MARKING SCHEME
MATHEMATICS (BASIC)

SECTION A

1. The prime factorisation of natural number 288 is

(a) $2^4 \times 3^3$ (b) $2^4 \times 3^2$

(c) $2^5 \times 3^2$ (d) $2^5 \times 3^1$

Ans. (c) $2^5 \times 3^2$ 1

---

2. If $2 \cos\theta = 1$, then the value of θ is

(a) 45° (b) 60°

(c) 30° (d) 90°

Ans. (b) 60° 1

---

3. A card is drawn at random from a well-shuffled deck of 52 cards. The probability of getting a red card is :

(a) $\frac{1}{26}$ (b) $\frac{1}{13}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Ans. (d) $\frac{1}{2}$ 1

---

4. The discriminant of the quadratic equation $2x^2 - 5x - 3 = 0$ is

(a) 1 (b) 49

(c) 7 (d) 19

Ans. (b) 49 1

---

5. The distance between the points (3, 0) and (0, –3) is

(a) $2\sqrt{3}$ units (b) 6 units

(c) 3 units (d) $3\sqrt{2}$ units

Ans. (d) $3\sqrt{2}$ units 1

---

6. The seventh term of an A.P. whose first term is 28 and common difference -4, is

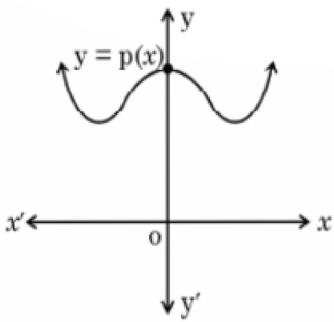
(a) 0 (b) 4

(c) 52 (d) 56

Ans. (b) 4 *1*

---

7. The graph of $y = p(x)$ is shown in the figure for some polynomial $p(x)$. The number of zeroes of $p(x)$ is/are :



(a) 0 (b) 1

(c) 2 (d) 3

Ans. (a) 0 *1*

---

8. The sides of two similar triangles are in the ratio 4 : 7. The ratio of their perimeters is

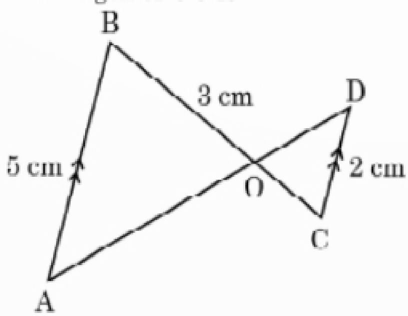
(a) 4 : 7 (b) 12 : 21

(c) 16 : 49 (d) 7 : 4

Ans. (a) 4 : 7 *1*

---

9. In the given figure, AB || CD. If AB = 5 cm, CD = 2 cm and OB = 3 cm then the length of OC is



(a) $\frac{15}{2}$ cm

(b) $\frac{10}{3}$ cm

(c) $\frac{6}{5}$ cm

(d) $\frac{3}{5}$ cm

Ans. (c) $\frac{6}{5}$ cm *1*

---

10. The sum and the product of zeroes of the polynomial $p(x) = x^2 + 5x + 6$ are respectively

(a) 5, – 6

(b) – 5, 6

(c) 2, 3

(d) –2, –3

Ans. (b) – 5, 6 *1*

---

11. A die is thrown once. Find the probability of getting a number less than 7.

(a) $\frac{5}{6}$

(b) 1

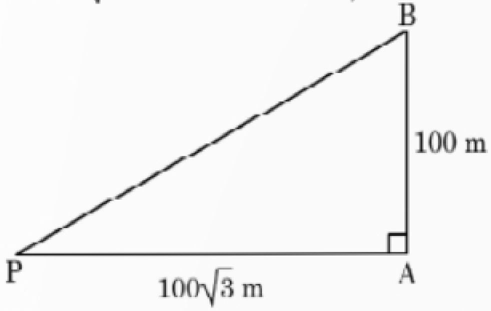
(c) $\frac{1}{6}$

(d) 0

Ans. (b) 1 *1*

---

12. The angle subtended by a vertical pole of height 100 m at a point on the ground $100\sqrt{3}$ m from the base is, has measure of



