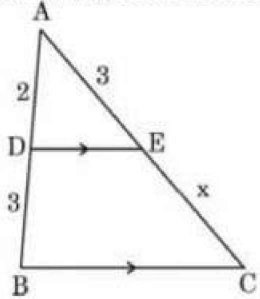
MARKING SCHEME

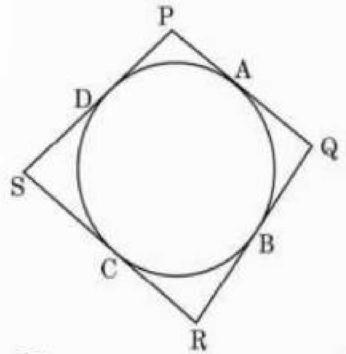
MATHEMATICS (Subject Code–041)

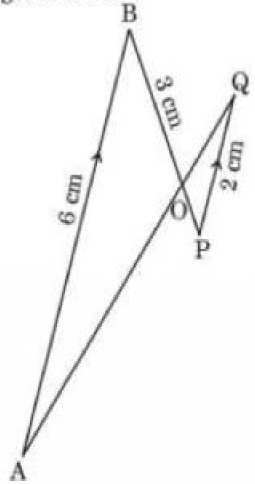
(PAPER CODE: 30/2/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each	
1.	Which of the following quadratic equations has sum of its roots as 4 ? (a) $2x^2 - 4x + 8 = 0$ (b) $-x^2 + 4x + 4 = 0$ (c) $\sqrt{2}x^2 - \frac{4}{\sqrt{2}}x + 1 = 0$ (d) $4x^2 - 4x + 4 = 0$	
Sol.	(b) $-x^2 + 4x + 4 = 0$	1
2.	What is the length of the arc of the sector of a circle with radius 14 cm and of central angle 90° ? (a) 22 cm (b) 44 cm (c) 88 cm (d) 11 cm	
Sol.	(a) 22 cm	1
3.	If $\Delta ABC \sim \Delta PQR$ with $\angle A = 32°$ and $\angle R = 65°$, then the measure of $\angle B$ is : (a) 32° (b) 65° (c) 83° (d) 97°	
Sol.	(c) 83°	1
4.	If 'p' and 'q' are natural numbers and 'p' is the multiple of 'q', then what is the HCF of 'p' and 'q' ? (a) pq (b) p (c) q (d) p + q	
Sol.	(c) q	1
5.	The coordinates of the vertex A of a rectangle ABCD whose three vertices are given as B(0, 0), C(3, 0) and D(0, 4) are : (a) (4, 0) (b) (0, 3) (c) (3, 4) (d) (4, 3)	
Sol.	(c) (3, 4)	1

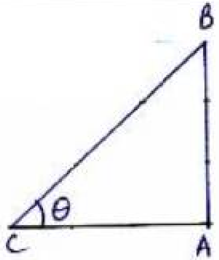
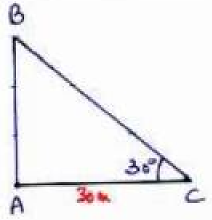
6.	If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represent coincident lines, then the value of 'r' is : (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) -2 (d) 2	
Sol.	(d) 2	1
7.	A bag contains 100 cards numbered 1 to 100. A card is drawn at random from the bag. What is the probability that the number on the card is a perfect cube ? (a) $\frac{1}{20}$ (b) $\frac{3}{50}$ (c) $\frac{1}{25}$ (d) $\frac{7}{100}$	
Sol.	(c) $\frac{1}{25}$	1
8.	The pair of equations $x = a$ and $y = b$ graphically represents lines which are : (a) parallel (b) intersecting at (b, a) (c) coincident (d) intersecting at (a, b)	
Sol.	(d) intersecting at (a, b)	1
9.	If one zero of the polynomial $6x^2 + 37x - (k - 2)$ is reciprocal of the other, then what is the value of k ? (a) -4 (b) -6 (c) 6 (d) 4	
Sol.	(a) -4	1
10.	What is the total surface area of a solid hemisphere of diameter 'd' ? (a) $3\pi d^2$ (b) $2\pi d^2$ (c) $\frac{1}{2}\pi d^2$ (d) $\frac{3}{4}\pi d^2$	
Sol.	(d) $\frac{3}{4}\pi d^2$	1

