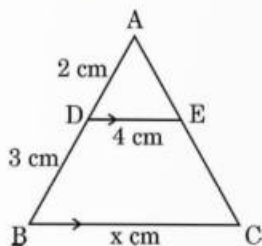


4. In the given figure, $DE \parallel BC$. The value of x is :



- (a) 6
(b) 12.5
(c) 8
(d) 10

Sol. (d) 10

1

5. A quadratic equation whose roots are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is :

- (a) $x^2 - 4x + 1 = 0$
(b) $x^2 + 4x + 1 = 0$
(c) $4x^2 - 3 = 0$
(d) $x^2 - 1 = 0$

Sol. (a) $x^2 - 4x + 1 = 0$

1

6. If $\tan \theta = \frac{5}{12}$, then the value of $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$ is :

- (a) $-\frac{17}{7}$
(b) $\frac{17}{7}$
(c) $\frac{17}{13}$
(d) $-\frac{7}{13}$

Sol. (a) $-\frac{17}{7}$

1

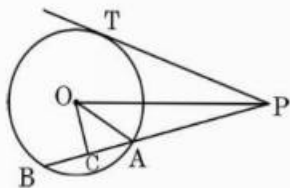
7. The distance between the points $P\left(-\frac{11}{3}, 5\right)$ and $Q\left(-\frac{2}{3}, 5\right)$ is :

- (a) 6 units
(b) 4 units
(c) 2 units
(d) 3 units

18.	<p>If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data :</p> <p>(a) decreases by 2 (b) remains unchanged (c) decreases by $2n$ (d) decreases by 1</p>	
Sol.	(a) decreases by 2	1
	<p><i>Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.</i></p> <p>(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.</p>	
19.	<p><i>Assertion (A) :</i> If the points $A(4, 3)$ and $B(x, 5)$ lie on a circle with centre $O(2, 3)$, then the value of x is 2.</p> <p><i>Reason (R) :</i> Centre of a circle is the mid-point of each chord of the circle.</p>	
Sol.	(c) Assertion (A) is true, but Reason (R) is false	1
20.	<p><i>Assertion (A) :</i> The number 5^n cannot end with the digit 0, where n is a natural number.</p> <p><i>Reason (R) :</i> Prime factorisation of 5 has only two factors, 1 and 5.</p>	

22.

In the given figure, PT is a tangent to the circle centered at O. OC is perpendicular to chord AB. Prove that $PA \cdot PB = PC^2 - AC^2$.



Sol.

$$\begin{aligned} PA \cdot PB &= (PC - AC)(PC + BC) \\ &= (PC - AC)(PC + AC) \quad [AC = BC] \\ &= PC^2 - AC^2 \end{aligned}$$

1

 $\frac{1}{2}$ $\frac{1}{2}$

23.

Using prime factorisation, find HCF and LCM of 96 and 120.

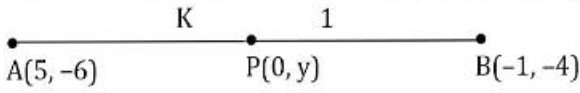
ol.

$$\begin{aligned} 96 &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \\ &= 2^5 \times 3 \\ 120 &= 2 \times 2 \times 2 \times 3 \times 5 \\ &= 2^3 \times 3 \times 5 \\ \text{HCF} &= 24 \\ \text{LCM} &= 480 \end{aligned}$$

 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

24.

Find the ratio in which y-axis divides the line segment joining the points (5, -6) and (-1, -4).

<p>Sol.</p>  <p>Let the point of division be P(0, y) which divides AB in the ratio K : 1</p> $0 = \frac{-K+5}{K+1} \Rightarrow K = 5$ <p>Ratio is 5:1</p>		<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
<p>25(a).</p>	<p>If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.</p>	
<p>Sol.</p>	$m^2 + n^2 = (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2$ $= a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta)$ $= a^2 + b^2$	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
	<p>OR</p>	
<p>25(b).</p>	<p>Prove that :</p> $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$	
<p>Sol.</p>	$\text{LHS} = \frac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \frac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}}$ $= \frac{\sec A - 1 + \sec A + 1}{\sqrt{\sec^2 A - 1}}$	<p>1</p>

$$= \frac{2 \sec A}{\tan A}$$

$$= 2 \operatorname{cosec} A = \text{RHS}$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

SECTION C

This section comprises of short answer (SA) type questions of 3 marks each.