(a) 90°

(b) 60°

(c) 45°

(d) 30°

Ans. (d) 30° *1*

---

13. The volume of a cone of radius 'r' and height '3r' is :

(a) $\frac{1}{3}\pi r^3$

(b) $3\pi r^3$

(c) $9\pi r^3$

(d) πr^3

Ans. (d) πr^3 *1*

---

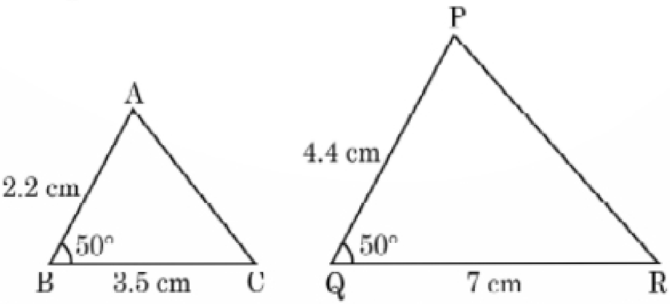
14. The distance between two parallel tangents of a circle of diameter 7 cm is :

(a) 7 cm (b) 14 cm

(c) $\frac{7}{2}$ cm (d) 28 cm

Ans. (a) 7 cm *1*

15.



In the above figure, the criterion of similarity by which $\Delta ABC \sim \Delta PQR$ is :

(a) SSA (Side – Side – Angle) Similarity

(b) ASA (Angle – Side – Angle) Similarity

(c) SAS (Side – Angle – Side) Similarity

(d) AA (Angle – Angle) Similarity

Ans. (c) SAS (Side – Angle – Side) Similarity *1*

---

16. The larger of two supplementary angles exceeds the smaller by 18 degrees. What is the measure of larger angle ?

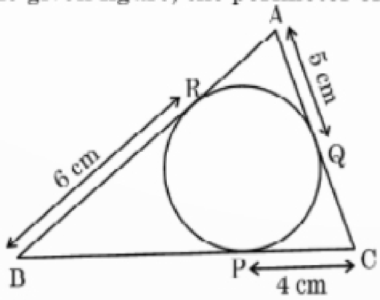
(a) 81° (b) 99°

(c) 36° (d) 54°

Ans. (b) 99° *1*

---

17. In the given figure, the perimeter of ΔABC is :



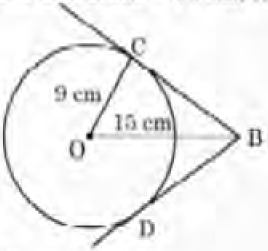
(a) 30 cm (b) 15 cm

(c) 45 cm (d) 60 cm

Ans. (a) 30 cm *1*

---

18. In the given figure, BC and BD are tangents to the circle with centre O and radius 9 cm. If OB = 15 cm, then the length (BC + BD) is :



(a) 18 cm (b) 12 cm

(c) 24 cm (d) 36 cm

Ans. (c) 24 cm *1*

(Assertion – Reason based questions)

Directions for Q.19 & Q.20 : In question numbers **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option :

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** A tangent to a circle is perpendicular to the radius through the point of contact.

Reason (R) : The lengths of tangents drawn from the external point to a circle are equal.

Ans. (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A) *1*

---

20. **Assertion (A) :** The system of linear equations $3x + 5y - 4 = 0$ and $15x + 25y - 25 = 0$ is inconsistent.

Reason (R) : The pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is inconsistent if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Ans. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A) *1*

SECTION B

21. (a) Find the coordinates of the point which divides the line segment joining the points (7, –1) and (–3, 4) internally in the ratio 2 : 3.

