MARKING SCHEME

MATHEMATICS (BASIC) 430/6/1

SECTION A

1. The time, in seconds, taken by 150 athletes to run a 100 m hurdle race are tabulated below :

Time (sec.)	13-14	14-15	15-16	16-17	17-18	18-19
Number of Athletes	2	4	5	71	48	20

The number of athletes who completed the race in less than 17 seconds is

(a) 11 (b) 71

(c) 82 (d) 68

Answer (c) 82 1

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2. The distance of the point (5, 0) from the origin is

(a) 0 (b) 5

(c) $\sqrt{5}$ (d) 5^2

Answer (b) 5 1

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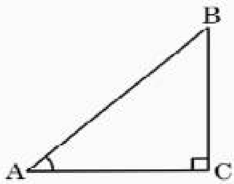
3. In ΔABC, right angled at C, if $\tan A = \frac{8}{7}$, then the value of cot B is



(a) $\frac{7}{8}$ (b) $\frac{8}{7}$

(c) $\frac{7}{\sqrt{113}}$ (d) $\frac{8}{\sqrt{113}}$

Answer (b) $\frac{8}{7}$ 1

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4. Area of a quadrant of a circle of radius 7 cm is

(a) $154\ cm^2$ (b) $77\ cm^2$

(c) $\frac{77}{2}\ cm^2$ (d) $\frac{77}{4}\ cm^2$

Answer (c) $\frac{77}{2}\ cm^2$ 1

5. If HCF (72, 120) = 24, then LCM (72, 120) is

(a) 72 (b) 120

(c) 360 (d) 9640

Answer (c) 360 1

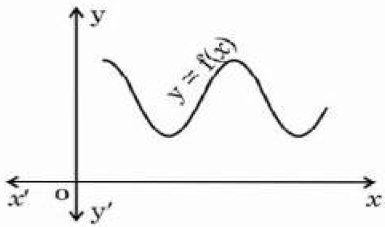
6. One card is drawn at random from a well-shuffled deck of 52 playing cards. What is the probability of getting a black king ?

(a) $\frac{1}{26}$ (b) $\frac{1}{13}$

(c) $\frac{1}{52}$ (d) $\frac{1}{2}$

Answer (a) $\frac{1}{26}$ 1

7. The graph of y = f(x) is shown in the figure for some polynomial f(x).



The number of zeroes of f(x) is

(a) 0 (b) 2

(c) 3 (d) 4

Answer (a) 0 1

8. The value of k, if (6, k) lies on the line represented by $x - 3y + 6 = 0$, is

(a) -4 (b) 12

(c) -12 (d) 4

Answer (d) 4 1

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9. The prime factorisation of the number 2304 is

(a) $2^8 \times 3^2$ (b) $2^7 \times 3^3$

(c) $2^8 \times 3^1$ (d) $2^7 \times 3^2$

Answer (a) 2^8 x 3^2 1

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10. If n is a natural number, then 8^n cannot end with digit

(a) 0 (b) 2

(c) 4 (d) 6

Answer (a) 0 1

---

11. The median of first seven prime numbers is

(a) 5 (b) 7

(c) 11 (d) 13

Answer (b) 7 1

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12. If (2, 4) is the mid-point of the line-segment joining (6, 3) and (a, 5), then the value of a is

(a) 2 (b) 4

(c) -4 (d) -2

Answer (d) -2 1

---

13. The value of 'k' for which the system of equations $kx + 2y = 5$ and $3x + 4y = 1$ have no solution, is

(a) $k = \frac{3}{2}$ (b) $k \neq \frac{3}{2}$

(c) $k \neq \frac{2}{3}$ (d) $k = 15$

Answer (a) $k = \frac{3}{2}$ 1

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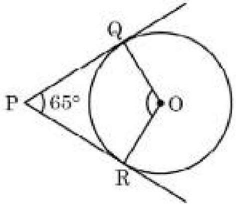
14. In the given figure, PQ and PR are tangents drawn from P to the circle with centre O such that $\angle QPR = 65°$. The measure of $\angle QOR$ is.



