X'MAS EXAMINATION
+1 MATHEMATICS ANSWER KEY

AMC

1, (a) $A = \{0, 1, 2, 3\}$

(b) $A \cup B = \{0, 1, 2, 3, 4, 5\}$, $A \cap B = \{2, 3\}$

2 (a) $x + 1 = 3$ $\quad$ $y - 2 = 1$

$x = 2$ $\quad$ $y = 3$

(b) $A \times B = \{(1,5), (1,6), (1,7), (2,5), (2,6), (2,7)\}$

$B \times A = \{(5,1), (5,2), (6,1), (6,2), (7,1), (7,2)\}$

3, (a) 2^8

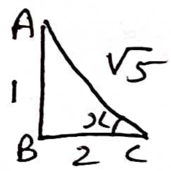
(b) $[f+g](x) = f(x) + g(x) = x^2 + x + 2$

$[fg](x) = f(x) \cdot g(x) = x^2(x+2) = x^3 + 2x^2$

4, (a) $\sin 420^\circ = \sin(360 + 60) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

(b) $AC = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\sin x = -\frac{1}{\sqrt{5}}$, $\cos x = \frac{-2}{\sqrt{5}}$



5, (a) $i^9 = i^8 \times i = i$, $i^{19} = i^{16} \times i^3 = -i$

$i^9 + i^{19} = i - i = 0$

(b) $z_1 + z_2 = 3 + 4i + 7 + 6i = 10 + 10i$

6, Let x be the marks in the third examination

$\frac{62 + 48 + x}{3} \geq 60$

$110 + x \geq 180$

$x \geq 70$

7, (a) $n = 5$, (b) $21_{C_2} = 210$

8, $(2x+3)^5 = 5_{C_0}(2x)^5 + 5_{C_1}(2x)^4 \times 3 + 5_{C_2}(2x)^3 \times 3^2 + 5_{C_3}(2x)^2 \times 3^3$

$+ 5_{C_4}(2x) \times 3^4 + 5_{C_5}(3)^5$

$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$

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9, (a) $A' = \{1,3,5,8\}$, $B' = \{1,2,7,8\}$

(b) $A \cup B = \{2,3,4,5,6,7\}$

$LHS = (A \cup B)' = \{1,8\}$

$RHS = A' \cap B' = \{1,8\}$

10, (a) $A \cap B = \phi$

(b) $A - B = \{1,2\}$, $B - A = \{7,8\}$

(ii) $(A-B) \cup (B-A) = \{1,2,7,8\}$

11, (a) $R = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

(b) $R = \{(x,y) : y = x+1, x \in A, y \in A\}$

(c) domain $= \{1,2,3,4,5\}$

Range $= \{2,3,4,5,6\}$

12, (a) $\frac{2\pi}{3}$ radian $= \frac{2 \times 180}{3} = 120^\circ$

(b) $LHS = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \frac{2\sin 4x \cdot \cos x}{2\cos 4x \cdot \cos x}$

$= \tan 4x$

13, (a) $z = 2+2i+i-1 = 1+3i$

(b) $\bar{z} = 1-3i$, $|z| = \sqrt{1+9} = \sqrt{10}$

$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{1-3i}{10} = \frac{1}{10} - \frac{3}{10}i$

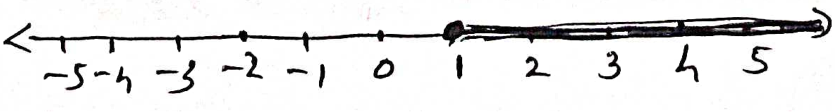
14, (a) $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

xing by 8 , $4(3x-4) \geq 2(x+1) - 8$

$12x - 16 \geq 2x + 2 - 8$

$10x \geq 10$

$x \geq 1$



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15, (a) $\frac{1}{8!} + \frac{1}{9!} = \frac{x}{10!}$

$$\frac{10!}{8!} + \frac{10!}{9!} = \frac{10!\,x}{10!}$$

$9 \times 10 + 10 = x \implies x = 100$

(b) Number of ways $= {}^{5}C_{3} = 10$

16, (a) $10+1 = 11$

(b) $(a+b)^4 = {}^4C_0\, a^4 + {}^4C_1\, a^3 b + {}^4C_2\, a^2 b^2 + {}^4C_3\, a b^3 + {}^4C_4\, b^4$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

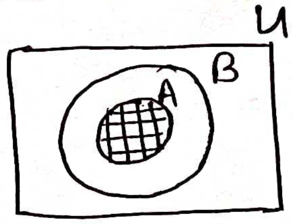
$(a+b)^4 - (a-b)^4 = 8a^3b + 8ab^3$

$= 8ab(a^2+b^2)$

$(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4 = 8\sqrt{2}\sqrt{3}\,(3+2) = 40\sqrt{6}$

17, (a) $(1, 3]$ (b) (i) $A \cap B = A$

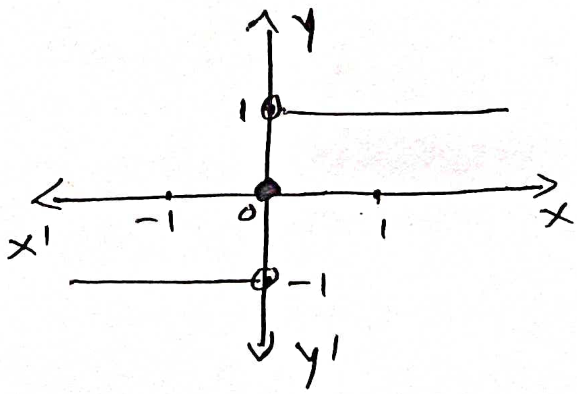
(ii)



(c) $\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

18, (a) domain $= \mathbb{R}$, Range $= \{-1, 0, 1\}$

(b)



(c) $f(2) = 1,\ f(-2) = -1 \implies f(2) + f(-2) = 0$

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19, (a) $\sin 75^\circ = \sin(45+30)$

$= \sin 45 \cos 30 + \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(b) $LHS = \cos(\pi/4 + x) + \cos(\pi/4 - x)$

$= \cos\pi/4 \cos x - \sin\pi/4 \sin x + \cos\pi/4 \cos x + \sin\pi/4 \sin x$

$= 2\cos\pi/4 \cos x = 2 \cdot \frac{1}{\sqrt{2}} \cdot \cos x = \sqrt{2}\cos x = RHS$

(c) Let $\tan 3x = \tan(2x + x)$

$\tan 3x = \dfrac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x}$

$\tan 3x(1 - \tan 2x \cdot \tan x) = \tan 2x + \tan x$

$\tan 3x - \tan 3x \cdot \tan 2x \cdot \tan x = \tan 2x + \tan x$

$\tan 3x - \tan 2x - \tan x = \tan 3x \cdot \tan 2x \cdot \tan x$

20, (a) Number of 3 digits numbers $= {}^5P_3$

$= \frac{5!}{2!} = 5 \times 4 \times 3 = 60$

(b) $n = 11$, $P_1 = 2$, $P_2 = 2$, $P_3 = 2$

Total number of arrangements $= \dfrac{n!}{P_1!\,P_2!\,P_3!}$

$= \dfrac{11!}{2!\,2!\,2!} = \dfrac{11!}{8} = 4989600$

(c) Number of arrangements in which vowels occur together $= \dfrac{8!}{2!\,2!} \times \dfrac{4!}{2!}$

$= 120960$