2 : 3

Solution: A **(7, -1)** P(x, y) B**(-3, 4)**

Let P(x, y) divide AB internally in the ratio 2 : 3

$x = \dfrac{2 \times -3 + 3 \times 7}{2+3} = \dfrac{15}{5} = 3$ *1*

$y = \dfrac{2 \times 4 + 3 \times -1}{2+3} = \dfrac{5}{5} = 1$ *1*

Coordinates of the required point P are (3, 1)

OR

(b) Find the value(s) of y for which the distance between the points A(3, –1) and B(11, y) is 10 units.

Solution: AB = 10 units $\Rightarrow AB^2 = 100$

$\Rightarrow (11-3)^2 + (y+1)^2 = 100$ *1*

$\Rightarrow y + 1 = \pm 6$

$\Rightarrow y = 5, -7$ $\frac{1}{2} + \frac{1}{2}$

---

22. Evaluate : $\tan^2 60° - 2 \operatorname{cosec}^2 30° - 2 \tan^2 30°$.

Solution: $\tan^2 60° - 2 \operatorname{cosec}^2 30° - 2 \tan^2 30°$

$= (\sqrt{3})^2 - 2(2)^2 - 2\left(\frac{1}{\sqrt{3}}\right)^2$ $1\frac{1}{2}$

$= -\frac{17}{3}$ $\frac{1}{2}$

---

23. Find the LCM and HCF of 92 and 510, using prime factorisation.

Solution: $92 = 2 \times 2 \times 23$ $\frac{1}{2}$

$510 = 2 \times 3 \times 5 \times 17$ $\frac{1}{2}$

HCF = 2 $\frac{1}{2}$

$LCM = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$ $\frac{1}{2}$

---

24. (a) Solve for x and y : $x + y = 6$, $2x - 3y = 4$.

Solution: On solving the given equations and getting

$x = \frac{22}{5}$ and $y = \frac{8}{5}$ *1+1*

OR

(b) Find out whether the following pair of linear equations are consistent or inconsistent :

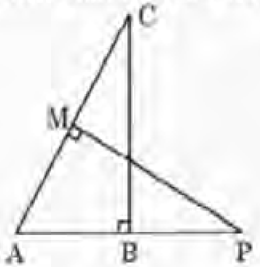
$5x - 3y = 11, \quad -10x + 6y = 22$

Solution: $-\frac{5}{10} = -\frac{3}{6} \neq \frac{11}{22}$ or $-\frac{1}{2} = -\frac{1}{2} \neq \frac{1}{2}$ $1\frac{1}{2}$

$\Rightarrow$ given pair of linear equations is inconsistent $\frac{1}{2}$

---

25. In the given figure, ABC and AMP are two right triangles, right angled at B and M, respectively. Prove that $\Delta ABC \sim \Delta AMP$.



Solution: In Δ ABC and Δ AMP,

$\angle ABC = \angle AMP$ (90° each) $\frac{1}{2}$

$\angle BAC = \angle MAP$ (common) *1*

By AA Similarity

$\Delta ABC \sim \Delta AMP$ $\frac{1}{2}$

SECTION C

26. (a) Prove that

$$\sec\theta\,(1 - \sin\theta)\,(\sec\theta + \tan\theta) = 1$$

Solution: LHS = $\sec\theta\,(1 - \sin\theta)\,(\sec\theta + \tan\theta)$

$$= \frac{1}{\cos\theta}\,(1 - \sin\theta)\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right) \qquad 1$$

$$= \frac{1}{\cos\theta}\,(1 - \sin\theta)\left(\frac{1 + \sin\theta}{\cos\theta}\right) \qquad 1$$

$$= \frac{1 - \sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1 = \text{RHS} \qquad \frac{1}{2} + \frac{1}{2}$$

OR

(b) Prove that

$$\frac{1 + \sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1 - \cos\theta}$$