26(a). Prove that $\sqrt{3}$ is an irrational number.

Sol. Let $\sqrt{3}$ be a rational number.

$$\therefore \sqrt{3} = \frac{p}{q}, \text{ let } p \text{ \& } q \text{ be co-primes and } q \neq 0$$

$$3q^2 = p^2 \Rightarrow p^2 \text{ is divisible by } 3 \Rightarrow p \text{ is divisible by } 3$$

$$\Rightarrow p = 3a, \text{ where 'a' is some integer} \quad \text{---- (i)}$$

$$9a^2 = 3q^2 \Rightarrow q^2 = 3a^2 \Rightarrow q^2 \text{ is divisible by } 3 \Rightarrow q \text{ is divisible by } 3$$

$$\Rightarrow q = 3b, \text{ where 'b' is some integer} \quad \text{---- (ii)}$$

(i) and (ii) leads to contradiction as 'p' and 'q' are co-primes.

$$\therefore \sqrt{3} \text{ is an irrational number.}$$

$$\frac{1}{2}$$

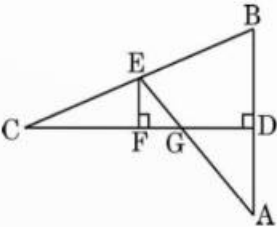
$$1$$

$$\frac{1}{2}$$

$$1$$

OR

26(b) The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change together next ?

Sol.	<p>LCM = 432</p> <p>i.e. $\frac{432}{60} = 7 \text{ min } 12 \text{ sec.}$</p> <p>$\Rightarrow$ traffic lights will change simultaneously again at 7 : 7 : 12 a.m.</p>	<p>2</p> <p>1</p>
27.	<p>If p^{th} term of an A.P. is q and q^{th} term is p, then prove that its n^{th} term is $(p + q - n)$.</p>	
Sol.	<p>$a_p = a + (p - 1)d = q$ _____ (i)</p> <p>$a_q = a + (q - 1)d = p$ _____ (ii)</p> <p>Solving (i) and (ii)</p> <p>$d = -1, a = q + p - 1$</p> <p>$a_n = (q + p - 1) + (n - 1)(-1) = q + p - n$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2} + \frac{1}{2}$</p> <p>1</p>
28(a).	<p>In the given figure, CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G. Prove that $\frac{CF}{CD} = \frac{FG}{DG}$.</p> 	

Sol.

$\Delta EFG \sim \Delta ADG$

$$\Rightarrow \frac{EF}{AD} = \frac{FG}{DG} \text{ _____ (i)}$$

$\Delta EFC \sim \Delta BDC$

$$\Rightarrow \frac{EF}{BD} = \frac{CF}{CD}$$

$$\Rightarrow \frac{EF}{AD} = \frac{CF}{CD} \quad \{\text{BD} = \text{AD}\} \text{ _____ (ii)}$$

Using (i) and (ii)

$$\frac{FG}{DG} = \frac{CF}{CD}$$

1

1

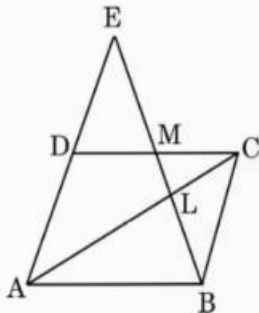
$\frac{1}{2}$

$\frac{1}{2}$

OR

28(b).

In the given figure, ABCD is a parallelogram. BE bisects CD at M and intersects AC at L. Prove that $EL = 2BL$.



<p>Sol.</p>	<p>$\Delta ALE \sim \Delta CLB$</p> <p>$\Rightarrow \frac{AL}{CL} = \frac{EL}{BL}$ _____ (i)</p> <p>Also $\Delta CLM \sim \Delta ALB$</p> <p>$\Rightarrow \frac{AL}{CL} = \frac{AB}{CM}$</p> <p>$\Rightarrow \frac{AL}{CL} = \frac{CD}{CM}$ {AB = CD} _____ (ii)</p> <p>Using (i) and (ii)</p> <p>$\frac{EL}{BL} = \frac{2CM}{CM}$</p> <p>$\Rightarrow EL = 2BL$</p>	<p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
<p>29.</p>	<p>Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they will meet in 2 hours. Had they walked in the same direction with same speeds as before, they would have met in 8 hours. Find their walking speeds.</p>	
<p>Sol.</p>	<p>Let walking speeds be x km/hr. and y km/hr. ($x > y$)</p> <p>ATQ, $2x + 2y = 16$</p> <p>and $8x - 8y = 16$</p> <p>Solving to get $x = 5$, $y = 3$</p> <p>Speeds are 5 km/hr. 3 km/hr.</p>	<p>1</p> <p>1</p> <p>$\frac{1}{2} + \frac{1}{2}$</p>