(a) 65° (b) 125°

(c) 115° (d) 90°

Answer (c) 115° 1

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15. The zeroes of the quadratic polynomial $16x^2 - 9$ are :

(a) $\frac{3}{4}, \frac{3}{4}$ (b) $-\frac{3}{4}, \frac{3}{4}$

(c) $\frac{9}{16}, \frac{9}{16}$ (d) $-\frac{3}{4}, -\frac{3}{4}$

Answer (b) $-\frac{3}{4}, \frac{3}{4}$ 1

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16. If –5, x, 3 are three consecutive terms of an A.P., then the value of x is

(a) –2 (b) 2

(c) 1 (d) –1

Answer (d) – 1 1

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17. An unbiased die is thrown. The probability of getting an odd prime number is

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

Answer (d) $\frac{1}{3}$ 1

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18. If the mean of 6, 7, x, 8, y, 14 is 9, then

(a) $x + y = 21$ (b) $x + y = 19$

(c) $x - y = 19$ (d) $x - y = 21$

Answer (b) $x + y = 19$ 1

Directions for Q. 19 & Q. 20 : In question numbers **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option :

(a) Both Assertion (A) and Reason (R) are true; and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

Reason (R) : The probability that a non-leap year has 53 Sundays is $\frac{1}{7}$.

Answer (b)Both Assertion(A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A). 1

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20. **Assertion (A) :** For $0 < \theta \leq 90^\circ$, cosec θ – cot θ and cosec θ + cot θ are reciprocal of each other.

Reason (R) : $\cot^2 \theta - \text{cosec}^2 \theta = 1$

Answer (c) Assertion(A) is true, but Reason (R) is false. 1

SECTION B

21. Evaluate : $5 \operatorname{cosec}^2 45^\circ - 3 \sin^2 90^\circ + 5 \cos 0^\circ$.

Solution $5 \operatorname{cosec}^2 45^\circ - 3 \sin^2 90^\circ + 5 \cos 0^\circ$

$= 5(\sqrt{2})^2 - 3(1)^2 + 5(1)$ $1\frac{1}{2}$

$= 12$ $\frac{1}{2}$

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22. (a) Find a quadratic polynomial whose zeroes are 6 and -3.

Solution (a) Sum of zeroes = 6+ (−3) = 3 $\frac{1}{2}$

Product of zeroes = 6(− 3) = − 18 $\frac{1}{2}$

Quadratic polynomial is $(x^2 - 3x - 18)$ or $k(x^2 - 3x - 18)$ *1*

OR

(b) Find the zeroes of the polynomial $x^2 + 4x - 12$.

Solution (b) $x^2 + 4x - 12 = (x + 6)(x - 2)$ *1*

Zeroes are − 6, 2 *1*

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23. (a) Find the value of k for which the roots of the quadratic equation $5x^2 - 10x + k = 0$ are real and equal.

Solution a = 5, b = − 10, c = k ½

Roots are real and equal

$D = 0 \Rightarrow b^2 - 4ac = 0$ ½

$(-10)^2 - 4(5)(k) = 0 \Rightarrow 100 - 20k = 0$ ½

$k = 5$ ½

OR

(b) If one root of the quadratic equation $3x^2 - 8x - (2k + 1) = 0$ is seven times the other, then find the value of k.

Solution Let roots be α, 7α ½

$\alpha + 7\alpha = -\left(\frac{-8}{3}\right) = \frac{8}{3} \Rightarrow 8\alpha = \frac{8}{3}$ gives $\alpha = \frac{1}{3}$ ½

$\alpha(7\alpha) = -\frac{(2k+1)}{3} \Rightarrow 7\alpha^2 = -\frac{(2k+1)}{3}$ ½

$k = -\frac{5}{3}$ ½

24. A box contains 20 discs which are numbered from 1 to 20. If one disc is drawn at random from the box, then find the probability that the number on the drawn disc is a

(i) 2-digit number

(ii) number less than 10

Solution P(2 digit number) = $\frac{11}{20}$ *1*

P(number less than 10) = $\frac{9}{20}$ *1*

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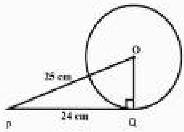
25. From a point P, the length of the tangent to a circle is 24 cm and the distance of P from the centre of the circle is 25 cm. Find the radius of the circle.



Solution

figure ½

$OQ = \sqrt{25^2 - 24^2}$ 1

$OQ = 7$ cm ½

SECTION C

26. The sum of the reciprocals of Varun's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Solution Let Varun's present age = x years

ATQ, $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$ 1

$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$

$6x + 6 = x^2 + 2x - 15 \Rightarrow x^2 - 4x - 21 = 0$ 1

$(x - 7)(x + 3) = 0$ ½

$x = 7, x = -3$ (rejecting) ½

$\therefore$ Varun's present age = 7 years

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27. A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household :

Family size	1-3	3-5	5-7	7-9	9-11
Number of Families	7	8	2	2	1

Find the median of this data.