Solution: LHS $= \dfrac{1 + \sec\theta}{\sec\theta} = \dfrac{1 + \dfrac{1}{\cos\theta}}{\dfrac{1}{\cos\theta}} = 1 + \cos\theta$ $\qquad 1$

$$= \frac{(1 - \cos\theta)(1 + \cos\theta)}{(1 - \cos\theta)} \qquad 1$$

$$= \frac{1 - \cos^2\theta}{1 - \cos\theta} = \frac{\sin^2\theta}{1 - \cos\theta} = \text{RHS} \qquad 1$$

---

27. Show that the points A(1, 7), B(4, 2) C(–1, –1) and D(– 4, 4) are vertices of the square ABCD.

Solution:

$$AB = \sqrt{(4-1)^2 + (2-7)^2} = \sqrt{34}$$
$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{34}$$
$$CD = \sqrt{(-4+1)^2 + (4+1)^2} = \sqrt{34}$$
$$DA = \sqrt{(-4-1)^2 + (4-7)^2} = \sqrt{34} \qquad 2$$

$\therefore$ AB = BC = CD = DA

$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{68}$$
$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{68} \qquad 1$$

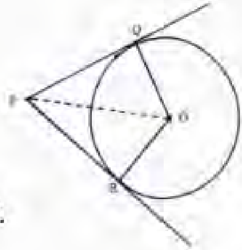
$\therefore$ AC = BD

Hence, □ABCD is a square.

---

28. Prove that the tangents drawn from an external point to a circle are equal in length.

Solution:



1 (For Fig.)

Given: A circle with centre O and PQ, PR are tangents to the circle from an external point P.
To Prove: PQ = PR

Construction: Join OP, OQ, OR $\frac{1}{2}$

Proof : In Δ OPQ and Δ OPR
OP = OP (common)
OQ = OR (radii of the same circle)
∠ OQP = ∠ ORP (each 90°)
⇒ Δ POQ ≅ Δ POR (RHS congruence) 1

∴ PQ = PR $\frac{1}{2}$

---

29. If α, β are zeroes of the quadratic polynomial $x^2 + 3x + 2$, find a quadratic polynomial whose zeroes are α + 1, β + 1.

Solution: $p(x) = x^2 + 3x + 2$

α, β are its zeroes

$\therefore \alpha + \beta = -3,\ \alpha\beta = 2$ $\frac{1}{2}+\frac{1}{2}$

Now,

$(\alpha + 1) + (\beta + 1) = \alpha + \beta + 2 = -3 + 2 = -1$ $\frac{1}{2}$

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1 = +2 - 3 + 1 = 0$ 1

∴Required Polynomial is $k(x^2 + x)$ or $x^2 + x$ $\frac{1}{2}$

---

30. Prove that $3 + 7\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number.

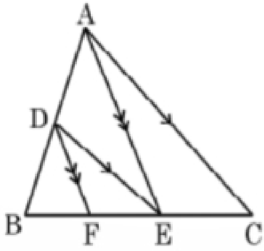
Solution: Let us assume that $3+7\sqrt{2}$ is a rational number.

$\Rightarrow 3 + 7\sqrt{2} = \frac{p}{q}$, p, q are integers and $q \neq 0$ 1

$\Rightarrow \sqrt{2} = \frac{p-3q}{7q}$ 1

RHS is rational but LHS is irrational
∴Our assumption is wrong
Hence, $3 + 7\sqrt{2}$ is an irrational number } 1

31. (a) In the given figure, DE ∥ AC and DF ∥ AE



Prove that

$\frac{BF}{FE} = \frac{BE}{EC}$

Solution: In Δ ABE, DF ∥ AE (given), hence by BPT

$\frac{BD}{DA} = \frac{BF}{FE}$ ________ (i) $1\frac{1}{2}$

In Δ ABC, DE ∥ AC (given), hence by BPT

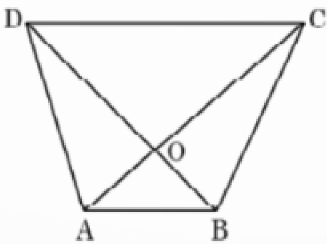
$\frac{BD}{DA} = \frac{BE}{EC}$ ________ (ii) 1

From (i) and (ii)

$\frac{BF}{FE} = \frac{BE}{EC}$ $\frac{1}{2}$

OR

(b) The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{OD}$



Show that quadrilateral ABCD is a trapezium.