30. Prove that :

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

Sol.

$$\begin{aligned} \text{LHS} &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)} \\ &= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta (\sin \theta - \cos \theta)} \\ &= \frac{\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} + 1 \\ &= 1 + \operatorname{cosec} \theta \sec \theta = \text{RHS} \end{aligned}$$

1

1

$\frac{1}{2}$

$\frac{1}{2}$

31. Find the mean of the following frequency distribution :

Classes	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
Frequency	14	22	16	6	5	3	4

Sol.

C.I.	x	f	$u = \frac{x - 42.5}{5}$	fu
25 – 30	27.5	14	-3	-42
30 – 35	32.5	22	-2	-44
35 – 40	37.5	16	-1	-16
40 – 45	42.5	6	0	0
45 – 50	47.5	5	1	5
50 – 55	52.5	3	2	6
55 – 60	57.5	4	3	12
		70		-79

For
correct
table

2 Marks

$$\text{Mean} = 42 \cdot 5 - \frac{79}{70} \times 5 = 36 \cdot 86$$

1

SECTION D

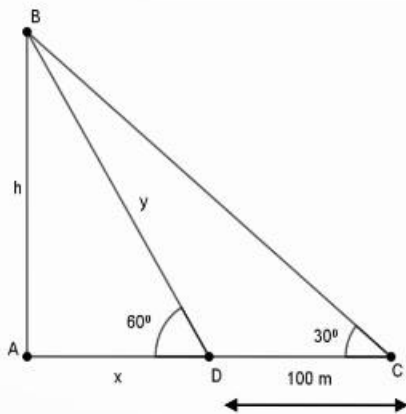
This section comprises of long answer (LA) type questions of 5 marks each.

32.

One observer estimates the angle of elevation to the basket of a hot air balloon to be 60° , while another observer 100 m away estimates the angle of elevation to be 30° . Find :

- (a) The height of the basket from the ground.
- (b) The distance of the basket from the first observer's eye.
- (c) The horizontal distance of the second observer from the basket.

Sol.



Let B is the basket of hot air balloon

$$\tan 60^\circ = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \text{ _____ (i)}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{x + 100} \Rightarrow x = h\sqrt{3} - 100 \text{ _____ (ii)}$$

Correct Figure

1 Mark

1

1

using (i) and (ii)

$$(a) h = (h\sqrt{3} - 100)\sqrt{3} = 3h - 100\sqrt{3} \Rightarrow h = 50\sqrt{3} \text{ m}$$

$$(b) \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{h}{y} = \frac{50\sqrt{3}}{y} \Rightarrow y = 100 \text{ m}$$

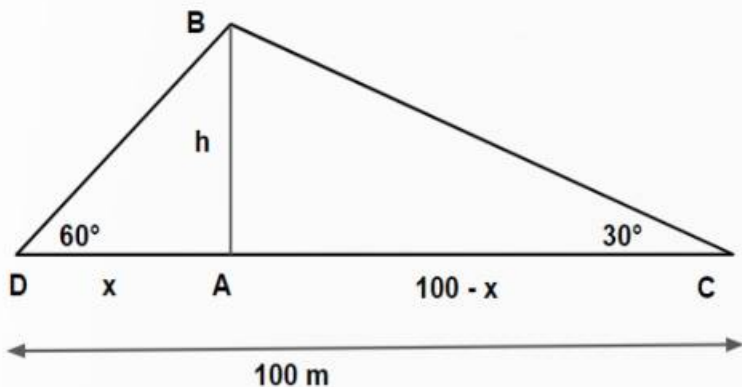
$$(c) x = \frac{h}{\sqrt{3}} = 50 \text{ m} \Rightarrow x + 100 = 150 \text{ m}$$

