MARKING SCHEME

MATHEMATICS (BASIC)

SECTION A

1. A quadratic polynomial the sum and product of whose zeroes are –3 and 2 respectively, is :

(a) $x^2 + 3x + 2$ (b) $x^2 - 3x + 2$ (c) $x^2 - 3x - 2$ (d) $x^2 + 3x - 2$

Ans. (a) $x^2 + 3x + 2$ *1*

---

2. (HCF × LCM) for the numbers 70 and 40 is :

(a) 10 (b) 280 (c) 2800 (d) 70

Ans. (c) 2800 *1*

---

3. If the radius of a semi-circular protractor is 7cm, then its perimeter is :

(a) 11 cm (b) 14 cm (c) 22 cm (d) 36 cm

Ans. (d) 36 cm *1*

---

4. The number $(5 - 3\sqrt{5} + \sqrt{5})$ is :

(a) an integer (b) a rational number

(c) an irrational number (d) a whole number

Ans. (c) an irrational number *1*

---

5. If $p(x) = x^2 + 5x + 6$, then $p(-2)$ is :

(a) 20 (b) 0 (c) – 8 (d) 8

Ans. (b) 0 1

---

6. Which of the following **cannot** be the probability of an event ?

(a) 0.1 (b) $\frac{5}{3}$ (c) 3% (d) $\frac{1}{3}$

Ans. (b) $\frac{5}{3}$ *1*

---

7. The pair of linear equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has :

(a) a unique solution (b) exactly two solutions

(c) infinitely many solutions (d) no solution

Ans. (d) no solution *1*

---

8. If $\Delta ABC \sim \Delta DEF$ and $\angle A = 47^\circ$, $\angle E = 83^\circ$, then $\angle C$ is equal :

(a) 47° (b) 50° (c) 83° (d) 130°

Ans. (b) 50° *1*

---

9. If the pair of linear equations $x - y = 1$, $x + ky = 5$ has a unique solution $x = 2, y = 1$, then the value of k is :

(a) -2 (b) -3 (c) 3 (d) 4

Ans. (c) 3 *1*

---

10. The value of $5 \sin^2 90^\circ - 2 \cos^2 0^\circ$ is :

(a) -2 (b) 5 (c) 3 (d) -3

Ans. (c) 3 *1*

---

11. The length of the arc of a circle of radius 14 cm which subtends an angle of 60° at the centre of the circle is :

(a) $\frac{44}{3}$ cm (b) $\frac{88}{3}$ cm (c) $\frac{308}{3}$ cm (d) $\frac{616}{3}$ cm

Ans. (a) $\frac{44}{3}$ cm *1*

---

12. The angle of elevation of the top of a 30 m high tower at a point 30 m away from the base of the tower is :

(a) 30° (b) 45° (c) 60° (d) 90°

Ans. (b) 45° *1*

---

13. The mode of the numbers 2, 3, 3, 4, 5, 4, 4, 5, 3, 4, 2, 6, 7 is :

(a) 2 (b) 3 (c) 4 (d) 5

Ans. (c) 4 *1*

---

14. From a well-shuffled deck of 52 playing cards, a card is drawn at random. What is the probability of getting a red queen ?

(a) $\frac{1}{52}$ (b) $\frac{1}{26}$ (c) $\frac{1}{13}$ (d) $\frac{12}{13}$

Ans. (b) $\frac{1}{26}$ *1*

---

15. A quadratic equation whose one root is 2 and the sum of whose roots is zero, is :

(a) $x^2 + 4 = 0$ (b) $x^2 - 2 = 0$ (c) $4x^2 - 1 = 0$ (d) $x^2 - 4 = 0$

Ans. (d) $x^2 - 4 = 0$ *1*

---

16. Which of the following is **not** a quadratic equation ?

(a) $2(x-1)^2 = 4x^2 - 2x + 1$

(b) $2x - x^2 = x^2 + 5$

(c) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

(d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Ans. (c) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ *1*

---

17. How many tangents can be drawn to a circle from a point on it ?

(a) One (b) Two (c) Infinite (d) Zero

Ans. (a) One *1*

---

18. The length of the tangent from an external point A to a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is :

(a) 7 cm (b) 5 cm (c) $\sqrt{7}$ cm (d) 25 cm

Ans. (b) 5 cm *1*

(Assertion - Reason type questions)

In question numbers **19** and **20**, a statement of **Assertion (A)** is followed by a statement of **Reason (R).** Choose the correct option :

(a) Both Assertion (A) and Reason (R) are true and Reason (R) gives the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) does not give the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** A tangent to a circle is perpendicular to the radius through the point of contact.