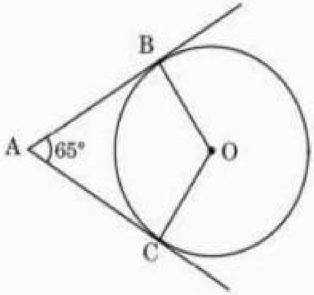
11.	If three coins are tossed simultaneously, what is the probability of getting at most one tail ? (a) $\frac{3}{8}$ (b) $\frac{4}{8}$ (c) $\frac{5}{8}$ (d) $\frac{7}{8}$	
Sol.	(b) $\frac{4}{8}$	1
12.	In the given figure, DE ∥ BC. If AD = 2 units, DB = AE = 3 units and EC = x units, then the value of x is :  (a) 2 (b) 3 (c) 5 (d) $\frac{9}{2}$	
Sol.	(d) $\frac{9}{2}$	1
13.	The hour-hand of a clock is 6 cm long. The angle swept by it between 7:20 a.m. and 7:55 a.m. is : (a) $\left(\frac{35}{4}\right)^{\circ}$ (b) $\left(\frac{35}{2}\right)^{\circ}$ (c) 35° (d) 70°	
Sol.	(b) $\left(\frac{35}{2}\right)^{\circ}$	1
14.	The zeroes of the polynomial $p(x) = x^2 + 4x + 3$ are given by : (a) 1, 3 (b) −1, 3 (c) 1, −3 (d) −1, −3	
Sol.	(d) −1, −3	1

15.	In the given figure, the quadrilateral PQRS circumscribes a circle. Here PA + CS is equal to :  (a) QR (b) PR (c) PS (d) PQ	
Sol.	(c) PS	1
16.	If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - ax - b$, then the value of $\alpha^2 + \beta^2$ is : (a) $a^2 - 2b$ (b) $a^2 + 2b$ (c) $b^2 - 2a$ (d) $b^2 + 2a$	
Sol.	(b) $a^2 + 2b$	1
17.	The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is : (a) ab (b) $\frac{1}{2}ab$ (c) $\frac{1}{4}ab$ (d) 2ab	
Sol.	(b) $\frac{1}{2}$ ab	1

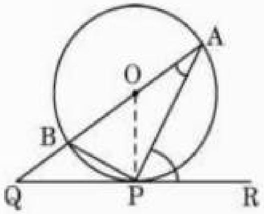
18.	In the given figure, AB ∥ PQ. If AB = 6 cm, PQ = 2 cm and OB = 3 cm, then the length of OP is :  (a) 9 cm (b) 3 cm (c) 4 cm (d) 1 cm	
Sol.	(d) 1 cm	1
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* A tangent to a circle is perpendicular to the radius through the point of contact. *Reason (R) :* The lengths of tangents drawn from an external point to a circle are equal.	
Sol.	(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).	1
20.	*Assertion (A) :* The polynomial $p(x) = x^2 + 3x + 3$ has two real zeroes. *Reason (R):* A quadratic polynomial can have at most two real zeroes.	
Sol.	(d) Assertion (A) is false, but Reason (R) is true.	1

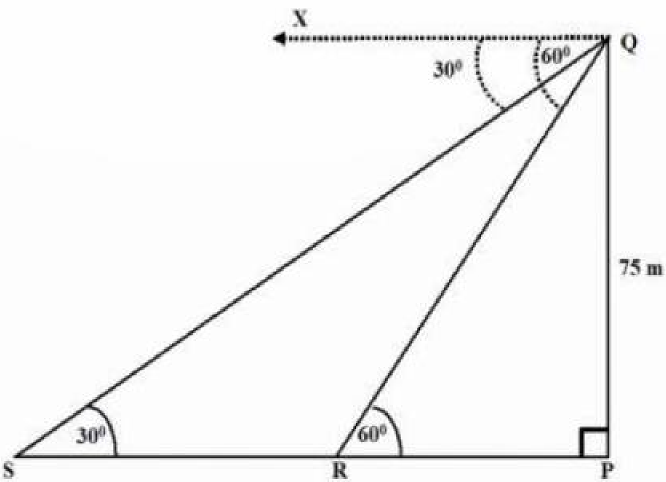
	SECTION B This section comprises of Very Short Answer (VSA) type questions of 2 marks each.	
21.	**Prove that $2 + \sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.**	
Sol.	Let us assume that $2 + \sqrt{3}$ is rational Let $2 + \sqrt{3} = \frac{p}{q}$; $q \neq 0$ and p, q are integers $\Rightarrow \sqrt{3} = \frac{p-2q}{q}$ p and q are integers, $\therefore$ p – 2q is an integer $\Rightarrow \frac{p-2q}{q}$ is a rational number $\Rightarrow \sqrt{3}$ is a rational number which contradicts our assumption that $\sqrt{3}$ is an irrational number. $\Rightarrow 2 + \sqrt{3}$ is an irrational number	 ½ ½ ½ ½
22(a).	**If $4\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}$, then find the value of p.**	
Sol.	$4\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}$ $\Rightarrow 4(1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + p = \frac{3}{4}$ $\Rightarrow 4 - 4 + \frac{3}{4} + p = \frac{3}{4}$ $\Rightarrow p = 0$	 1 ½ ½
	OR	
22(b).	**If $\cos A + \cos^2 A = 1$, then find the value of $\sin^2 A + \sin^4 A$.**	
Sol.	$\cos A + \cos^2 A = 1 \Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$ $\therefore \sin^2 A + \sin^4 A = \cos A + \cos^2 A$ ($\because \sin^2 A = \cos A$) $= 1$	1 1
23.	**Show that the points (– 2, 3), (8, 3) and (6, 7) are the vertices of a right-angled triangle.**	
Sol.	Let the given points be A (– 2, 3), B (8, 3) and C (6, 7) Then, AB = 10, BC = $\sqrt{4+16} = \sqrt{20}$, AC = $\sqrt{64+16} = \sqrt{80}$	 1 ½