$\frac{1}{2}$

1

$\frac{1}{2}$

ANOTHER SOLUTION AS PER BELOW FIGURE IS ALSO POSSIBLE



Correct
Figure

1 Mark

Let B is the basket of hot air balloon. D and C be the positions of the first and second observer's respectively.

$$\tan 60^\circ = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \text{ _____ (i)}$$

1

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{100-x} \Rightarrow \sqrt{3}h = 100 - x \text{ _____ (ii)}$$

1

(a) using (i) and (ii)

$$h = \sqrt{3} (100 - \sqrt{3}h) \Rightarrow h = 25\sqrt{3} \text{ m}$$

$$(b) \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{h}{BD} \Rightarrow BD = 50 \text{ m}$$

$$(c) x = \frac{h}{\sqrt{3}} = 25 \text{ m}$$

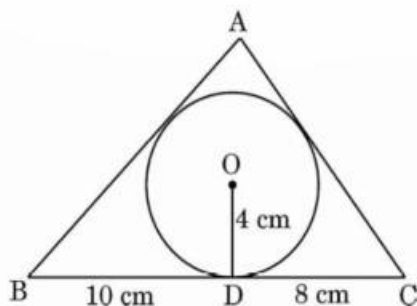
$$\Rightarrow AC = 100 - x = 75 \text{ m}$$

 $\frac{1}{2}$

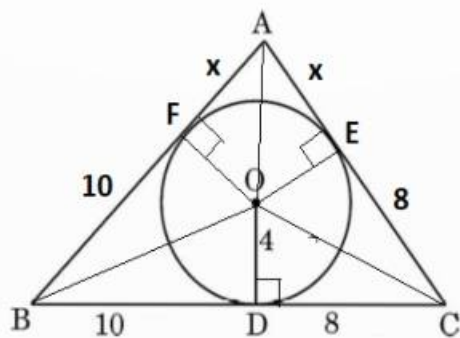
1

 $\frac{1}{2}$

- 33(a). A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 10 cm and 8 cm respectively. Find the lengths of the sides AB and AC, if it is given that area $\Delta ABC = 90 \text{ cm}^2$.



Sol.



Join OA, OB, OC and draw OE \perp AC and OF \perp AB.

BF = 10 cm, CE = 8 cm, Let AF = AE = x

ar Δ ABC = ar Δ BOC + ar Δ COA + ar Δ AOB

$$90 = \frac{1}{2} \cdot 4 (BC + CA + AB)$$

$$90 = 2(18 + 8 + x + 10 + x)$$

$$90 = 4(18 + x)$$

$$x = 4.5$$

AB = 14.5 cm and AC = 12.5 cm

$1\frac{1}{2}$

$1\frac{1}{2}$

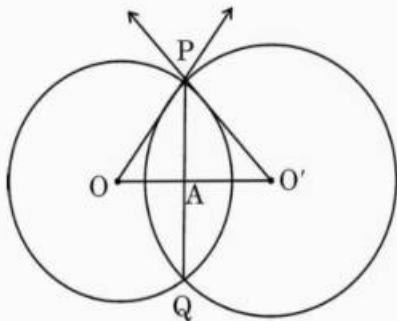
1

1

OR

33(b).

Two circles with centres O and O' of radii 6 cm and 8 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.



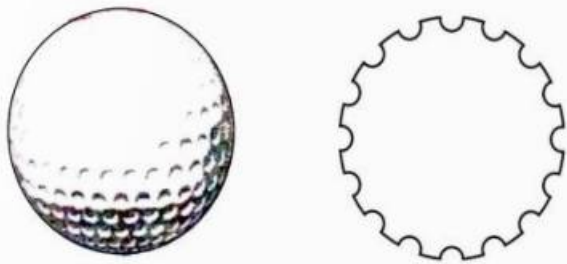
<p>Sol.</p>	<p>$OO' = \sqrt{6^2 + 8^2} = 10 \text{ cm}$ $\{OP \perp O'P\}$</p> <p>Let $OA = x$, $O'A = 10 - x$</p> <p>$AP^2 = 36 - x^2$</p> <p>Also $AP^2 = 64 - (10 - x)^2$</p> <p>Therefore $36 - x^2 = 64 - (10 - x)^2$</p> <p>$\Rightarrow 36 - x^2 = 64 - 100 - x^2 + 20x$</p> <p>$\Rightarrow x = 3.6$</p> <p>In ΔPAO, $AP^2 = 36 - (3.6)^2 = 23.04$</p> <p>$\Rightarrow AP = 4.8$</p> <p>Length $PQ = 2 \times AP = 9.6 \text{ cm}$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>2</p> <p>1</p> <p>$\frac{1}{2}$</p>
<p>34(a).</p>	<p>A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed ?</p>	
<p>Sol.</p>	<p>Let first average speed of the train be x km/hr.</p> <p>$\frac{54}{x} + \frac{63}{x+6} = 3$</p> <p>$\Rightarrow 54x + 324 + 63x = 3x^2 + 18x$</p>	<p>2</p>