Reason (R) : The lengths of tangents drawn from an external point to a circle are equal.

Ans. (b) Both Assertion (A) and Reason (R) are true but Reason (R) does not give the correct explanation of Assertion (A) *1*

---

20. **Assertion (A) :** If one root of the quadratic equation $4x^2 - 10x + (k - 4) = 0$ is reciprocal of the other, then value of k is 8.

Reason (R) : Roots of the quadratic equation $x^2 - x + 1 = 0$ are real.

Ans. (c) Assertion (A) is true but Reason (R) is false *1*

SECTION B

21. If $\sin\alpha = \frac{1}{2}$, then find the value of $(3 \cos \alpha - 4 \cos^3 \alpha)$.

Solution: $\sin \alpha = \frac{1}{2} \Rightarrow \alpha = 30°$ *$\frac{1}{2}$*

$\therefore 3 \cos \alpha - 4 \cos^3 \alpha = 3 \cos 30° - 4 \cos^3 30°$

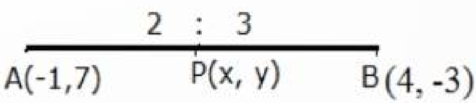
$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{2} - \frac{4(3\sqrt{3})}{8}$ *1*

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$ *$\frac{1}{2}$*

---

22. **(A)** Find the coordinates of the point which divides the join of A (−1, 7) and B (4, −3) in the ratio 2 : 3.

Solution:



$x = \frac{2(4) + 3(-1)}{2 + 3} = \frac{8 - 3}{5} = 1$ *1*

$y = \frac{2(-3) + 3(7)}{2 + 3} = \frac{15}{5} = 3$ *1*

Coordinates of the required point are (1, 3)

OR

(B) If the points A (2, 3), B (−5, 6), C (6, 7) and D (p, 4) are the vertices of a parallelogram ABCD, find the value of p.

Solution: Mid point of AC = Mid point of BD

$\therefore \left(\frac{2+6}{2}, \frac{3+7}{2}\right) = \left(\frac{-5+p}{2}, \frac{6+4}{2}\right)$ *1*

$\frac{-5+p}{2} = 4 \Rightarrow p = 13$ *1*

---

23. **(A)** Find the discriminant of the quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots.

Solution: $3x^2 - 2x + \frac{1}{3} = 0$

$a = 3, b = -2, c = \frac{1}{3}$ $\frac{1}{2}$

Discriminant (D) $= b^2 - 4ac = (-2)^2 - 4(3)\left(\frac{1}{3}\right) = 0$ 1

$\therefore$ Roots are real and equal $\frac{1}{2}$

OR

(B) Find the roots of the quadratic equation $x^2 - x - 2 = 0$.

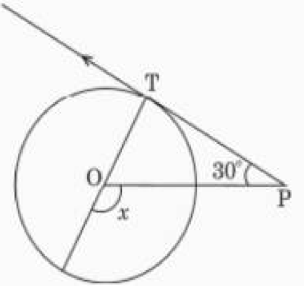
Solution: $x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$ 1

$x = 2, x = -1$ $\frac{1}{2} + \frac{1}{2}$

---

24. In the adjoining figure, PT is a tangent at T to the circle with centre O. If $\angle TPO = 30°$, find the value of x.



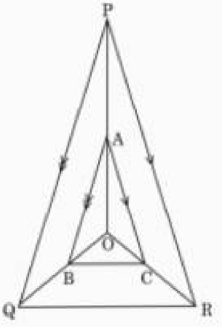
Solution:

$\angle OTP = 90°$ (tangent $\perp$ radius at the point of contact) 1

Getting $x = 120°$ 1

---

25. In the adjoining figure, A, B and C are points on OP, OQ and OR respectively such that AB||PQ and AC||PR. Show that BC||QR.



