V10M Virudhunagar District Common Examinations
Common Half Yearly Examination - December 2022

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Standard 10

Time: 3.00 Hrs. **MATHEMATICS** Marks: 100

PART - I

Note: i) Answer ALL the questions. **14×1=14**
ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1) If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is
a) 1 b) 2 c) 3 d) 6

2) If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to set B, then the number of elements in B is
a) 3 b) 2 c) 4 d) 8

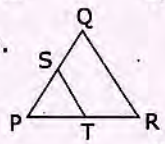
3) Using Euclid's division lemma, if the cube of any positive integer is divided by 9, then the possible remainders are
a) 0, 1, 8 b) 1, 4, 8 c) 0, 1, 3 d) 1, 3, 5

4) If $A = 2^{65}$ and $B = 2^{64}+2^{63}+2^{62}+.....+2^0$, which of the following is true?
a) B is 2^{64} more than A b) A and B are equal
c) B is larger than A by 1 d) A is larger than B by 1

5) $x^2-2x-24$ and x^2-Kx-6 has the GCD $(x-6)$ then the value of K is
a) 3 b) 5 c) 6 d) 8

6) Graph of a linear equation is a
a) Straight line b) Circle c) Parabola d) Hyperbola

7) In a given figure ST || QR, PS = 2 cm and SQ = 3 cm. Then the ratio of the area of ΔPQR to the area of ΔPST is



a) 25:4 b) 25:7 c) 25:11 d) 25:13

8) The area of triangle formed by the points (−5, 0), (0, −5) and (5, 0) is
a) 0 sq.units b) 25 sq.units c) 5 sq.units d) None of these

9) The slope of the line joining (12, 3), (4, a) is 1/8. The values of 'a' is
a) 1 b) −5 c) 4 d) 2

10) The ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, then the angle of elevation of the sun has measure.
a) 30° b) 45° c) 60° d) 90°

11) The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to
a) $\tan^2\theta$ b) 1 c) $\cot^2\theta$ d) 0

12) The total surface area of a cylinder whose radius is 1/3 of its height is
a) $\frac{9\pi h^2}{8}$ sq.units b) $24\pi h^2$ sq.units c) $\frac{8\pi h^2}{9}$ sq.units d) $\frac{56\pi h^2}{9}$ sq.units

13) If the standard deviation of x, y, z is p then the standard deviation of 3x+5, 3y+5, 3z+5 is
a) 3p+5 b) 3p c) p+5 d) 9p+5

14) The probability of getting a job for a person is x/3. If the probability of not getting the job is 2/3 then the value of x is
a) 2 b) 1 c) 3 d) 1.5

PART - II

Note: i) Answer any TEN questions. ii) Question No. 28 is compulsory. 10×2=20

15) Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$ which of the following are relations from A to B? (i) $R_1 = \{(2, 1), (7, 1)\}$ (ii) $R_2 = \{(2, -1), (7, 7), (1, 3)\}$

16) If $f(x) = 2x-1$, $g(x) = \frac{x+1}{2}$, show that fog = gof = x.

17) If $13824 = 2^a \times 3^b$ then find a and b.

18) If 3+K, 18−K, 5K+1 are in A.P then find K.

19) Find the excluded values of the expression $\frac{x^2+6x+8}{x^2+x-2}$.

20) Find the values of x, y and z from the following: $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

21) In ΔABC, AD is the bisector of $\angle A$. If BD = 4 cm, DC = 3 cm and AB = 6 cm, find AC.

22) If the radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.

23) Show that the given points are collinear (−3, −4), (7, 2) and (12, 5).

24) Find the intercepts made by the line $4x+3y+12 = 0$ on the Co-ordinate axes.

25) Prove that $\frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A} = 2 \operatorname{cosec} A$.

26) A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

27) Find the standard deviation of first 21 Natural numbers.

28) If the total surface area of a cone of radius 7 cm is 704 cm^2, then find its height.

PART - III

Note: i) Answer any TEN questions. ii) Question No. 42 is compulsory. 10×5=50

29) Let A = {6, 9, 15, 18, 21}: B = {1, 2, 3, 4, 5, 6} and f : A→B be defined by $f(x) = \frac{x-3}{3}$. Represent f by (i) an arrow diagram (ii) a set of ordered pairs (iii) a table (iv) a graph.

