Mathematics X

Answers of First Terminal Examination 2023

2 score

Answer any three

1) This is an open question.Students can write their own sequence and its 10 th term . The first term should be 3.
An example is given below
   a) $3, 5, 7 \cdots$
   b) $x_n = 2n + 1 \rightarrow x_{10} = 2 \times 10 + 1 = 21$

   (or)

   $x_{10} = f + 9d = 3 + 9 \times 2 = 21$

2) Side of the enlarged square is $= \sqrt{81} = 9$
   Side of the first square $9 - 2 = 7$m

   (or)

   If side of the first square x then $(x + 2)^2 = 81$
   $x + 2 = \pm\sqrt{81}, x = 9, -9$
   $x + 2 = 9 \rightarrow x = 7$.Side of the first square is 7m

3) a) 9
   b) $\frac{3}{9}$

4) a) 60°
   b) $\frac{1}{2} \times 60 = 30^\circ$

3 score

Answer any four

5) a) 19times common difference should be added.
   b) $103 - 4 = 99$. 1
   99 is a multiple of common difference . 103 is a term of the sequence .

   (or)

   Common difference is 3.

   $4 \div 3 \rightarrow$remainder 1 ,
   $103 \div 3 \rightarrow$remainder 1
   103 is a term of the sequence.

   (or)

   Algebraic form of the sequence is $3n + 1$
   $103 = 34 \times 3 + 1$.
   103 is the 34 th term of the sequence .

6) a) Consecutive multiples of 8 are x and $x + 8$
   $x(x + 8) = 384$
   $x^2 + 8x + 16 = 384 + 16, (x + 4)^2 = 400, x + 4 = \pm\sqrt{400}$.
   $x + 4 = 20 \rightarrow x = 16$ (Multiples are positive integers)
   Multiples of 8 are $16, 24$

   (or)

   Two consecutive multiples are $x - 4$ and $x + 4$
   $(x - 4)(x + 4) = 384, x^2 - 4^2 = 384$
   $x^2 - 16 = 384, x^2 = 400, x = 20$

7) a) $\frac{4}{10}$
   b) $\frac{3}{10}$
   c) $\frac{6}{11}$

8) Steps of construction.Students are expected to follow these steps for the constucting the triangle.
   - ⋆ Draw a circle of radius 4cm.
   - ⋆ Divide the angle around the centre as $2 \times 60 = 120^\circ, 2 \times 80 = 160^\circ$ by drawing radii
   - ⋆ Join the ends of the radii. It makes the triangle.

9) a) Sum of first 30 terms $= \frac{30(30+1)}{2} = 15 \times 31 = 465$

b) Sum $= 2(1+2+3\cdots+30) = 2\times 465 = 930$

c) $5+7+9+\cdots+63 = (2+3)+(4+3)+(6+3)+\cdots+(60+3)$
$= (2+4+6+\cdots+60)+30\times 3 = 930+90 = 1020$

(or)

There are 30 terms in the sequence $5,7,9,\cdots 63$ as each term is 3 more than the terms of $2,4,6\cdots 60$

Terms can make into 15 pairs taking terms equidistant from both ends . Pair sum is $5+63 = 68$

Sum of terms $15\times 68 = 1029$

(or)

Sum $= (x_1+x_n)\times\frac{n}{2}$

Sum$= (5+63)\times\frac{30}{2} = 68\times 15 = 1020$

10) a) $PB = 16-10 = 6$

b) $PC\times PD = PA\times PB = 16\times 6 = 96$

c) $PC\times PD = 96\to PC\times 8 = 96, PC = \frac{96}{8} = 12$

4 score

Answer any eight

11) a) $1-\frac{1}{3} = \frac{2}{3}$

b) Number of green beads is $\frac{1}{3}\times 27 = 9$

c) When 5 blue beads are added, number of beads becomes 32.