	$\Rightarrow 3x^2 - 99x - 324 = 0$ or $x^2 - 33x - 108 = 0$ $\Rightarrow (x - 36)(x + 3) = 0$ $\Rightarrow x = 36, -3$ (rejected) Therefore, first average speed of the train was 36 km/hr.	2 1
	OR	
34(b).	Two pipes together can fill a tank in $\frac{15}{8}$ hours. The pipe with larger diameter takes 2 hours less than the pipe with smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separately.	
Sol.	Let the time taken by smaller diameter tap be x hrs. Time taken by larger diameter tap is $(x - 2)$ hrs. Therefore $\frac{1}{x-2} + \frac{1}{x} = \frac{8}{15}$ $\Rightarrow 15(2x - 2) = 8x(x - 2)$ $\Rightarrow 8x^2 - 46x + 30 = 0$ $\Rightarrow 4x^2 - 23x + 15 = 0$ $\Rightarrow (4x - 3)(x - 5) = 0$ $\Rightarrow x = \frac{3}{4}, x = 5$	2 1 1

	$x \neq \frac{3}{4}$ as $x - 2 < 0$ Smaller diameter tap fills in 5 hrs. Larger diameter tap fills in 3 hrs.	1
35.	A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. (Use $\pi = 3.14$)	
Sol.	<p>Area of that part of the field in which the horse can graze by means of a 5 m long rope $= \frac{1}{4} \times 3.14 \times (5)^2$</p> $= 19.625 \text{ m}^2$ <p>Area of that part of the field in which the horse can graze by means of a 10 m long rope $= \frac{1}{4} \times 3.14 \times (10)^2$</p> $= 78.5 \text{ m}^2$ <p>Increase in grazing area $= 78.5 \text{ m}^2 - 19.625 \text{ m}^2 = 58.875 \text{ m}^2$</p>	1 1 1 1 1
	SECTION E This section comprises of 3 case-study based questions of 4 marks each.	

36.

A golf ball is spherical with about 300 – 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.



Based on the above, answer the following questions :

- (i) Find the surface area of one such dimple.
- (ii) Find the volume of the material dug out to make one dimple.
- (iii) (a) Find the total surface area exposed to the surroundings.

OR

- (iii) (b) Find the volume of the golf ball.

Sol. (i) $SA = 2\pi r^2 = 2 \times \frac{22}{7} \times 4 = \frac{176}{7} \text{ mm}^2$ or 25.1 mm^2

1

(ii) Volume of material dug out to make one dimple $= \frac{2}{3} \times \frac{22}{7} \times 8$

$= \frac{352}{21} \text{ mm}^3$ or 16.76 mm^3

1

(iii)(a) radius of ball = 21 mm

Total surface area exposed to surroundings

$$= 4\pi(21)^2 - 315 \times \pi(2)^2 + 315 \times 2\pi(2)^2$$

1

$$= 4 \times \frac{22}{7} \times 21 \times 21 + \frac{22}{7} \times 315 \times 4$$

1

$$= 9504 \text{ mm}^2$$

OR

$$(iii) (b) \text{ Volume of the golf ball} = \frac{4}{3}\pi(21)^3 - 315 \times \frac{2}{3}\pi(2)^3$$

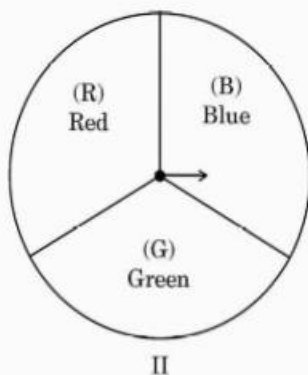
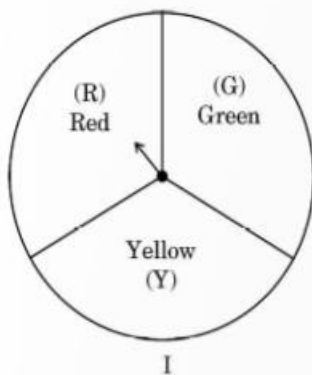
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$$= 33528 \text{ mm}^3$$

1

37.