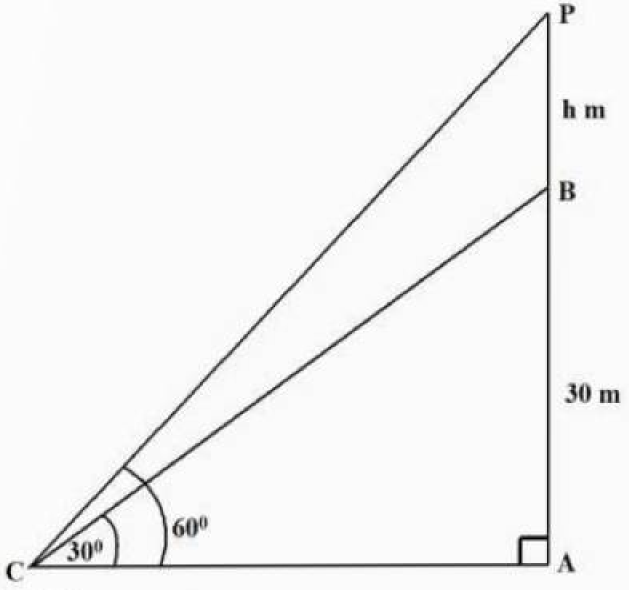
	$\therefore AB^2 = BC^2 + AC^2$ ∴ the given points are the vertices of a right angled triangle.	½
24(a).	**The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the sun.**	
Sol.	 Let AB be the tower of height 'h'. $\therefore AC = \sqrt{3}\ h$ In Δ ABC, $\tan\theta = \frac{AB}{AC} = \frac{h}{\sqrt{3}\ h}$ $\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$ $\Rightarrow \theta = 30°$	1 ½ ½
	OR	
24(b).	**The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.**	
Sol.	 Height of tower = AB In Δ ABC, $\tan 30° = \frac{AB}{30}$ $\Rightarrow AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ ∴ Height of Tower is $10\sqrt{3}$ m	1 1

25.	In the given figure, O is the centre of the circle. AB and AC are tangents drawn to the circle from point A. If $\angle BAC = 65°$, then find the measure of $\angle BOC$. 	
Sol.	$\angle BAC + \angle BOC = 180°$ $\Rightarrow \angle BOC = 180° - 65°$ $\Rightarrow \angle BOC = 115°$	1 1
	SECTION C This section comprises of Short Answer (SA) type questions of 3 marks each.	
26(a).	**Find by prime factorisation the LCM of the numbers 18180 and 7575. Also, find the HCF of the two numbers.**	
Sol.	$18180 = 2^2 \times 3^2 \times 5 \times 101$ $7575 = 3 \times 5^2 \times 101$ $LCM = 2^2 \times 3^2 \times 5^2 \times 101 = 90900$ $HCF = 3 \times 5 \times 101 = 1515$	½ ½ 1 1
	OR	
26(b).	**Three bells ring at intervals of 6, 12 and 18 minutes. If all the three bells rang at 6 a.m., when will they ring together again ?**	
Sol.	LCM of 6, 12, 18 = 36 So, all the three bells ring together after 36 minutes at 6 : 36 AM	2 1
27.	**Prove that :** $\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1}{\tan\theta + \cot\theta}$.	
Sol.	$LHS = \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)$	