30) If ℓ^{th}, m^{th} and n^{th} terms of an A.P are x, y, z respectively then show that (i) $x(m-n)+y(n-\ell)+z(\ell-m) = 0$ (ii) $(x-y)n+(y-z)\ell+(z-x)m = 0$.

31) In a G.P the product of three consecutive terms is 27 and the sum of the product of two terms taken at a time is $\frac{57}{2}$. Find the three terms.

32) Find the values of m and n if $36x^4-60x^3+61x^2-mx+n$ is a perfect square.

33) If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ show that $A^2-5A+7I_2 = 0$.

34) State and prove Thales Theorem.

35) The hypotenuse of a right triangle is 6m more than twice of the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

36) Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (−5, 12) and (−4, 3).

37) Find the equation of the median of ΔABC through A where the vertices are A(6, 2), B(−5, −1) and C(1, 9).

38) From the top of tower 50m high, the angles of depression of the top and bottom of a tree are observed to be 30° and 45° respectively. Find the height of the tree. $\left(\sqrt{3} = 1.732\right)$

39) An industrial metallic bucket is in the shape of a frustm of a right circular cone whose top and bottom diameters are 10m and 4m and whose height is 4m. Find the curved and total surface area of the bucket.

40) The rainfall recorded in various places of five districts in a week are given below:

Rainfall in mm	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Find its standard deviation.

41) Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

42) Find x if g f f(x) = f g g(x). Given f(x) = 3x+1 and g(x) = x+3.

PART - IV

Note: Answer the following questions. 2×8=16

43) Construct a ΔPQR in which PQ = 8 cm, $\angle R = 60°$ and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ. **(OR)**
Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also measure the lengths of the tangents.

44) The following table shows the data about the number of pipes and the time taken to fill the same tank.

Number of pipes (x)	2	3	6	9
Time taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence
i) Find the time taken to fill the tank when five pipes are used.
ii) Find the number of pipes when the time is 9 minutes. **(OR)**
Draw the graph of $y = x^2+x-2$ and hence solve $x^2+x-2 = 0$.

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Standard 10 MATHS

PART - I

① c) 3
② b) 2
③ a) 0, 1, 8
④ d) A is larger than B by 1
⑤ b) 5
⑥ a) straight line.
⑦ a) 25 : 4
⑧ b) 25 Sq. units
⑨ d) 2
⑩ c) 60°
⑪ b) 1
⑫ c) $\frac{8\pi h^2}{9}$ Sq.u
⑬ b) 3P
⑭ b) 1.

PART - II

⑮ (2,1); (7,1) ∈ A×B ∴ R_1 is not relation — ①
$R_2 \subseteq A \times B$ ∴ R_2 is relation from A to B — ①

⑯ $fog = f\left(\frac{x+1}{2}\right)$
$= 2\left(\frac{x+1}{2}\right) - 1$
$= x$ — ①

$gof = g(2x-1)$
$= \frac{2x-1+1}{2} = x$
$= \frac{2x}{2} = x$ — ①

⑰ $13824 = 2^9 \times 3^3 = 2^a \times 3^b$ ∴ a = 9 ; b = 3 — ① ①

⑱ $t_2 - t_1 = t_3 - t_2$
$18 - k - (3+k) = 5k + 1 - (18 - k)$
$15 - 2k = 6k - 17$ — ①

$8k = 32$
$k = 4$ — ①

⑲ $\frac{x^2+6x+8}{x^2+x-2} = \frac{(x+2)(x+4)}{(x+2)(x-1)} = \frac{x+4}{x-1}$ — ①

$(x-1) = 0$; $x = 1$ ∴ The excluded value = 1. — ①

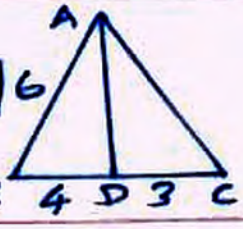
(20) $x+y+2=9$; $x+2=5$; $y+2=7$

$y+5=9$ ① $x+3=5$ $4+2=7$

$\boxed{y=4}$ $\boxed{x=2}$ $\boxed{z=3}$ —①

(21) $\frac{AB}{AC}=\frac{BD}{DC}$ ① $4AC=18$

$\frac{6}{AC}=\frac{4}{3}$ $\boxed{AC=4.5cm}$ ①



(22) $OA=5cm$; $OC=4cm$ $AC^2=25-16$

$OA^2=OC^2+AC^2$ $AC^2=9$; $AC=3cm$

$5^2=4^2+AC^2$ —① $\therefore \boxed{AB=3+3=6cm}$ —①

(23) $A(-3,-4)$; $B(7,2)$; $C(12,5)$.