Solution: In Δ POQ, AB || PQ

$\Rightarrow \frac{OA}{AP} = \frac{OB}{BQ}$ (By Thales Theorem) ________ (i) $\frac{1}{2}$

In Δ POR, AC $\parallel$ PR

$\Rightarrow \frac{OA}{AP} = \frac{OC}{CR}$ (By Thales Theorem) ________ (ii) $\frac{1}{2}$

From (i) and (ii) $\frac{OB}{BQ} = \frac{OC}{CR}$ $\frac{1}{2}$

$\therefore$ In Δ QOR, BC $\parallel$ QR (By converse of Thales theorem) $\frac{1}{2}$

SECTION C

26. Find the zeroes of the quadratic polynomial $x^2 + 6x + 8$ and verify the relationship between the zeroes and the coefficients.

Solution: $x^2 + 6x + 8 = (x + 4)(x + 2)$ 1

$\Rightarrow$ Zeroes are $-4, -2$ 1

Sum of zeroes $= -4 + (-2) = -6 = \frac{-6}{1} = \frac{-\text{Coeff. of } x}{\text{Coeff. of } x^2}$ $\frac{1}{2}$

Product of zeroes $= (-4)(-2) = 8 = \frac{8}{1} = \frac{\text{Constant term}}{\text{Coeff. of } x^2}$ $\frac{1}{2}$

---

27. Prove that $\frac{1+\tan^2 A}{1+\cot^2 A} = \sec^2 A - 1$

Solution: $\text{LHS} = \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}}$ 1

$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}}$ $\frac{1}{2}$

$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \frac{1 - \cos^2 A}{\cos^2 A}$ 1

$= \frac{1}{\cos^2 A} - 1 = \sec^2 A - 1 = \text{RHS}$ $\frac{1}{2}$

---

28. (A) A lending library has a fixed charge for first three days and an additional charge for each day thereafter. Rittik paid ₹ 27 for a book kept for 7 days and Manmohan paid ₹ 21 for a book kept for 5 days. Find the fixed charges and the charge for each extra day.

Solution: Let fixed charge be ₹ x

Let charge for each extra day be ₹ y

ATQ, $x + 4y = 27$ *1*

$x + 2y = 21$ *1*

On solving ; $x = 15,\ y = 3$ $\frac{1}{2}+\frac{1}{2}$

∴ Fixed charge = ₹ 15

and charge for each extra day = ₹ 3

OR

(B) Find the values of 'a' and 'b' for which the system of linear equations $3x + 4y = 12,\ (a + b)\,x + 2\,(a - b)\,y = 24$ has infinite number of solutions.

Solution: For Infinite number of solutions $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{24}$ 1

$\frac{3}{a+b} = \frac{1}{2} \Rightarrow a + b = 6$ $\frac{1}{2}$

$\frac{2}{a-b} = \frac{1}{2} \Rightarrow a - b = 4$ $\frac{1}{2}$

On solving, $a = 5,\ b = 1$ $\frac{1}{2}+\frac{1}{2}$

---

29. A die is rolled once. Find the probability of getting :

(i) an even prime number.

(ii) a number greater than 4.

(iii) an odd number.

Solution: $S = \{1, 2, 3, 4, 5, 6\}$

(i) P(an even prime number) $= \frac{1}{6}$ *1*

(ii) P(a number greater than 4) $= \frac{2}{6}$ or $\frac{1}{3}$ *1*

(iii) P(an odd number) $= \frac{3}{6}$ or $\frac{1}{2}$ *1*

---

30. Find the area of the sector of a circle of radius 7 cm and of central angle 90°. Also, find the area of corresponding major sector.

Solution: $r = 7$ cm, $\theta = 90°$

Area of sector $= \frac{\pi r^2 \theta}{360} = \frac{22}{7} \times 7 \times 7 \times \frac{90}{360}$ *1*

$= \frac{77}{2}$ cm^2 or 38·5 cm^2 *1/2*

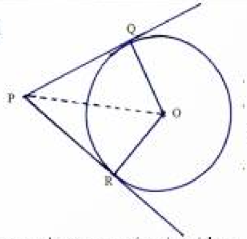
Area of circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154$ cm^2 *1*

Area of major sector $= 154 - 38{\cdot}5 = 115{\cdot}5$ cm^2 *1/2*

---

31. (A) Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:



1 (For Fig.)

Given: A circle with centre O and PQ, PR are tangents to the circle from an external point P.