	$= \left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$	½
	$= \frac{\sin^2\theta}{\cos\theta} \times \frac{\cos^2\theta}{\sin\theta}$	1
	$= \sin\theta\cos\theta$	
	$\text{RHS} = \frac{1}{\tan\theta + \cot\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$	½
	$= \frac{\cos\theta\sin\theta}{\sin^2\theta + \cos^2\theta}$	1
	$= \sin\theta\cos\theta$	
	$\therefore$ LHS = RHS	
28.	**If Q(0, 1) is equidistant from P(5, – 3) and R(x, 6), find the values of x.**	
Sol.	$PQ = QR \Rightarrow PQ^2 = QR^2$	
	$(5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$	1
	$\Rightarrow 25 + 16 = x^2 + 25$	1
	$\Rightarrow x^2 = 16$	
	$\Rightarrow x = 4, x = -4$	½ + ½
29.	**A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the total area cleaned at each sweep of the two blades.**	
Sol.	Area cleaned by 1 blade $= \frac{22}{7} \times 21 \times 21 \times \frac{120^\circ}{360^\circ}$	1½
	$= 462$	1
	$\Rightarrow$ Total area cleaned $= 2 \times 462 = 924$	½
	$\therefore$ Total area cleaned is 924 cm^2	
30 (a).	**If the system of linear equations** **$2x + 3y = 7$ and $2ax + (a + b)y = 28$** **have infinite number of solutions, then find the values of 'a' and 'b'.**	
Sol.	system has infinite number of solutions	
	$\therefore \frac{2}{2a} = \frac{3}{a+b} = \frac{7}{28}$	1
	$\Rightarrow \frac{1}{a} = \frac{1}{4} \Rightarrow a = 4$	1
	and $a + b = 12 \Rightarrow b = 8$	1
	OR	

30(b).	If $217x + 131y = 913$ and $131x + 217y = 827$, then solve the equations for the values of x and y.	
Sol.	$\left.\begin{array}{l}217x + 131y = 913\\131x + 217y = 827\end{array}\right]$ Adding $348(x + y) = 1740$ $x + y = 5$ Subtracting, $86(x - y) = 86$ $x - y = 1$ $\Rightarrow x = 3, y = 2$	 1 1 ½ + ½
31.	In the given figure, O is the centre of the circle and QPR is a tangent to it at P. Prove that $\angle QAP + \angle APR = 90°$.  	
Sol.	$OA = OP$ $\therefore$ In ΔOAP, $\angle OPA = \angle OAP$ ... (i) $\Rightarrow \angle OPA + \angle APR = 90°$ $\Rightarrow \angle OAP + \angle APR = 90°$ Using (i) $\Rightarrow \angle QAP + \angle APR = 90°$	 1 1 ½ ½
	SECTION D This section comprises of Long Answer (LA) type questions of 5 marks each.	
32.	How many terms of the arithmetic progression 45, 39, 33, ... must be taken so that their sum is 180 ? Explain the double answer.	
Sol.	45, 39, 33, ... $a = 45, d = -6$ $S_n = 180$ $180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)]$ $\Rightarrow 180 = \frac{n}{2}[90 - 6n + 6]$	 ½ 1

	$\Rightarrow 360 = 96n - 6n^2$ $\Rightarrow 6n^2 - 96n + 360 = 0$ $\Rightarrow n^2 - 16n + 60 = 0 \Rightarrow (n-10)(n-6) = 0$ $n - 10 = 0, n - 6 = 0 \Rightarrow n = 10, 6$ We get two values of 'n' as sum of 7^{th} term to 10^{th} term is zero as some terms are negative and some are positive.	 1 1 1 ½
33(a).	**As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 60°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.** **(Use $\sqrt{3} = 1 \cdot 73$)**	
Sol.	 PQ = Height of Light house = 75 m $\angle XQS = \angle QSP = 30°$ $\angle XQR = \angle QRP = 60°$ R and S are position of ships. In Δ PQR, $\frac{75}{PR} = \tan 60° = \sqrt{3} \Rightarrow PR = \frac{75}{\sqrt{3}} = 25\sqrt{3}$ In Δ PQS, $\frac{75}{PS} = \tan 30°$ $\Rightarrow PS = 75\sqrt{3}$	1 for correct figure 1½ 1