Slope of $AB=\frac{2+4}{7+3}=\frac{6}{10}=\frac{3}{5}$ — ① —①

Slope of $BC=\frac{5-2}{12-7}=\frac{3}{5}$ — ② —①

$\therefore$ ① = ②

(24) $4x+3y=12$ $\frac{4x}{-12}+\frac{3y}{-12}=\frac{-12}{-12}$ ①

$\frac{x}{-3}+\frac{y}{-4}=1$ $\therefore \boxed{a=-3}$; $\boxed{b=-4}$ —①

(25) $L.H.S=\frac{\sin A(1-\cos A)+\sin A(1+\cos A)}{1-\cos^2 A}$ —①

$=\frac{2\sin A}{\sin^2 A}=\frac{2}{\sin A}=2\operatorname{cosec} A=R.H.S.$ —①

(26) $n(S)=12$; $n(A)=3$; $P(A)=\frac{3}{12}=\frac{1}{4}$ —①

(27) $S.D\ \sigma=\sqrt{\frac{n^2-1}{12}}=\sqrt{\frac{441-1}{12}}=\sqrt{36.6}\simeq 6.05$ —①

(28) TSA of a cone $= 704$

$\pi r(l+r) = 704$

$\frac{22}{7} \times 7(l+7) = 704$ — (1)

$l+7 = 32$

$l = 25$ cm

$l^2 = h^2 + r^2$

$625 = h^2 + 49$

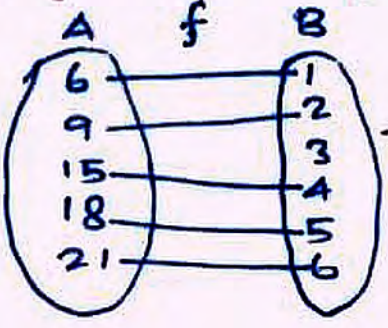
$h = 24$ cm — (1)

PART-III

(29) $A = \{6, 9, 15, 18, 21\}$; $B = \{1, 2, 3, 4, 5, 6\}$

$f(x) = \frac{x-3}{3}$;

i) an arrow diagram



— (1)

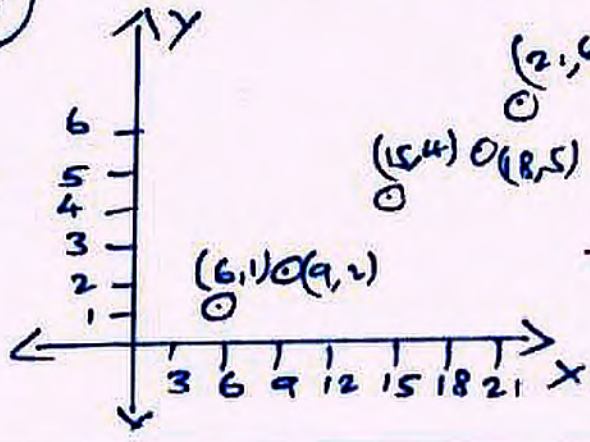
ii) $f = \{(6,1); (9,2); (15,4); (18,5); (21,6)\}$. — (2)

iii) a table

x	6	9	15	18	21
$f(x)$	1	2	4	5	6

— (1)

iv)



— (1)

(30) $t_l = x$; $t_m = y$, $t_n = z$ Example: 2.28

$a + (l-1)d = x$

$a + (m-1)d = y$ — (1)

$a + (n-1)d = z$

$x - y = (l-m)d$ — (1)

$y - z = (m-n)d$ $z - x = (n-l)d$

i) $x(m-n) + y(n-l) + z(l-m) = 0$ — (2)

ii) $(x-y)n + (y-z)l + (z-x)m = 0$ — (1)

(31) Let the three terms are $\frac{a}{r}, a, ar$

$\frac{a}{r} \times a \times ar = 27$ $\quad (ar \times \frac{a}{r}) + (a \times ar) + (ar \times \frac{a}{r}) = \frac{57}{2}$