Probability of getting green beads $\frac{9}{32}$

12) Consider first 13 terms. Sum of the terms equidistant from both ends are equal. Middle term is half of the pair sum .

a) $x_7 = \frac{x_5+x_9}{2} = \frac{58}{2} = 29$

b) $x_1+x_{13} = x_5+x_9 = 58$

c) Sum of first 13 terms $= x_7\times 13 = 29\times 13 = 377$

13) a) $\angle CBO = 40°$.

b) $\angle BOC = 180-80 = 100°$

c) $\angle BAC = \frac{1}{2}\times 100 = 50°$

d) $\angle BDC = 180-50 = 130$

14) a) 12cm

b) $12-x$

c) $x(12-x) = 35\to x^2-12x = -35$

) $x^2-12x+36 = -35+36\to (x-6)^2 = 1, x-6 = 1, x = 7$

Sides are 7cm and 5cm

15) a) $PA\times PB = PC^2\to 5\times 3 = PC^2$
$PC = \sqrt{15}$

b) Steps of onstruction

Draw a line of length 8 cm , say $AB = 8$ cm

Draw a semicircle with diameter AB

Mark a point on AB such that $AP = 5, BP = 3$.Draw perpendicu lar to AB at P

The perpendicular line cut the semicircle at C. Draw the equilateral triangle with side PC which is $\sqrt{15}$cm

16) $f = 5\times 1-3 = 2, d = 5$

a) $2,7,12\cdots$

b) $5n-3 = 122\to 5n = 125, n = \frac{125}{5} = 25$

c) 13 th term will be the middle term. $x_{13} = 5\times 13-3 = 62$

Sum of first 25 terms $= x_{13}\times 25 = 62\times 25 = 1550$

(or)

$x_{25} = 5\times 25-3 = 122$

Sum $= (x_1+x_{25})\times\frac{25}{2} = (2+122)\times\frac{25}{2} = 124\times\frac{25}{2} = 62\times 25 = 1550$

(or)

Consider first 25 terms. By pairing terms equidistant from both ends pair sum will be 124. Middle term will be 62 without pair .

Sum $= 124\times 12+62 = 1550$

17) a) $PC = 15-3 = 12$

b) $PA = 13-x$

c) $PA\times PB = PC\times PD$

$(13-x)\times x=12\times 3, 13x-x^2=36$

$-x^2+13x=36, x^2-13x=-36$

$x^2-13x+(\frac{13}{2})^2=-36+\frac{169}{4}$

$(x-\frac{13}{2})=\pm\sqrt{\frac{25}{4}}=\frac{-5}{2},\frac{5}{2}$

$x-\frac{13}{2}=\frac{5}{2}\rightarrow x=\frac{18}{2}=9$

$x-\frac{13}{2}=\frac{-5}{2}\rightarrow x=\frac{8}{2}=4$

Since PB is shorter length in the figure $PB=4$cm

(or)

The equation $x^2-13x+36=0$ can be solved using formula.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\rightarrow x=4,9$

Since PB is the shorter length in the figure , $PB=4$cm

18) Let x be the side of the square

$x^2+4x=221, x^2+4x+4=221+4\rightarrow (x+2)^2=225$

$x+2=\pm\sqrt{225}\rightarrow x=15,-15$

$x=15-2=13$. Side of the square is 13

Area of the square is $13^2=169$ itemize

19) Number of chairs in the semicircles are in arithmetic sequence.

a) $100,110,120\cdots$ is the sequence . $f=100, d=10$

$x_{15}=f+14d=100+14\times 10=240$

b) Total number of chairs $=(x_1+x_{15})\times\frac{15}{2}=(100+240)\times\frac{15}{2}=340\times\frac{15}{2}=170\times 15=2250$

(or)

In this sequence $x_n=10n+90$

Middle term is $x_8=10\times 8+90=170$

Sum of terms(total number of chairs)$x_8\times 15=170\times 15=2250$ itemize

20) a) $\angle ABC=180-75=105^\circ$

b) $\angle BCD=180-70=110^\circ$

c) $\angle ADC=180-105=75^\circ$

d) $\angle ADQ=180-75=105^\circ$

21) a) $x_{15}=4+14d=4+14\times 5=74$

Sum $=(x_1+x_{15})\times 15=(4+74)\times\frac{15}{2}=39\times 15=585$

(or)

$x_8=4+7d=4+7\times 5=39$

Sum $=x_8\times 15=39\times 15=585$

b) The terms of the arithmetic sequence $7,12,17\cdots$ are 3 more than the terms in the same position of the sequence $4,9,14\cdots$.