Solution: In Δ AOB and Δ COD,

$\frac{AO}{BO} = \frac{CO}{OD} \Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

∠ AOB = ∠ COD (vertically opp. angles)

⇒ Δ AOB ~ Δ COD (SAS Similarity) 2

⇒ ∠ CAB= ∠ACD (or ∠ DBA= ∠ BDC)

But, these are alternate interior angles

∴ AB ∥ CD ⇒ □ABCD is a trapezium 1

SECTION D

32. (a) The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 80 m more than the shorter side, find the length of the sides of the field.

Note: There is an error in the question, so full marks to be awarded to the Candidate, who attempted.

OR

(b) The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present age.

Solution: Let age of father = x years

and age of son = (45 – x) years 1

Five years ago, age of father = (x – 5) years

Age of son = (40 – x) years 1

A. T. Q., (x – 5) (40 – x) = 124 1

$x^2 - 45x + 324 = 0$

$(x - 36)(x - 9) = 0$ 1

x = 36, x = 9 (rejected) $\frac{1}{2}$

⇒ Father's age = 36 years and son's age = 9 years $\frac{1}{2}$

---

33. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. Also, find the volume of the vessel.

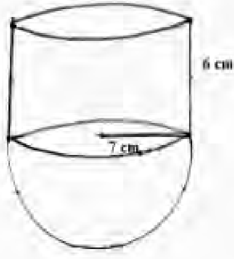
Solution: Radius of hemispherical bowl = radius of cylinder = 7 cm

Height of cylinder = 13 – 7 = 6 cm 1

Inner surface area of the vessel = $2\pi rh + 2\pi r^2$

$= 2\pi r(h + r) = 2 \times \frac{22}{7} \times 7(6 + 7)$

$= 44 \times 13 = 572 \text{ cm}^2$ 2



Volume of the vessel = $\pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2 (h + \frac{2}{3} r)$

$= \frac{22}{7} \times 7 \times 7 (6 + \frac{14}{3})$

$= \frac{4928}{3} \text{ cm}^3$ or 1642.67 cm^3 2

34. The table given below shows the daily expenditure on food of 25 households in a locality :

Daily expenditure (₹)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of household	4	5	12	2	2

Find the mean daily expenditure on food. Also, find the mode of the data.

Solution:

Daily Exp. (₹)	No. of household (f_i)	x_i	f_ix_i
100 – 150	4	125	500
150 – 200	5	175	875
200 – 250	12	225	2700
250 – 300	2	275	550
300 – 350	2	325	650
	25		5275

For Table: $1\frac{1}{2}$

Mean $x = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{5275}{25} = 211$ $1\frac{1}{2}$

Mode: Modal Class = 200 – 250 $\frac{1}{2}$

$l = 200,\ f_1 = 12,\ f_0 = 5,\ f_2 = 2,\ h = 50$

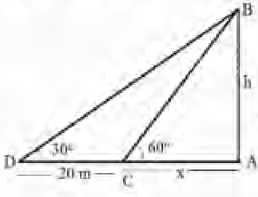
$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 200 + \left(\frac{12-5}{24-5-2}\right) \times 50$$ 1

$$= \frac{3750}{17} \text{ or } 220{\cdot}59$$ $\frac{1}{2}$

35. (a) A TV tower stands vertically on the bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from the point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

Solution:



For figure 1

In Δ ABC, $\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}\, x$ $1+\frac{1}{2}$

In Δ ABD, $\tan 30° = \frac{h}{20+x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$ $1+\frac{1}{2}$