$a^3 = 27$

$\boxed{a = 3}$ —(1) $\quad \frac{1 + r^2 + r}{r} = \frac{19}{6}$

$(6r - a)(6r - 4) = 0$ $\quad \boxed{6r^2 - 13r + 6 = 0}$ —(1)

$\boxed{r = \frac{3}{2} \; ; \; r = \frac{2}{3}}$ —(1)

$\therefore$ three terms are $a = 3$ & $r = \frac{3}{2} = 2, 3, \frac{9}{2}$ —(1)

" $a = 3$ & $r = \frac{2}{3} = \frac{9}{2}, 3, 2$ —(1)

(32)

$$\begin{array}{r|l} & 6x^2 - 5x + 3 \\ \hline 6x^2 & 36x^4 - 60x^3 + 61x^2 - mx + n \\ & 36x^4 \\ \hline & -60x^3 + 61x^2 \\ & -60x^3 + 25x^2 \\ \hline & 36x^2 - mx + n \\ & 36x^2 - 30x + 9 \\ \hline & 0 \end{array}$$

$m = 30$; —(1)

$n = 9$ —(1)

—(3)

(33) $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ —(1) $\quad -5A = \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix}$ —(1)

$A^2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$ $\quad 7I_2 = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$

$\therefore A^2 - 5A + 7I_2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} + \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$ —(2)

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$= 0$. —(1)

(34) State and prove Thales Theorem (1)

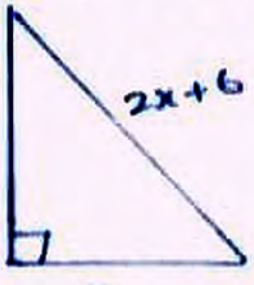
In $\triangle ABC$, $\angle A = 90^\circ$ Proof :- (3)

Proof : $AB^2 + AC^2 = BC^2$, Draw AD ⊥ BC (4)

(35) The Shorter side $= x$

hypotenuse $= 2x+6$ } (1)

The third side $= 2x+4$



By Pythagoras theorem,

$x^2 + (2x+4)^2 = (2x+6)^2$ — (2)

$x^2 - 8x - 20 = 0$ $\therefore x = 10, -2$

$\therefore$ The side of the triangle $x = 10$ cm :- (1)

$2x+4 = 24$ cm ; — (1)

$2x+6 = 26$ cm — (1)

(36) Let A (8, 6) ; B (5, 11) ; C (−5, 12) D (−4, 3)

Area of the quadrilateral ABCD

$= \frac{1}{2} \left| \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix} \right|$ Sq. units (1)

$= \frac{1}{2} \left| \begin{matrix} 8 & 5 & -5 & -4 & 8 \\ 6 & 1 & 12 & 3 & 6 \end{matrix} \right|$ — (1)

$= \frac{1}{2} [(88 + 60 - 15 - 24) - (30 - 55 - 48 + 24)]$ (1)

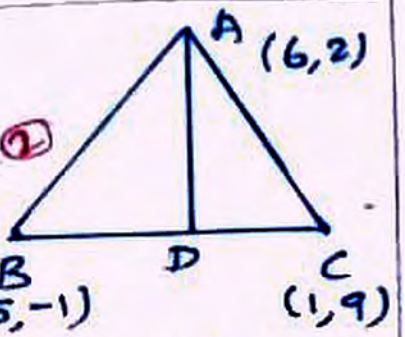
$= \frac{1}{2} \{109 + 49\}$

$= \frac{1}{2} (158)$ — (1)

$= 79$ Sq. units. — (1)

Area of the quadrilateral ABCD $= 79$ Sq. units

(37) AD is median.



midpoint of BC

$D = \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right) = (-2,4)$ — (2)