Solution

Family size	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11
Number of families	7	8	2	2	1
Cf	7	15	17	19	20

For correct cf 1

Median class 3 – 5 ½

Median = $l + \frac{\frac{N}{2} - C}{f} \times h$

$= 3 + \frac{10-7}{8} \times 2$ 1

$= 3 \cdot 75$ ½

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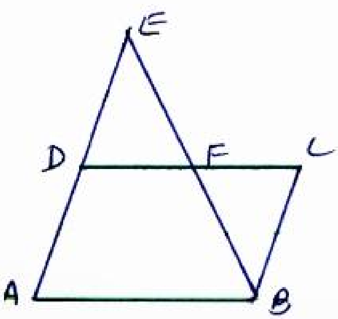
28. (a) E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.

Solution



(a)ABCD is a parallelogram (1 for figure)

To prove: Δ ABE ~ Δ CFB

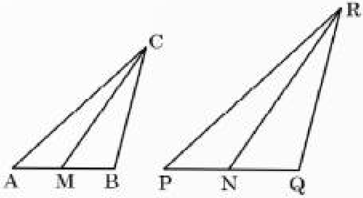
In Δ ABE and Δ CFB,

∠ A = ∠ C (opp. angles of parallelogram) 1/2

∠ AEB = ∠ CBF (alt. int. angles) 1/2

∴ Δ ABE ~ Δ CFB (AA similarity) 1

OR



(b) In the given figure, CM and RN are respectively the medians of ΔABC and ΔPQR. If $\Delta ABC \sim \Delta PQR$, then prove that $\Delta AMC \sim \Delta PNR$.

Solution $\Delta ABC \sim \Delta PQR$

$\frac{AB}{PQ} = \frac{AC}{PR} \Rightarrow \frac{2AM}{2PN} = \frac{AC}{PR}$ ½+1

$\frac{AM}{PN} = \frac{AC}{PR}$

Also $\angle A = \angle P$ ($\Delta ABC \sim \Delta PQR$)

$\therefore \Delta AMC \sim \Delta PNR$ (SAS similarity) 1½

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29. Find the co-ordinates of the points of trisection of the line-segment joining the points (5, 3) and (4, 5).

Solution

Let C divides AB in the ratio 1 : 2 ½

$\therefore C\left(\frac{1\times4+2\times5}{1+2}, \frac{1\times5+2\times3}{1+2}\right)$, i.e., $C\left(\frac{14}{3}, \frac{11}{3}\right)$ 1

Let D divides AB in the ratio 2 : 1 ½

$\therefore D\left(\frac{2\times4+1\times5}{2+1}, \frac{2\times5+1\times3}{2+1}\right)$, i.e., $D\left(\frac{13}{3}, \frac{13}{3}\right)$ 1

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30. Prove that $3-2\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number.

Solution Let us assume that $3-2\sqrt{5}$ is a rational number.

$\therefore 3-2\sqrt{5} = \frac{p}{q}$, $q \neq 0$, p and q are integers 1

$\Rightarrow \sqrt{5} = \frac{3q-p}{2q}$ 1

Now RHS is rational but LHS is irrational

$\therefore$ Our assumption is wrong 1

$\therefore$ $3-2\sqrt{5}$ is an irrational number.

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31. (a) Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\cos^2 A}{(1+\sin A)^2}$

Solution LHS = $\frac{cot\ A - \cos A}{cot\ A + \cos A} = \frac{\frac{cos\ A}{\sin A} - \cos A}{\frac{cos\ A}{\sin A} + \cos A}$ ½

$= \frac{1-sin\ A}{1+sin\ A}$ 1

$= \frac{(1-\sin A)(1+\sin A)}{(1+\sin A)(1+\sin A)}$ 1

$= \frac{1-\sin^2 A}{(1+\sin A)^2} = \frac{cos^2 A}{(1+\sin A)^2}$ ½

OR

(b) Prove that $(\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$

Solution LHS = $(\sec\theta + \tan\theta)(1 - \sin\theta)$

$= \left(\frac{1}{cos\ \theta} + \frac{\sin\theta}{\cos\theta}\right)(1-\sin\theta)$ 1

$= \left(\frac{1+\sin\theta}{\cos\theta}\right)(1-\sin\theta) = \frac{(1-\sin^2\theta)}{\cos\theta}$ ½ + ½

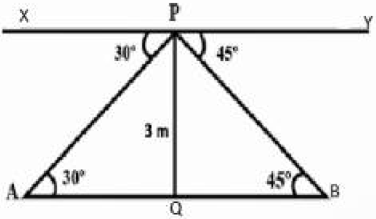
$= \frac{cos^2\ \theta}{\cos\theta} = \cos\theta = \text{RHS}$ 1