The difference is $15\times 3=45$

5 score

Answer any six

22) a) $60\times 50=3000$

b) Class A: Boys 30 , Girls 30

Class B : Boys 30, Girls 20

Probability of selecting one boy and one girl $=\frac{30\times 20+30\times 30}{3000}=\frac{1500}{3000}=\frac{1}{2}$

c) Probability of selecting both boys $=\frac{30\times 30}{3000}=\frac{900}{3000}=\frac{3}{10}$

d) Probability of selecting atleast one girl$=1-\frac{3}{10}=\frac{7}{10}$

(or)

Probability of selecting atleast one girls $=\frac{30\times 20+30\times 20+30\times 30}{3000}=\frac{2100}{3000}=\frac{7}{10}$

23) Steps of construction

Draw the rectangle $ABCD$ in which $AB=6$cm, $BC=4$cm

Produce AB to E such that $BC=BE$.

Draw a semicircle with AE as the diameter.Extent BC to the semicircle at F.

$BF^2=BA\times BE\rightarrow BF^2=BA\times BC$ Draw a square with BF as the side.

Area of rectangle is same as the area of the square.

24) a) $x_n = dn + (f - d) = 4n + (7 - 4) = 4n + 3$

b) $7 \div 4 \rightarrow$ gives the remainder 3. When the terms are divided by 4 the remainder will be 3

c) Proof

$x_n = 4n + 3$. Square of n th term is $(4n + 3)^2 = 16n^2 + 24n + 9$

$16n^2 + 24n + 9 = 16n^2 + 24n + 8 + 1 \rightarrow 4(4n^2 + 6n + 2) + 1$. This is not in the form of $4n + 3$.

Moreover ,square of n th term gives the remainder 1 on dividing by 4.Because it is 1 more than a multiple of 4 for any natural number n

Since the remainder is not 3 we can say square of a term is not in the sequence.

There is a mistake in english medium paper. For c part another sequence is given. It is $4, 7, 10 \cdots$. Its n th term is $3n + 1$. Square of n th term is $9n^2 + 6n + 1 = 3(3n^2 + 2n) + 1$. This is also in the form of its n th term. So we can say square of its n th term is also a term of the sequence.

25) a) In right triangle QPC, $PC^2 = 13^2 - 5^2 = 144, PC = 12$cm

b) In right triangle CPB, $PB^2 = 15^2 - 12^2 = 81, PB = 9$cm

c) $PA \times PB = PC^2 = 144$

d) $PA \times 9 = 144, PA = 16$cm

$AB = PA + PB = 25$cm

26) a) $f = 8$

b) $x_n = 248$

Sequence is $8, 12, 16 \cdots 248$

$4n + 4 = 248 \rightarrow 4n = 244, n = 61$

d) Since there are 61 terms 31 st term will be the middle term. $x_{31} = 4 \times 31 + 4 = 124 + 4 = 128$

Sum of 61 terms $= x_{31} \times 61 = 128 \times 61 = 7808$

(or)

$f = 8, x_n = 248, n = 61$

Sum of first n terms $= (x_1 + x_n) \times \frac{n}{2} = 256 \times \frac{61}{2} = 128 \times 61 = 7808$

27) a) $\angle PRQ = 50^\circ$

b) $\angle PRS = 65^\circ$

c) $\angle RQS = 35^\circ$

d) $\angle QPR = 180 - (50 + 35 + 65) = 30^\circ$

e) $180 - (65 + 35) = 180 - 100 = 80^\circ$

28) a) $f = 2 \times 1^2 + 3 \times 1 = 5$

b) $d = 2 \times 2 = 4$

c) $x_n = dn + (f - d) = 4n + (5 - 4) = 4n + 1$

d) 13 th term is the middle term. $x_{13} = 4 \times 13 + 1 = 53$

Sum of first 25 terms $= x_{13} \times 25 = 53 \times 25 = 1325$

(or)

$x_{25} = 4 \times 25 + 1 = 101$

Sum of first 25 terms $= (x_1 + x_{25}) \times \frac{25}{2} = (5 + 101) \times \frac{25}{2} = 1325$

29)

a) $2^0 + 2^1 + 2^2 + 2^3 + 2^4 = 1 + 2 + 4 + 8 + 16 = 31 = 2^5 - 1$

b) $2^0 + 2^1 + 2^2 + \cdots + 2^{10} = 2^{11} - 1$

c) $2^0 + 2^1 + 2^2 + \cdots + \boxed{2^{20}} = 2^{21} - 1$

d) $2^0 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$

e) $63 = 64 - 1 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 2^6 - 1$

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