Equation of median

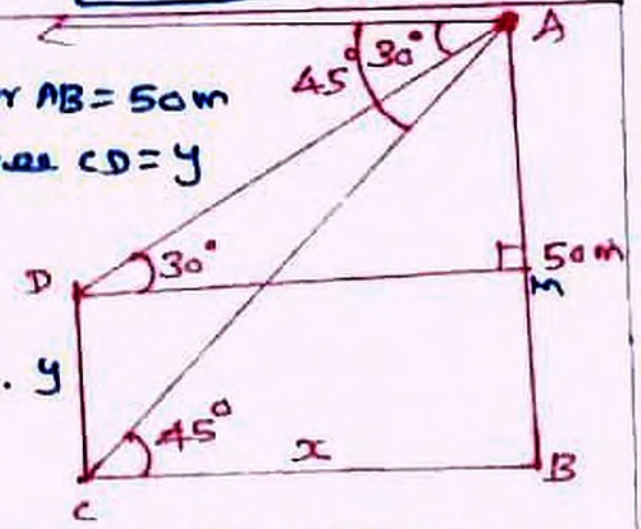
AD is $\frac{y-2}{4-2} = \frac{x-6}{-2-6}$; $\frac{y-2}{2} = \frac{x-6}{-8}$

$-8(y-2) = 2(x-6)$; $\boxed{2x+4y-14=0}$ — (3)

(38) height of the tower AB = 50m

Let the height of the trees CD = y

BD = x



$\tan 45^\circ = \frac{AB}{BD}$

$\therefore x = 50m$ — (1)

$\tan 30^\circ = \frac{AM}{DM}$

$AM = \frac{50\sqrt{3}}{3} = 28.87$ m ; — (2)

$\therefore CD = AB - AM$

$= 50 - 28.87$

$= \boxed{21.13\ m}$ — (2)

(39) diameter of the top = 10m ; $\therefore R = 5m$

$r = 2m$; $h = 4m$. $l = \sqrt{h^2+(R-r)^2}$ — (1)

$l = \sqrt{16+9} = 5m$

C.S.A $= \pi(R+r)l = \frac{22}{7}(5+2)5 = \boxed{110m^2}$ — (2)

T.S.A $= \pi(R+r)l + \pi R^2 + \pi r^2$ Sq. units.

$= \frac{22}{7}[(5+2)5 + 25 + 4] = \frac{1408}{7} = \boxed{201.14m^2}$ — (2)

(40) $A = 60$; $N = 40$; $\Sigma fd = -160$ $\Sigma fd^2 = 3050$ — (1)

S.D $\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \frac{\Sigma fd^2}{N}}$ — (1)

$= \sqrt{\frac{3050}{40} - \left(\frac{-160}{40}\right)^2}$ — (1) $= \sqrt{76.25 - 16}$ — (1)

$= \sqrt{60.25} = 7.76$

$\therefore$ S.D $\simeq 7.76$ — (1)

(41) $n(S) = 36$; $n(A) = 6$; $n(B) = 3$; — (1)

$n(A \cap B) = 1$.

$P(A) = \frac{6}{36}$; $P(B) = \frac{3}{36}$; $P(A \cap B) = \frac{1}{36}$ .. — (2)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{6+3-1}{36} = \frac{8}{36} = \frac{2}{9}$ — (2)

(42) $f(x) = 3x+1$; $g(x) = x+3$

$gff(x) = gof(3x+1) = g(9x+4) = 9x+7$ — (1)

$fgg(x) = fog(x+3) = f(x+6) = 3x+19$ — (1)

$gofof(x) = fogog(x)$

$9x+7 = 3x+19$ — (1)

$6x = 12$

$x = \frac{12}{6}$

$\boxed{x = 2}$ — (2)

PART - IV

(43) Given: PQ = 8cm
$\angle P = 60^\circ$
median RG = 5.8cm

Rough diagram — (1)
Fair diagram — (6)
The length of the altitude RM = 3.8cm — (1)

(OR)

Given: diameter = 6cm
r = 3cm
OP = 5cm

Rough diagram — (1)
Fair diagram — (6)

∴ The length of the tangents = 4cm — (1)

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(44) type of variation = Inverse variation — (1)

$xy = k \quad \therefore k = 2 \times 45 = 90$ — (1)

Graph — (4)

i) The time taken when five pipers are used = 18 mins. — (1)

ii) No of pipes used is 10 when the time is = 9 mins. — (1)

(OR)

$y = x^2 + x - 2$

x	-3	-2	-1	0	1	2
y	4	0	-2	-2	0	4

— (2)

Graph — (4)

The solution of the equation $x^2 + x - 2 = 0$ is $\{-2, 1\}$ — (2)

HEAD MASTER'S Sign