To Prove: PQ = PR

Construction: Join OP, OQ, OR *1/2*

Proof : In Δ OPQ and Δ OPR

OP = OP (common)

OQ = OR (radii of the same circle)

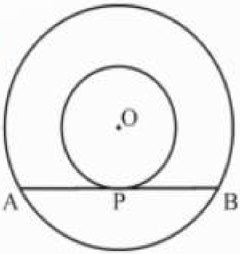
∠ OQP = ∠ ORP (each 90°)

⇒ Δ POQ ≅ Δ POR (RHS congruence) *1*

∴ PQ = PR *1/2*

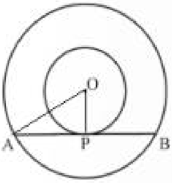
OR

(B) Two concentric circles with centre O are of radii 3 cm and 5 cm. Find the length of chord AB of the larger circle which touches the smaller circle at P.



Solution: Join OA and OP

OP ⊥ AB (radius ⊥ tangent at the point of contact) *1/2*



OP is the radius of smaller circle and AB is tangent at P.

AB is chord of larger circle and OP ⊥ AB

∴ AP = PB (⊥ from centre bisects the chord) *1*

In right Δ AOP, $AP^2 = OA^2 - OP^2$

$= (5)^2 - (3)^2 = 16$ *1/2*

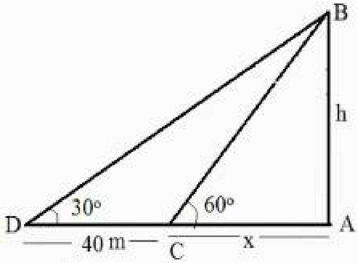
AP = 4 cm = PB *1/2*

∴ AB = 8 cm *1/2*

SECTION D

32. (A) The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower.

Solution:



1 (For Fig.)

In Δ ABC, $\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x$ *1+1/2*

In Δ ABD, $\tan 30° = \frac{h}{x + 40} \Rightarrow x + 40 = \sqrt{3}\,h$ *1+1/2*

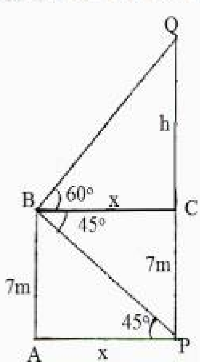
Getting x = 20 m *1/2*

and h= $20\sqrt{3}$ m (Height of tower) *1/2*

OR

(B) From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution:



1 (For Fig.)

In Δ ABP, $\tan 45° = \frac{7}{x} \Rightarrow x = 7$ $1+\frac{1}{2}$

In Δ BCQ, $\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x$ $1+\frac{1}{2}$

$h = 7\sqrt{3}$ m $\frac{1}{2}$

$\therefore$ Height of tower = PQ = 7 + h

$= 7 + 7\sqrt{3} = 7(1 + \sqrt{3})$ m $\frac{1}{2}$

---

33. (A) Find the sum of first 25 terms of the A.P. whose n^{th} term is given by $a_n = 5 + 6n$. Also, find the ratio of 20^{th} term to 45^{th} term.

Solution: $a_n = 5 + 6n$

$n = 1, \quad a_1 (1^{st} \text{ term}) = 5 + 6(1) = 11$ $\frac{1}{2}$

$n = 2, \quad a_2 (2^{nd} \text{ term}) = 5 + 6(2) = 17$ $\frac{1}{2}$

$\Rightarrow d = a_2 - a_1 = 17 - 11 = 6$ $\frac{1}{2}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{25} = \frac{25}{2}[2(11) + (25-1)6]$ 1

$= \frac{25}{2}[22 + 144]$

$S_{25} = 2075$ $\frac{1}{2}$

$\frac{a_{20}}{a_{45}} = \frac{5+6(20)}{5+6(44)} = \frac{125}{275} = \frac{5}{11}$ 1+1

$\therefore$ The required ratio is 5:11

OR

(B) In an A.P., if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

Solution:

