TRICHY

COMMON FIRST MID-TERM TEST - 2019

TC

Standard XII

Reg.No. | | | | | |

Time: 1.30 hours.

MATHEMATICS

Marks: 50

Part - I

I. Choose the correct answer: $10 \times 1 = 10$

1. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A, then $\frac{|\text{adj } B|}{|C|} =$

   a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) $\frac{1}{4}$ d) 1

2. If $A^T A^{-1}$ is symmetric, then $A^2 =$

   a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

   a) 17 b) 14 c) 19 d) 21

4. If a matrix contains at-least one non-zero element, then

   a) $\rho(A) \le 1$ a) $\rho(A) = 0$ a) $\rho(A) \ge 1$ a) $\rho(A) \le 0$

5. $i^n + i^{n+1} + i^{n+2} + i^{n+2}$ is

   a) 0 b) 1 c) −1 d) i

6. If $|Z_1| = 1$, $|Z_2| = 2$, $|Z_3| = 3$ and $|9Z_1Z_2 + 4Z_1Z_3 + Z_2Z_3| = 12$, then the value of $|Z_1 + Z_2 + Z_3|$ is

   a) 1 b) 2 c) 3 d) 4

7. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

   a) −2 b) −1 c) 1 d) 2

8. If $Z = \bar{Z}$, then Z is a

   a) imaginery number b) complex number c) real number d) none of the above

9. A zero of x^3+64 is

   a) 0 b) 4 c) 4i d) −4

10. If f and g are polynomials of degrees m and n respectively, and if h(x) = (fog) (x), then the degree of h is

    a) mn b) m + n c) m^n d) n^m

Part - II

II. Answer any 5 questions: (Ques.No.17 is compulsory) $5 \times 2 = 10$

11. If $\text{adj } A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.
12. Solve the system of linear equations by matrix inversion method: $2x-y=8$, $3x+2y=-2$
13. Solve the system of linear equations by Cramer's rule: $5x-2y+16=0$, $x+3y-7=0$
14. Simplify : $i^{-1924} + i^{2018}$.
15. Find the square root of $-6+8i$
16. Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x.
17. Write in polar form of the complex number $3 - i\sqrt{3}$.

Part - III

III. Answer any 5 questions: (Ques.No.24 is compulsory) $5 \times 3 = 15$

18. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A + 7I_2 = O_2$. Hence find A^{-1}.

19. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$

20. State and prove triangle inequality.

21. Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$

22. Show that the points 1, $1, -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

23. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

24. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

Part - IV

IV. Answer all the questions: $3 \times 5 = 15$

25. a) A boy is walking along the path $y = ax^2 + bx + c$ through the points (-6,8), (-2,-12) and (3,8). He wants to meet his friend at P(7,60). Will he meet his friend? (Use Gaussian Elimination method)

(or)

b) Find the value of k for which the equations $kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have

i) no solution ii) unique solution iii) infinitely many solution

26. a) Investigate for what values of λ and μ the system of linear equations. $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has

i) no solution ii) a unique solution iii) an infinite number of solutions

(or)

b) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

i) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ ii) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha - n\beta)$.

27. a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$ *(or)*

b) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39 + 135$, if it is known that 1+2i and $\sqrt{3}$ are two of its zeros.

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