SECTION D

32. (a) From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, find the width of the river. (Use $\sqrt{3} = 1.73$)

Solution

For fig. 1



In Δ APQ, $\tan 30° = \frac{3}{AQ}$ 1

$\frac{1}{\sqrt{3}} = \frac{3}{AQ} \Rightarrow AQ = 3\sqrt{3}$ ½

In Δ PBQ, $\tan 45° = \frac{3}{BQ}$ 1

BQ = 3 ½

$\therefore AB = AQ + BQ = 3\sqrt{3} + 3$ ½

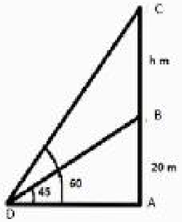
$= 3(1\cdot 73 + 1) = 8\cdot 19$

Width of river = 8·19 m ½

OR

(b) From a point on the ground, the angle of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. (Use $\sqrt{3} = 1.73$)

Solution BC = transmission tower = h and AD = x



For fig. 1

In Δ ABD, $\tan 45^\circ = \frac{20}{x}$ 1

$x = 20$ ½

In Δ ACD, $\tan 60^\circ = \frac{20 + h}{x}$ 1

$\sqrt{3}x = 20 + h$ ½

$\therefore h = 20(\sqrt{3} - 1)$ m ½

$h = 14.6$ m ½

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33. The first term of an A.P. is 22, the last term is –6 and the sum of all the terms is 64. Find the number of terms of the A.P. Also, find the common difference.

Solution $a = 22$, $a_n = -6$, $S_n = 64$

$S_n = 64 \Rightarrow \frac{n}{2}[22 - 6] = 64$ 2

$n = 8$ 1

$22 + (8-1)d = -6$ 1

$\Rightarrow d = -4$ 1

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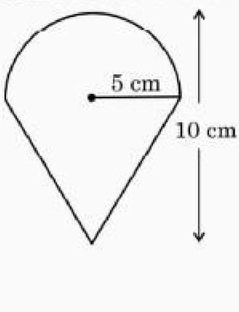
34. An ice-cream filled cone having radius 5 cm and height 10 cm is as shown in the figure. Find the volume of the ice-cream in 7 such cones.



Solution Height of conical part = 10 – 5 = 5 cm ½

Volume of 1 ice cream cone

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ 1

$= \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3} x \frac{22}{7} x 5 x 5[5 + 10]$ 1½

$= \frac{22 \times 25 \times 15}{21} cm^3$ 1

Volume of 7 ice cream cones

$= 7 \times \frac{22 \times 25 \times 15}{21}$ $\frac{1}{2}$

$= 2750 \ cm^3$ $\frac{1}{2}$

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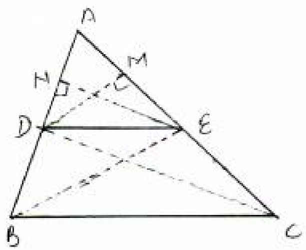
35. (a) Prove that a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, divides the two sides in the same ratio.

Solution



For figure1

Given In Δ ABC, DE // BC $\frac{1}{2}$

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$ $\frac{1}{2}$

Const. : Join BE, CD. Draw DM ⊥ AC and EN ⊥ AB $\frac{1}{2}$

Proof : $\frac{ar(\Delta\, ADE)}{ar(\Delta\, BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$ ________ (i) 1

similarly $\frac{ar(\Delta\, ADE)}{ar(\Delta\, CDE)} = \frac{AE}{EC}$ ________ (ii) $\frac{1}{2}$

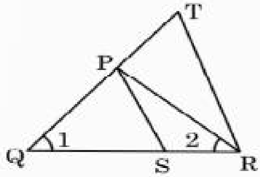
Δ BDE and Δ CDE are on the same base DE and between the same parallel lines BC and DE.

ar(Δ ADE) = ar(Δ CDE) ________ (iii) $\frac{1}{2}$

From (i), (ii) and (iii) $\frac{AD}{DB} = \frac{AE}{EC}$ $\frac{1}{2}$

OR

(b) In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$.