$S_n = 3n^2 + 5n$

$n = 1, \; S_1 (1^{st} \text{ term}) = 3(1)^2 + 5(1) = 8$ 1

$n = 2, \; S_2 (\text{sum of } 1^{st} \text{ two terms}) = 3(2)^2 + 5(2) = 22$ 1

$a_1 + a_2 = 22 \Rightarrow a_2 = 22 - 8 = 14$ $\frac{1}{2}$

$\therefore d = a_2 - a_1 = 14 - 8 = 6$ 1

$a_k = 164$

$\Rightarrow 8 + (k - 1)6 = 164$ 1

$\therefore k = 27$ $\frac{1}{2}$

---

34. The following table gives the monthly consumption of electricity of 100 families :

Monthly Consumption (in units)	130-140	140-150	150-160	160-170	170-180	180-190	190-200
Number of families	5	9	17	28	24	10	7

Find the median of the above data.

Solution:

Monthly Consumption	Number of families	cf
130 – 140	5	5
140 – 150	9	14
150 – 160	17	31
160 – 170	28	59
170 – 180	24	83
180 – 190	10	93
190 – 200	7	100
	100	

1 (For Table.)

$$\frac{N}{2} = \frac{100}{2} = 50$$

$\therefore$ Median class is 160 – 170 1

$l = 160,\ h = 10,\ cf = 31,\ f = 28$ 1

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 160 + \frac{50 - 31}{28} \times 10 = 160 + \frac{19}{28} \times 10$$ $1\frac{1}{2}$

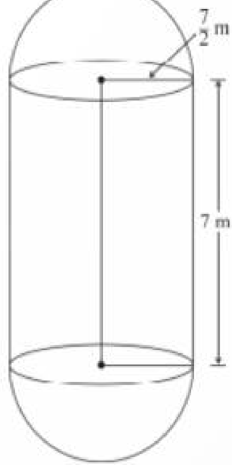
$= 166.8$ $\frac{1}{2}$

---

35. The boilers are used in thermal power plants to store water and then used to produce steam. One such boiler consists of a cylindrical part in middle and two hemispherical parts at its both ends.

Length of the cylindrical part is 7m and radius of cylindrical part is $\frac{7}{2}$m.

Find the total surface area and the volume of the boiler. Also, find the ratio of the volume of cylindrical part to the volume of one hemispherical part.



Solution:

$h = 7 \text{ m}, r = \frac{7}{2} \text{ m}$

Total surface area $= 2\pi rh + 2(2\pi r^2) = 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times \frac{7}{2}\left(7 + 2 \times \frac{7}{2}\right)$ $1\frac{1}{2}$

$= 308 \text{ m}^2$ $\frac{1}{2}$

Volume of the boiler $= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right)$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7 + \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$ $1\frac{1}{2}$

$= \frac{2695}{6} m^3$ or $449.16\, m^3$ $\frac{1}{2}$

$\frac{\text{Volume of cylindrical part}}{\text{Volume of one hemispherical part}} = \frac{\pi r^2 h}{\frac{2}{3}\pi r^3}$ $\frac{1}{2}$

$= \frac{3h}{2r} = \frac{3(7)}{2 \times \frac{7}{2}} = \frac{3}{1}$ or $3 : 1$ $\frac{1}{2}$

SECTION E

36. Use of mobile screen for long hours makes your eye sight weak and give you headaches. Children who are addicted to play "PUBG" can get easily stressed out. To raise social awareness about ill effects of playing PUBG, a school decided to start 'BAN PUBG' campaign, in which students are asked to prepare campaign board in the shape of a rectangle. One such campaign board made by class X student of the school is shown in the figure.

Based on the above information, answer the following questions :

(i) Find the coordinates of the point of intersection of diagonals AC and BD.

(ii) Find the length of the diagonal AC.

(iii) (a) Find the area of the campaign Board ABCD.

OR

(b) Find the ratio of the length of side AB to the length of the diagonal AC.

Solution:

We know that diagonals of a rectangle bisect each other.