$\sqrt{3}\, h = 20 + x$

$\sqrt{3}\left(\sqrt{3}x\right) = 20 + x$

$\Rightarrow x = 10$ $\frac{1}{2}$

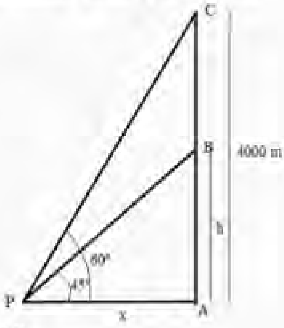
$\Rightarrow h = \sqrt{3}\, x = 10\sqrt{3}$

$\Rightarrow$ Height of tower $= 10\sqrt{3}$ m or 17·3 m $\frac{1}{2}$

OR

(b) An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$)

Solution:



For figure 1

In Δ APC, $\frac{h}{x} = \tan 45° \Rightarrow h = x$ $1+\frac{1}{2}$

In Δ APC, $\frac{4000}{x} = \tan 60° \Rightarrow x = \frac{4000}{\sqrt{3}}$ $1+\frac{1}{2}$

$\Rightarrow h = x = \frac{4000}{\sqrt{3}}$

Distance between the aeroplanes $= 4000 - \frac{4000}{\sqrt{3}}$ $\frac{1}{2}$

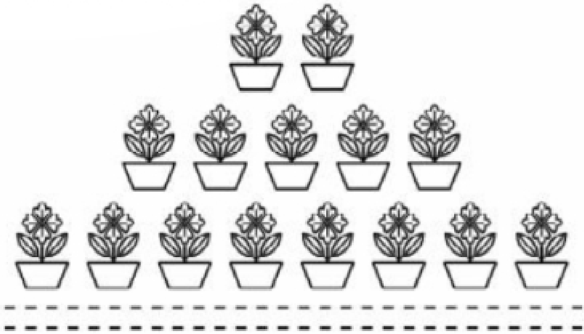
$= 4000\left(1 - \frac{1}{\sqrt{3}}\right)$

$= \frac{5080}{3}$ m or 1693.33 m (approx.) $\frac{1}{2}$

(**Note**: ½ mark to be deducted for not using $\sqrt{3}=1.73$)

SECTION E

36. Aahana being a plant lover decides to convert her balcony into beautiful garden full of plants. She bought few plants with pots for her balcony. She placed the pots in such a way that number of pots in the first row is 2, second row is 5, third row is 8 and so on.



Based on the above information, answer the following questions :

(i) Find the number of pots placed in the 10^{th} row.

(ii) Find the difference in the number of pots placed in 5^{th} row and 2^{nd} row.

(iii) If Aahana wants to place 100 pots in total, then find the total number of rows formed in the arrangement.

OR

(iii) If Aahana has sufficient space for 12 rows, then how many total number of pots are placed by her with the same arrangement ?

Solution: $a = 2, d = 3$

(i) Number of pots in the 10^{th} row

$$= a_{10} = a + 9d = 29$$ *1*

(ii) $a_5 - a_2 = (a + 4d) - (a + d) = 3d = 9$ *1*

(iii) $S_n = 100 \Rightarrow \frac{n}{2}[2(2) + (n-1)3] = 100$ *1*

$3n^2 + n - 200 = 0 \Rightarrow (3n + 25)(n - 8) = 0$

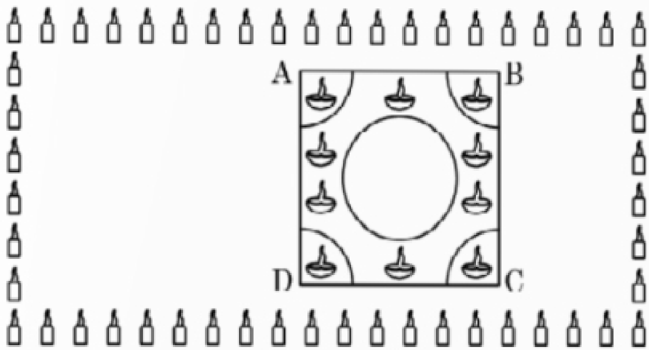
$\therefore n = 8 \quad (n = -\frac{25}{3}$ rejected), *1*

OR

(iii) $S_{12} = \frac{12}{2}[2(2) + 11(3)]$ *1*

$= 222$ *1*

37. Interschool Rangoli Competition was organized by one of the reputed schools of Odissa. The theme of the Rangoli Competition was Diwali celebrations where students were supposed to make mathematical designs. Students from various schools participated and made beautiful Rangoli designs. One such design is given below.