Solution In Δ PQR, $\angle 1 = \angle 2$

$\therefore$ PQ = PR (sides opposite to equal angles) 1

Now $\frac{QR}{QS} = \frac{QT}{PR}$

$\therefore \frac{QS}{QR} = \frac{PR}{QT} \Rightarrow \frac{QS}{QR} = \frac{PQ}{QT}$ (as PR = PQ) ________ (i) 2

In Δ PQS and Δ TQR,

$\angle Q = \angle Q$ (common)

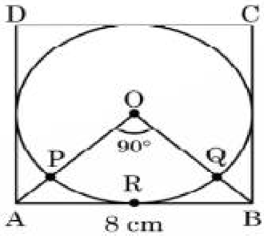
$\frac{QS}{QR} = \frac{PQ}{QT}$ (from (i)) $1\frac{1}{2}$

$\therefore \Delta PQS \sim \Delta TQR$ (SAS similarity) $\frac{1}{2}$

SECTION E

36. For the inauguration of 'Earth day' week in a school, badges were given to volunteers. Organisers purchased these badges from an NGO, who made these badges in the form of a circle inscribed in a square of side 8 cm.

O is the centre of the circle and $\angle AOB = 90°$:



Based on the above information, answer the following questions :

(i) What is the area of square ABCD ?

(ii) What is the length of diagonal AC of square ABCD ?

(iii) Find the area of sector OPRQO.

OR

(iii) Find the area of remaining part of square ABCD when area of circle is excluded.

Solution (i) Area of square ABCD = $(8)^2 = 64$ cm^2 1

(ii) AC = $\sqrt{(8)^2 + (8)^2} = \sqrt{128} = 8\sqrt{2}$ cm 1

(iii) We know that diagonals of square bisect each other at 90°

$\angle AOB = 90°$

Area of sector OPRQ = $\frac{\pi r^2 \theta}{360°}$

$= \frac{22}{7} \text{ x } 4 \text{ x } 4 \text{ x } \frac{90}{360}$ 1

$= \frac{88}{7}$ cm^2 1

OR

(iii) Area of circle = $\pi r^2 = \frac{22}{7} \text{ x } 4 \text{ x } 4 = \frac{352}{7}$ cm^2 1

Required area = $64 - \frac{352}{7} = \frac{96}{7}$ cm^2 1

37.

Lokesh, a production manager in Mumbai, hires a taxi everyday to go to his office. The taxi charges in Mumbai consists of a fixed charges together with the charges for the distance covered. His office is at a distance of 10 km from his home. For a distance of 10 km to his office, Lokesh paid ₹ 105. While coming back home, he took another route. He covered a distance of 15 km and the charges paid by him were ₹ 155.

Based on the above information, answer the following questions :

(i) What are the fixed charges ?

(ii) What are the charges per km ?

(iii) If fixed charges are ₹ 20 and charges per km are ₹ 10, then how much Lokesh have to pay for travelling a distance of 10 km ?

OR

(iii) Find the total amount paid by Lokesh for travelling 10 km from home to office and 25 km from office to home. [Fixed charges and charges per km are as in (i) & (ii).

Solution (i) Let fixed charge= ₹ x and charges per km = ₹ y

$x + 10y = 105, x + 15y = 155$ $\frac{1}{2}$

On solving, $x = 5$

$\therefore$Fixed charge = ₹ 5 $\frac{1}{2}$

(ii) on solving, we get $y = 10$

Charge per km = ₹10 1

(iii) $x + 10y = 20 + 10(10) =$ ₹ 120 1+1

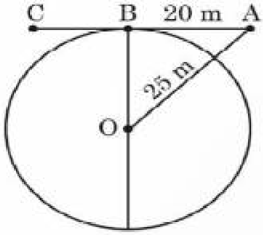
OR

(iii) Required amount $= x + 10y + x + 25y = 2x + 35y$ 1

$= 2(5) + 35(10) = 10 + 350 =$ ₹ 360 1

38.

Road



People of a circular village Dharamkot want to construct a road nearest to it. The road cannot pass through the village. But the people want the road at a shortest distance from the centre of the village. Suppose the road starts from A which is outside the circular village (as shown in the figure) and touch the boundary of the circular village at B such that AB = 20 m. Also the distance of the point A from the centre O of the village is 25 m.

Based on the above information, answer the following questions :

(i) If B is the mid-point of AC, then find the distance AC. **1**

(ii) Find the shortest distance of the road from the centre of the village. **1**

(iii) Find the circumference of the village. **2**

OR

(iii) Find the area of the village. **2**

Solution (i) AC = AB + BC = 20 + 20 = 40 m *1*

(ii) Shortest distance OB = $\sqrt{25^2 - 20^2} = 15$ m 1

(iii) Circumference = $2\pi(15) = 30\pi$ m or $\frac{660}{7}$ m 1+1

OR

Area = $\pi(15)^2 = 225\pi$ sq. m or $\frac{4950}{7}$ sq. m 1+1
