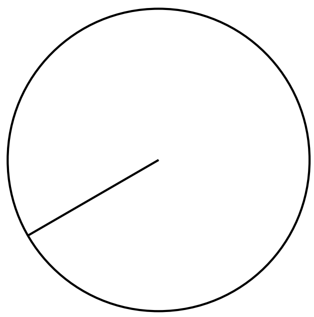
2023 Final (3)

Draw a circle of radius 4 centimetres.
Draw a triangle whose vertices are on this circle and two of the angles $40°$ and $60°$.

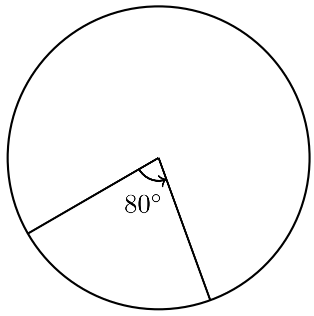
Answer:

To create a triangle with the given conditions of the angles $40°$ and $60°$, chords can be created with two times that of the required angles. To create a $40°$ angle, an angle of $80°$ can be created at the center. Similarly an adjacent angle of $120°$ can be created to form a $60°$ angle.

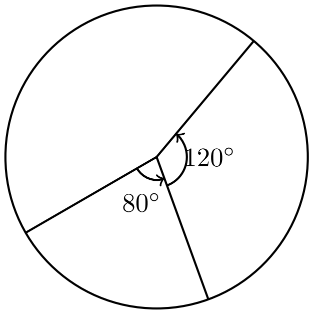
Draw a circle with 4 centimetres radius and a line from center to the circle.



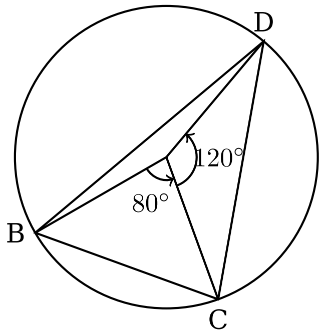
Draw another line from center to the circle at $80°$ angle.



Draw yet another line from center to the circle at $120°$ angle from the previous line.



Finally you can join the three points on the circle:



Here:

$$\angle BDC = 40^\circ$$
$$\angle DBC = 60^\circ$$