Rangoli is in the shape of square marked as ABCD, side of square being 40 cm. At each corner of a square, a quadrant of circle of radius 10 cm is drawn (in which diyas are kept). Also a circle of diameter 20 cm is drawn inside the square.

(i) What is the area of square ABCD ?

(ii) Find the area of the circle.

(iii) If the circle and the four quadrants are cut off from the square ABCD and removed, then find the area of remaining portion of square ABCD.

OR

(iii) Find the combined area of 4 quadrants and the circle, removed.

Solution: (i) Area of square ABCD $= (40)^2 = 1600 \text{ cm}^2$ *1*

(ii) Area of circle $= \pi r^2 = \frac{22}{7} \times 10 \times 10$

$$= \frac{2200}{7} \text{ cm}^2 \text{ or } 314.28 \text{ cm}^2$$ *1*

(iii) Area of 4 quadrants $= 4(\frac{1}{4}\pi r^2) = \frac{2200}{7} \text{ cm}^2$ *1*

Remaining area $= 1600 - \left(\frac{2200}{7} + \frac{2200}{7}\right)$

$= 1600 - \frac{4400}{7} = \frac{6800}{7}$ cm^2 or 971.43 cm^2 *1*

OR

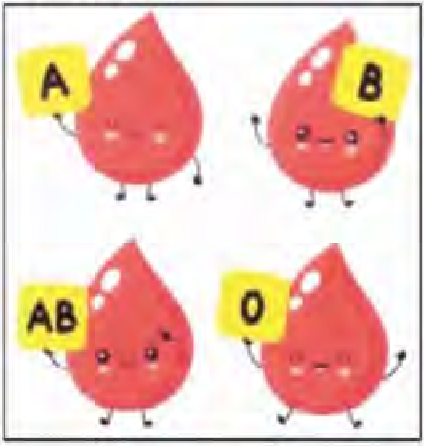
(iii) Area of 4 quadrants $= 4(\frac{1}{4}\pi r^2) = \frac{2200}{7}$ cm^2 *1*

Combined area of circle + 4 quadrants

$= \frac{2200}{7} + \frac{2200}{7} = \frac{4400}{7}$ cm^2 or 628.57 cm^2 *1*

38. Blood group describes the type of blood a person has. It is a classification of blood based on the presence or absence of inherited antigenic substances on the surface of red blood cells. Blood types predict whether a serious reaction will occur in a blood transfusion.

In a sample of 50 people, 21 had type O blood, 22 had type A, 5 had type B and rest had type AB blood group.



Based on the above, answer the following questions :

(i) What is the probability that a person chosen at random had type O blood ?

(ii) What is the probability that a person chosen at random had type AB blood group ?

(iii) What is the probability that a person chosen at random had neither type A nor type B blood group ?

OR

(iii) What is the probability that person chosen at random had either type A or type B or type O blood group ?

Solution: (i) P(type O) = $\frac{21}{50}$ *1*

(ii) No. of people with AB type blood group = 50 – (21 + 22 + 5) = 2 $\frac{1}{2}$

P(type AB) = $\frac{2}{50}$ or $\frac{1}{25}$ $\frac{1}{2}$

(iii) P(neither type A nor type B) = $\frac{21+2}{50} = \frac{23}{50}$ *1+1*

OR

(iii) P(type A or type B or type O) = $\frac{21+22+5}{50} = \frac{24}{25}$ *1+1*