	∴ Distance between the ships, RS = PS – PR $= 75\sqrt{3} - 25\sqrt{3} = 50\sqrt{3}$ $= 50 \times 1{\cdot}73 = 86{\cdot}5$ ∴ Distance between the ships is 86.5 m	1 ½
	OR	
33(b).	**From a point on the ground, the angle of elevation of the bottom and top of a transmission tower fixed at the top of 30 m high building are 30° and 60°, respectively. Find the height of the transmission tower. (Use $\sqrt{3} = 1{\cdot}73$)**	
Sol.	 Height of building AB = 30 m BP = transmission tower = h(say) ∠ ACB = 30°, ∠ ACP = 60° In Δ ABC, $\tan 30° = \frac{AB}{AC}$ $\Rightarrow \frac{1}{\sqrt{3}} = \frac{30}{AC} \Rightarrow AC = 30\sqrt{3}$ In Δ APC, $\tan 60° = \frac{AP}{AC}$	1 for correct figure 1½

	$\sqrt{3} = \frac{30 + h}{30\sqrt{3}} \Rightarrow 30\sqrt{3} \times \sqrt{3} = 30 + h$ $\Rightarrow h = 30\,(3 - 1)$ $\Rightarrow h = 60$ $\therefore$ Height of transmission tower = 60 m	1½ 1
34.	**A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mean and median of the following data.**	

Number of cars	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency (periods)	7	14	13	12	20	11	15	8

Sol.

Number of cars	x_i	f_i	$x_i f_i$	c.f.
0 – 10	5	7	35	7
10 – 20	15	14	210	21
20 – 30	25	13	325	34
30 – 40	35	12	420	46
40 – 50	45	20	900	66
50 – 60	55	11	605	77
60 – 70	65	15	975	92
70 – 80	75	8	600	100
Total		100	4070	

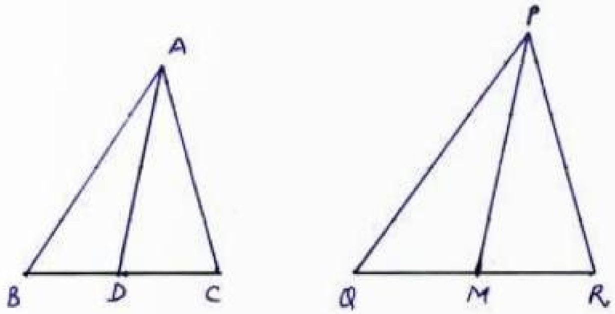
Correct table — 2

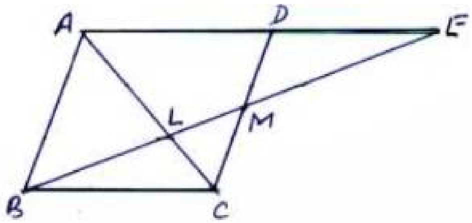
Mean $= \frac{\sum x_i f_i}{\sum f_i} = \frac{4070}{100} = 40\cdot 7$ — 1

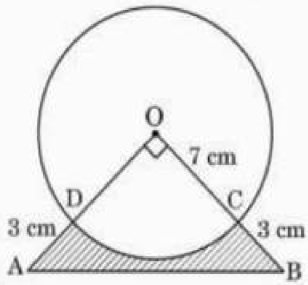
Median class : 40 – 50 — ½

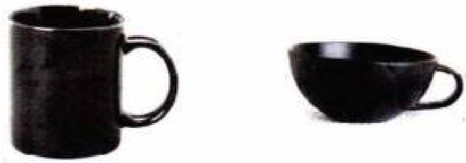
Median $= 40 + \frac{50 - 46}{20} \times 10 = 42$ — 1½

35(a). **Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR. Show that Δ ABC ~ Δ PQR.**

Sol.	 In Δ ABC and Δ PQR $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ $\frac{AB}{PQ} = \frac{2\,BD}{2\,QM} = \frac{AD}{PM}$ (∵ D is midpoint of BC and M is midpoint of QR) $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \Rightarrow \Delta ABD \sim \Delta PQM$ $\Rightarrow \angle B = \angle Q$ — (i) Now, In Δ ABC and Δ PQR $\frac{AB}{PQ} = \frac{BC}{QR}$ (given) $\angle B = \angle Q$ from (i) $\therefore\ \Delta ABC \sim \Delta PQR$	1 for correct figure 1 1 ½ ½ 1
	OR	
35(b).	**Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD (produced) in E. Prove that EL = 2BL.**	