A middle school decided to run the following spinner game as a fund-raiser on Christmas Carnival.



Making Purple : Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you 'win'. One such outcome is written as 'RB'.

Based on the above, answer the following questions :

- (i) List all possible outcomes of the game.
- (ii) Find the probability of 'Making Purple'.
- (iii) (a) For each win, a participant gets ₹ 10, but if he/she loses, he/she has to pay ₹ 5 to the school.
If 99 participants played, calculate how much fund could the school have collected.

OR

- (iii) (b) If the same amount of ₹ 5 has been decided for winning or losing the game, then how much fund had been collected by school ? (Number of participants = 99)

Sol. (i) All possible outcomes: RR, RG, RB, GR, GB, GG, YR, YB, YG

(ii) Number of favourable outcome (RB) = 1

1

$$P(\text{Making purple}) = \frac{1}{9}$$

1

$$\text{(iii)(a) As } P(\text{winning}) = \frac{1}{9}$$

$$\text{Therefore, number of people must win} = \frac{1}{9} \times 99 = 11$$

 $\frac{1}{2}$

\therefore Game lost by 88 persons.

 $\frac{1}{2}$

$$\text{Funds collected} = 5 \times 88 - 10 \times 11 = \text{₹ } 330$$

1

OR

$$\text{(iii)(b) Number of participants} = 99$$

$$P(\text{winning the game}) = \frac{1}{9}$$

$$\text{Number of persons won} = 11$$

 $\frac{1}{2}$

$$\text{Number of persons lost} = 88$$

 $\frac{1}{2}$

$$\text{Funds collected} = 88 \times 5 - 11 \times 5 = \text{₹ } 385$$

1

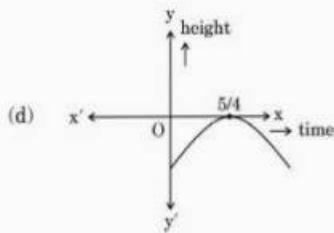
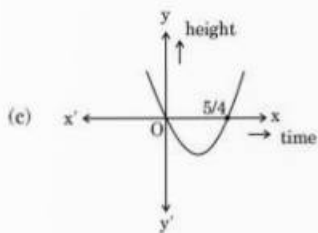
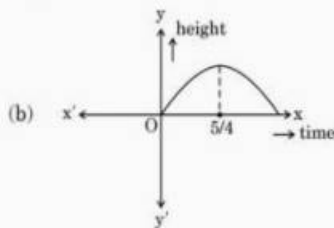
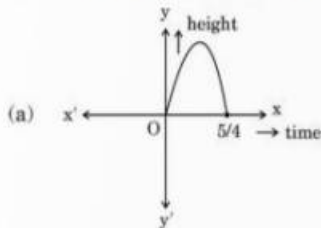
38.

In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.



Based on the above, answer the following questions :

- (i) Find zeroes of polynomial $p(t) = 20t - 16t^2$.
 (ii) Which of the following types of graph represents $p(t)$?



- (iii) (a) What would be the value of h at $t = \frac{3}{2}$? Interpret the result.

OR

- (iii) (b) How much distance has the dolphin covered before hitting the water level again ?

<p>Sol.</p>	<p>(i) $-16t^2 + 20t = 0 \Rightarrow 4t(-4t + 5) = 0$</p> <p>$t = 0, t = \frac{5}{4}$</p> <p>(ii) (a)</p> <p>(iii)(a) At $t = \frac{3}{2}$, $h = -16 \times \frac{9}{4} + 20 \times \frac{3}{2} = -36 + 30 = -6$</p> <p>It means after $\frac{3}{2}$ seconds, dolphin has reached 6 cm below water level.</p> <p>OR</p> <p>(iii)(b) Speed of dolphin = 20 cm per second.</p> <p>In one second, distance covered = 20 cm</p> <p>In $\frac{5}{4}$ seconds, distance covered = $20 \times \frac{5}{4} = 25$ cm</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p>
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