(i) Required point = Mid-point of AC = $\left(\frac{1+7}{2}, \frac{1+5}{2}\right) = (4, 3)$ *1*

(ii) $AC = \sqrt{(7-1)^2 + (5-1)^2} = 2\sqrt{13}$ *1*

(iii) (a) $AB = \sqrt{(7-1)^2 + (1-1)^2} = 6$ $\frac{1}{2}$

$BC = \sqrt{(7-7)^2 + (5-1)^2} = 4$ $\frac{1}{2}$

Area (ABCD) = AB × BC = 6 × 4 = 24 *1*

OR

(iii) (b) $AB = \sqrt{(7-1)^2 + (1-1)^2} = 6$ *1*

$\frac{AB}{AC} = \frac{6}{2\sqrt{13}} = \frac{3}{\sqrt{13}}$ *1*

$\therefore$ required ratio is $3 : \sqrt{13}$

37. Khushi wants to organize her birthday party. Being health conscious, she decided to serve only fruits in her birthday party. She bought 36 apples and 60 bananas and decided to distribute fruits equally among all.

Based on the above information, answer the following questions :

(i) How many guests Khushi can invite at the most ?

(ii) How many apples and bananas will each guest get ?

(iii) (a) If Khushi decides to add 42 mangoes, how many guests Khushi can invite at the most ?

OR

(b) If the cost of 1 dozen of bananas is ₹ 60, the cost of 1 apple is ₹ 15 and cost of 1 mango is ₹ 20, find the total amount spent on 60 bananas, 36 apples and 42 mangoes.

Solution:

(i) HCF (36, 60) = 12 *1*

Khushi can invite at the most 12 guests

(ii) $36 \div 12 = 3,\ 60 \div 12 = 5$

Each guest will get 3 apples and 5 bananas *1*

(iii) (a) HCF (36, 60, 42) = 6 *2*

Khushi can invite at the most 6 guests

OR

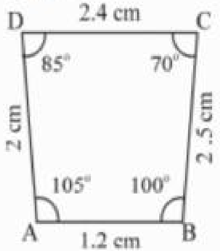
(iii) (b) Total cost $= 5 \times 60 + 36 \times 15 + 42 \times 20$ *1*

$= ₹1680$ *1*

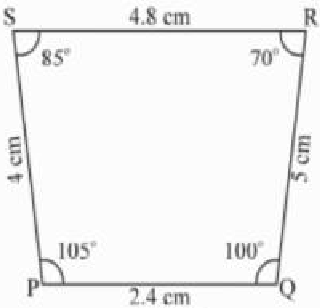
---

38. Observe the figures given below carefully and answer the questions :

Figure A

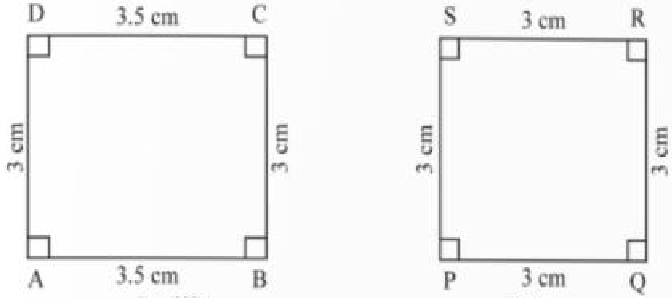


A (i)



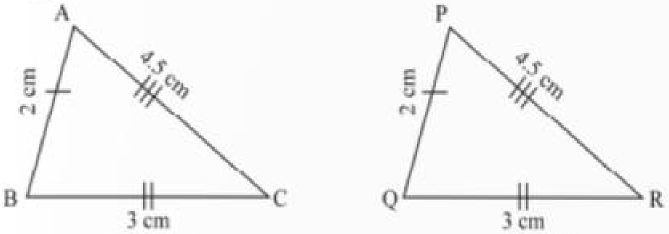
A (ii)

Figure B



B (iii) B (iv)

Figure C



C (v) C (vi)

(i) Name the figure(s) wherein two figures are similar.

(ii) Name the figure(s) wherein the figures are congruent.

(iii) (a) Prove that congruent triangles are also similar but not the converse.

OR

(b) What more is least needed for two similar triangles to be congruent ?

Solution:

(i)	Figure A and Figure C	$\frac{1}{2}+\frac{1}{2}$
(ii)	Figure C	1
(iii)	(a) Triangles are congruent ⇒ Corresponding angles are equal	1
	⇒ Triangles are similar.	$\frac{1}{2}$
	Conversely, if triangles are similar then ratio of corresponding sides is same which does not imply corresponding sides are equal	
	∴ Triangles may not be congruent.	$\frac{1}{2}$
	Note: Any suitable counter example can be given	
	OR	
(iii)	(b) One pair of corresponding side must be equal	2