Sol.		1 for correct figure
		
	In Δ BMC and Δ EMD	
	MC = MD	
	∠ CMB = ∠ EMD	
	∠ MBC = ∠ MED	
	∴ Δ BMC ≅ Δ EMD	1
	⇒ BC = DE	
	But AD = BC	
	∴ AD = DE	
	⇒ AE = 2 BC	1
	Δ AEL ~ Δ CBL	½
	∴ $\frac{EL}{BL} = \frac{AE}{BC}$	
	⇒ $\frac{EL}{BL} = \frac{2BC}{BC}$	½
	⇒ $\frac{EL}{BL} = 2$	
	⇒ EL = 2 BL	1

	SECTION E This section comprises of 3 case-study based questions of 4 marks each.	
36.	**Case Study – 1** In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the figure and its base ABCD is shown from the front side. The rate of silver plating is ₹ 20 per cm^2.  Based on the above, answer the following questions : (i) What is the area of the quadrant ODCO ? (ii) Find the area of Δ AOB. (iii) (a) What is the total cost of silver plating the shaded part ABCD ? **OR** (iii) (b) What is the length of arc CD ?	
Sol.	(i)Area of sector ODCO $= \frac{22}{7} \times 7 \times 7 \times \frac{90}{360} = \frac{77}{2}$ or 38·5 ∴ Area of sector ODCO is $\frac{77}{2}$ or 38·5 cm^2 (ii) ar (Δ AOB) $= \frac{1}{2} \times 10 \times 10 = 50$ ∴ ar (Δ AOB) is 50 cm^2 (iii) (a) Required cost = (50 – 38·5) × 20 = 230 ∴ required cost is ₹ 230. OR (iii) (b) Length of arc CD $= \frac{90}{360} \times 2 \times \frac{22}{7} \times 7$ = 11 ∴ Length of arc CD is 11 cm.	½ + ½ 1 1 1 1 1

37.	**Case Study – 2** In a coffee shop, coffee is served in two types of cups. One is cylindrical i shape with diameter 7 cm and height 14 cm and the other : hemispherical with diameter 21 cm.  Based on the above, answer the following questions : (i) Find the area of the base of the cylindrical cup. (ii) (a) What is the capacity of the hemispherical cup ? **OR** (ii) (b) Find the capacity of the cylindrical cup. (iii) What is the curved surface area of the cylindrical cup ?	
Sol.	(i) Area of base of the cylindrical cup $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2}$ or 38.5 $\therefore$ Area of base of the cylindrical cup is $\frac{77}{2}$ or 38.5 cm^2 (ii) (a) Capacity of hemispherical cup $= \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$ $= \frac{4851}{2}$ or 2425.5 $\therefore$ Capacity of hemispherical cup is $\frac{4851}{2}$ cm^3 or 2425.5 cm^3 OR (ii) (b) Capacity of cylindrical cup $= \frac{22}{7} \times (7)^2 \times 14$ $= 539$ $\therefore$ Capacity of cylindrical cup is 539 cm^3 (iii) External Curved surface area of cylindrical cup $= 2 \times \frac{22}{7} \times \frac{7}{2} \times 14 = 308$ $\therefore$ External Curved surface area of cylindrical cup is 308 cm^2	1 1 1 1 1 1

38.

Case Study – 3

Computer-based learning (CBL) refers to any teaching methodology that makes use of computers for information transmission. At an elementary school level, computer applications can be used to display multimedia lesson plans. A survey was done on 1000 elementary and secondary schools of Assam and they were classified by the number of computers they had.

Number of Computers	1 – 10	11 – 20	21 – 50	51 – 100	101 and more
Number of Schools	250	200	290	180	80

One school is chosen at random. Then :

(i) Find the probability that the school chosen at random has more than 100 computers.

(ii) (a) Find the probability that the school chosen at random has 50 or fewer computers.

OR

(ii) (b) Find the probability that the school chosen at random has no more than 20 computers.

(iii) Find the probability that the school chosen at random has 10 or less than 10 computers.

Sol.

(i) P (more than 100 computers) = $\frac{80}{1000}$ or 0·08 — 1

(ii)(a) 50 or fewer computers = 250 + 200 + 290 = 740 — 1

Required probability = $\frac{740}{1000}$ or 0·74 — 1

OR

(ii)(b) No more than 20 computers = 250 + 200 = 450 — 1

Required probability = $\frac{450}{1000}$ or 0·45 — 1

(iii) P (10 or less than 10 computer) = $\frac{250}{1000}$ or 0·25 — 1